ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Goal:

- to solve absolute value equations
- to solve absolute value inequalities

The absolute value of a number is a piecewise function:

$$|a| = \begin{cases} a \ \ if\ a \geq 0 \\ -a\, if\ a < 0 \end{cases}$$

Find:

a) $|4|$

$= 4$

b) $|-4|$

$= 4$

c) $|-5.7|$

$= 5.7$

d) $|8-10|$ $\neq |8|+|-10|$

$= |-2|$

$= 2$

similar properties to radicals

e) $|-6\times 3|$ $= |-6||3|$

$= |-18|$ $= 18$

$= 18$

Absolute value equations can have zero, one or two solutions.

No solution: absolute vale = neg

One solution: absolute value = zero

Two solutions: absolute value = positive

Solve:

$a)|x|=6$

$x=6$ or $-x=6$

$y=-6$

$x=\{-6,6\}$

$b)|x|=-6$

$\emptyset$ can't equal negative

$c)|x-2|=3$

$x-2=3$ or $-(x-2)=3$

$x=5$ $x-2=-3$

$x=-1$

$x=\{-1,5\}$

$d)|x+3|-1=7$

$|x+3|=8$

$x+3=8$ or $-(x+3)=8$

$x=5$ $x+3=-8$

$x=-11$

$x=\{-11,5\}$

$e)-2|x-5|+1=-13$

$-2|x-5|=-14$

$|x-5|=7$

$x-5=7$ $-(x-5)=7$

$x=12$ $x-5=-7$

$x=-2$

$x=\{-2,12\}$

$f)\frac{1}{2}|4-2x|-5=9$

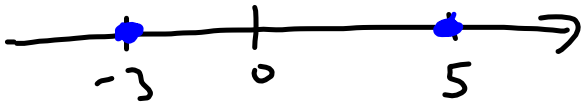
g) $2|x-1|-3>5$

For inequalities, solve equation and test point(s)

$2|x-1|-3=5$

$2|x-1|=8$

$|x-1|=4$



$x-1=4$ $\quad -(x-1)=4$

$x=5$ $\quad x-1=-4$

$x=-3$

$x=\{-3,5\}$

The solution to the inequality is:

~~$]-3,5[$~~ or $]-\infty,-3[\cup]5,\infty[$

Test a point $x=0$

$2|0-1|-3>5$

$2|-1|-3>5$

$2-3>5$

$-1>5$ X

h) $-|2x+3|-1>4$

$-|2x+3|-1=4$

$-|2x+3|=5$

$|2x+3|=-5$

$x = \{\}$

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test $x=0$

$-|2(0)+3|-1>4$

$-|3|-1>4$

$-3-1>4$

$-4>4$ X

i) $-|2x+3|-1<4$

$x \in \mathbb{R}$

or $x :]-\infty, \infty[$

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j) $-\frac{1}{4}|x+6|+2<0$

$-\frac{1}{4}|x+6|+2=0$

$-\frac{1}{4}|x+6|=-2$

$|x+6|=8$

$x+6=8$ $\quad -(x+6)=8$

$x=2$ $\quad x+6=-8$

$x=-14$

Critical points at $x=-14$ and $x=2$

test $x=0$

$-\frac{1}{4}|0+6|+2<0$

$-\frac{1}{4}|6|+2<0$

$-\frac{1}{4}(6)+2<0$

$-\frac{3}{2}+2<0$

$\frac{1}{2}<0$ X

Function is greater than zero between $x=-14$ and $x=2$, therefore

$X:]-\infty,-14[\cup]2,\infty[$