**2- Correction key**

1

A

2

D

8

C

9

C

11

The maximum possible revenue that this parking lot can earn in one hour is $**140**.

12

The coordinates of vertex R of this polygon of constraints are R**(9, 18)**.

13

|  |  |  |
| --- | --- | --- |
| There are | 2 🞏  3 🞏  4 ◼ | different combinations of child and adult passengers that will enable the guide to maximize his revenue for one trip. |
|

14

The coordinates of vertex P of this polygon of constraints are P(**2, 5**).

15

Work : (example)

a) The linear inequalities that express the constraints

(*x* ≥ 0 and *y* ≥ 0)

*x* + *y* ≥ 15

*x* + *y* ≤ 30

*y* > *x*

or equivalent linear inequalities

b) The polygon of constraints



Result : Yes \_\_\_\_\_ No X

17

Work : (example)

Inequalities representing the constraints

Given *x* : number of T-shirts

*y* : number of sweatshirts

*x* ≥ 0 *y* ≥ 0

*x* ≥ 2*y* *x* + *y* ≥ 450

4*x* + 10*y* ≥ 3600

Polygon of constraints and co-ordinates of the vertices



Function to be optimized

P = 3*x* + 8*y*

Value of the function to be optimized for each of the vertices

P = 3(900) + 8(0) = 2700

P = 3(450) + 8(0) = 1350

P = 3(400) + 8(200) = 2800

P = 3(300) + 8(150) = 2100

Profit is maximal for 400 T-shirts and 200 sweatshirts.

Result : The maximal profit that can be made from selling shirts is $2800

20

Example of an appropriate method

**Coordinates of the vertices**

P: intersection of *y* = 4 and *x* = 1

P(1, 4)

Q: intersection of *y* = 4 + 2*x* and *x* = 1

If *x* = 1 then *y* = 4 + 2(1) = 6

Q(1, 6)

R: intersection of *y* = 4 + 2*x* and *x* + *y* = 16

*y* = 4 + 2*x* and *y* = 16 − *x*

4 + 2*x* = 16 − *x*

*x* = 4

If *x* = 4 then *y* = 4 + 2(4) = 12

R(4, 12)

S: intersection of *y* = 4 and *x* + *y* = 16

If *y* = 4 then *x* + 4 = 16 therefore *x* = 12

S(12, 4)

**Maximum income**

|  |  |  |
| --- | --- | --- |
|  | Income | |
| Vertices | Option A  13*x* + 8*y* | Option B  9*x* + 9*y* |
| P(1, 4) | 45 | 45 |
| Q(1, 6) | 61 | 63 |
| R(4, 12) | 148 | 144 |
| S(12, 4) | 188 | 144 |
|  | Maximum income | |

Answer Option A will allow Oliver to maximize his income.

21

Example of an appropriate method

Let *x*: the number of type A containers

*y*: the number of type B containers

|  |  |
| --- | --- |
| System of inequalities  *x* ≥ 0  *y* ≥ 0  3*x* + *y* ≥ *7*  3*x* + 4*y* ≥ 19  *x* + 3*y* ≥ 8  The rule to be minimized  Transportation costs = 6000*x* + 5000*y* | The polygon of constraints |

Value of the rule to be minimized

|  |  |
| --- | --- |
| Vertex | 6000*x* + 5000*y* |
| (0, 7) | 35 000 |
| (1, 4) | 26 000 |
| (5, 1) | 35 000 |
| (8, 0) | 48 000 |

Answer The company must use 1 type A container and 4 type B containers.

22

Example of an appropriate method

Each member's maximum profit

|  |  |  |
| --- | --- | --- |
| Vertex | Profit: 1.50*x* + 2.50*y* |  |
| A(10, 14)  B(10, 4)  C(20, 4) | $50  $25  $40 | ← Maximum |

Each member made a maximum profit of $50.

Number of members

Total amount raised ÷ Each member's maximum profit

$800 ÷ $50 per member

16 members

Answer: This club has **16** members

23

Example of an appropriate method

Maximum possible weekly income without the additional constraint

|  |  |  |
| --- | --- | --- |
| Vertex | Income: 10*x* + 8*y* |  |
| P(5, 35)  Q(20, 5)  R(15, 5)  S(5, 15) | $330  $240  $190  $170 | ← Maximum |

Additional constraint

|  |  |
| --- | --- |
| *y* ≤ *x*  The coordinates of the 2 new vertices are (10, 10) and (15, 15) |  |

Maximum possible weekly income with the additional constraint

|  |  |  |
| --- | --- | --- |
| Vertex | Income: 10*x* + 8*y* |  |
| (10, 10)  (15, 15)  Q(20, 5)  R(15, 5) | $180  $270  $240  $190 | ← Maximum |

Difference between the maximum possible weekly incomes

$330 − $270 = $60

Answer: This constraint decreases Vincent's maximum possible weekly income by $**60**.

Name : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Group : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**568514 - Mathematics**

**Question Booklet**

1

The vertices of a polygon of constraints are shown on the graph below.



If the function to be optimized is *Z* = 2*x* + 3*y* − 10, what is the ordered pair of values that maximizes this function?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | (5, 10) | C) | (7, 3) |
| B) | (4, 8) | D) | (9, 7) |

2

A town's recreational department is publishing a guide listing the sports and cultural activities during the summer.

The printer charges $ 0.20 to print a page with photographs and $0.15 for a page without. The guide will have between 25 and 40 pages. At least 10 of these pages will have photographs.

If x represents the number of pages with photographs and y represents the number of pages without, what is the function to be optimized?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | *Z* = 0.15*x* + 0.20*y* | C) | *Z* = 0.40*x* + 0.25*y* |
| B) | *Z* = 0.25*x* + 0.40*y* | D) | *Z* = 0.20*x* + 0.15*y* |

8

The constraints related to the number of pine chairs and the number of oak chairs that a furniture factory manufactures in a week are represented by the polygon of constraints shown below.



The production costs of these two types of chairs is expressed by the equation :

*C* = 10*x* + 18*y*

How many chairs of each type must the company manufacture in a week to minimize its production cost?

|  |  |
| --- | --- |
| A) | 100 pine chairs and 250 oak chairs |
| B) | 200 pine chairs and 150 oak chairs |
| C) | 250 pine chairs and 100 oak chairs |
| D) | 400 pine chairs and 250 oak chairs |

9

The polygon of constraints of an optimization problem is illustrated below.



The function *Z* to be optimized is given by the equation

*Z* = 1.25*x* + 2.50*y*

where x and y are integers.

How may ordered pairs maximize the function *Z*?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | 0 | C) | 4 |
| B) | 1 | D) | 5 |

11

A parking lot is used to park motorcycles and cars. The polygon of constraints associated with this situation is represented below.



*x* : number of motorcycles in the parking lot

*y* : number of cars in the parking lot

The revenue that this parking lot can earn each hour is calculated using the expression *x* + 1.5 *y*.

What is the maximum possible revenue that this parking lot can earn in one hour?

12

The system of inequalities and the polygon of constraints below are associated with an optimization situation. Each side of the polygon and its related inequality are identified by the same number.

|  |  |
| --- | --- |
| ➀ *y* ≤ 2*x*  ➁ *x* ≤ 16  ➂ *y* ≥ -2*x* + 36 |  |

What are the coordinates of vertex R of this polygon of constraints?

13

There are constraints on how many children and how many adults can board a boat to go white-water rafting. The polygon of constraints below represents the different possible combinations of children and adults the boat can hold.



*x*: number of children in the boat

*y*: number of adults in the boat

To determine the maximum possible revenue that can be earned from one trip, the guide calculated the values indicated in the table below.

|  |  |
| --- | --- |
| Vertex of the polygon of constraints | Revenue |
| O(0, 0) | $0 |
| P(0, 9) | $180 |
| Q(6, 6) | $180 |

How many different combinations of child and adult passengers enable the guide to maximize his revenue for one trip.

14

The constraints associated with an optimization situation are represented by the system of inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

|  |  |
| --- | --- |
| ➀ *y* ≤ 2*x* + 1  ➁ 2*x* + 3*y* ≤ 35  ➂ *y* ≤ -5*x* + 42  ➃ *x* + 2*y* ≥ 12 |  |

What are the coordinates of vertex P of this polygon of constraints?

15

A town decides to hire some high school and CEGEP students for the summer. The following constraints must be observed :

‑ a minimum of 15 students are needed;

‑ a maximum of 30 students can be hired;

‑ more CEGEP students than high school students must be hired.

Draw the polygon of constraints and verify if the town can hire 10 high school students and 7 CEGEP students.

*x* : number of high school students

*y* : number of CEGEP students

17

The organizers of a festival want to have at least 450 shirts made with the festival's logo on them. They are going to have both T-shirts and sweatshirts. They expect to sell at least twice as many T‑shirts as sweatshirts. The organizers have a maximum budget of $3600 to have the shirts made. It costs them $4 for a T-shirt and $10 for a sweatshirt. The selling price of a T-shirts is $7 and that of a sweatshirt is $18.

What is the maximum profit the organizers can make from selling shirts?

20

Oliver works as a cashier and wrapper at a grocery store. There are different constraints that limit the number of hours he can work per week. These constraints are represented by the inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

|  |  |
| --- | --- |
| ➊ *y* ≥ 4  ➋ *x* + *y* ≤ 16  ➌ *y* ≤ 4 + 2*x*  ➍ *x* ≥ 1 |  |

*x*: number of hours he can work per week as a cashier

*y*: number of hours he can work per week as a wrapper

Oliver's boss suggests two ways of paying him:

Option A: $13 per hour as a cashier and $8 an hour as a wrapper;

Option B: $9 per hour, whether he works as a cashier or as a wrapper.

Which option will allow Oliver to maximize his income?

21

A mining company receives an order for 7 tonnes of iron, 19 tonnes of lead and 8 tonnes of copper. This order can be delivered using two types of containers.

Each type A container holds 3 tonnes of iron, 3 tonnes of lead and 1 tonne of copper.

Each type B container holds 1 tonne of iron, 4 tonnes of lead and 3 tonnes of copper.

Its costs $6000 to transport one type A container and $5000 to transport one type B container.

How many containers of each type must the company use to minimize transportation costs?

22

The members of a sports club participated in a fundraising campaign. They sold chocolate bars and packets of coffee. They made a profit of $1.50 for each chocolate bar sold and a profit of $2.50 for every packet of coffee sold.

The polygon of constraints below represents the different constraints faced by each member.



*x*: number of chocolate bars sold by each member

*y*: number of packets of coffee sold by each member

Each club member made the maximum possible profit. This campaign enabled them to raise a total of $800.

How many members does this club have?

23

Vincent works for a company that makes storage racks for compact disks. Each week, he divides his time between assembly work and finishing work.

The polygon of constraints below represents the different constraints that Vincent faces.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Coordinates of the vertices of the polygon of constraints |  |
|  | P(5, 35)  Q(20, 5)  R(15, 5)  S(5, 15) |  |
|  |  |  |

*x*: number of hours spent on assembly work each week

*y*: number of hours spent on finishing work each week

Vincent is told that from now on he faces the following additional constraint: the number of hours spent on finishing work must be less than or equal to the number of hours spent on assembly work.

He makes $10 an hour for assembly work and $8 an hour for finishing work.

By how many dollars does this constraint decrease Vincent's maximum possible weekly income?