Conic Sections (Conics)

Goal:

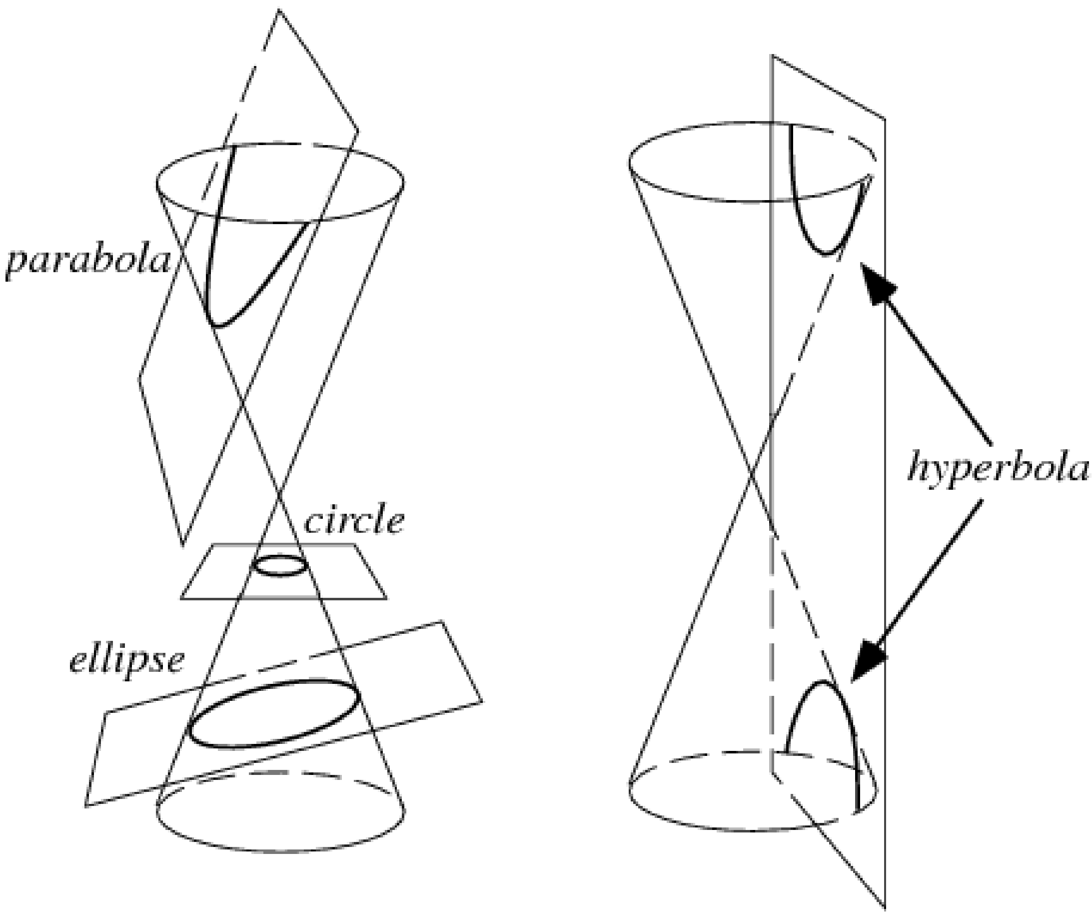
- to become familiar with the basic conics

- to understand the equation of circles and ellipses centered at the origin

The study of conics involves geometric loci (singular: locus). A geometric locus is a set of points that share a property. The result is usually a shape we can recognize.

Conics are geometric loci that are found by intersecting a cone with a plane.

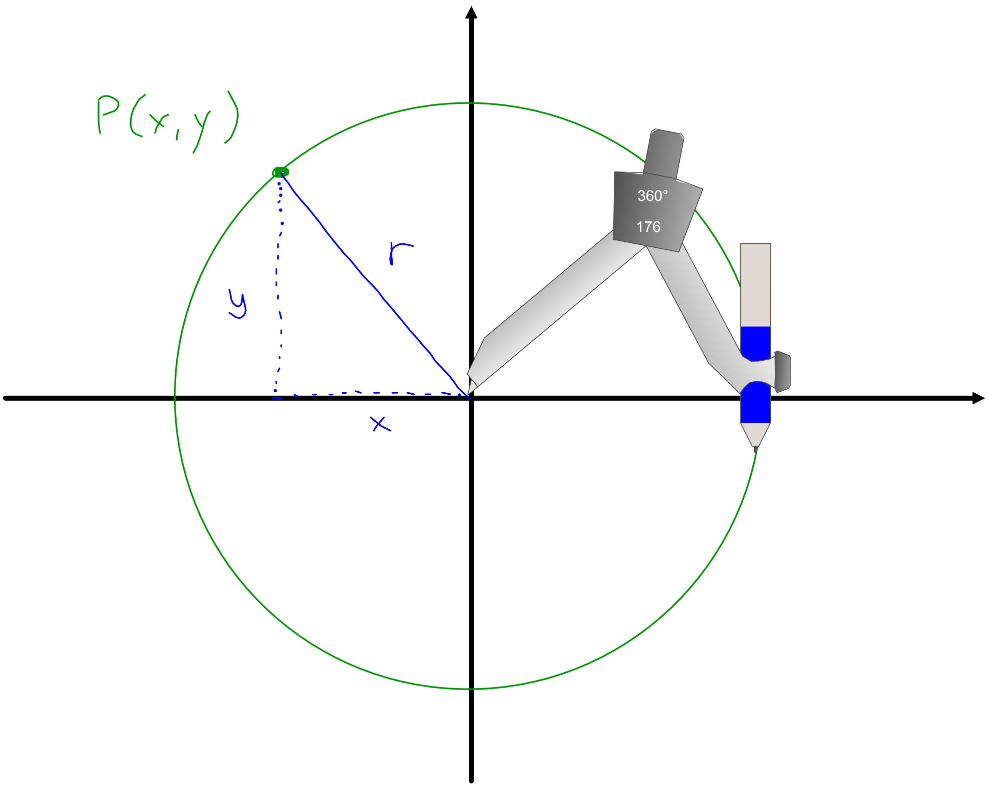
They are: circles, ellipses, parabolas and hyperbolas.




McDonald's

Since each conic section shares a property we are able to write an equation that represents it.

Circle definition: A set of points, all of which are equidistant from a fixed point (centre).



The equation for any circle centered at the origin is:

$$\boxed{x^2 + y^2 = r^2}$$

How far apart are two points on a circle, centered at the origin, with a diameter of 20 units, if they have x=7

$d = 2r = 20$

$r = 10$

$x^2 + y^2 = 10^2$

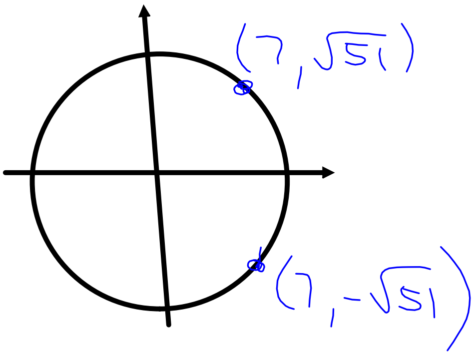
$x^2 + y^2 = 100$

$7^2 + y^2 = 100$

$49 + y^2 = 100$

$y^2 = 51$

$y = \pm\sqrt{51}$

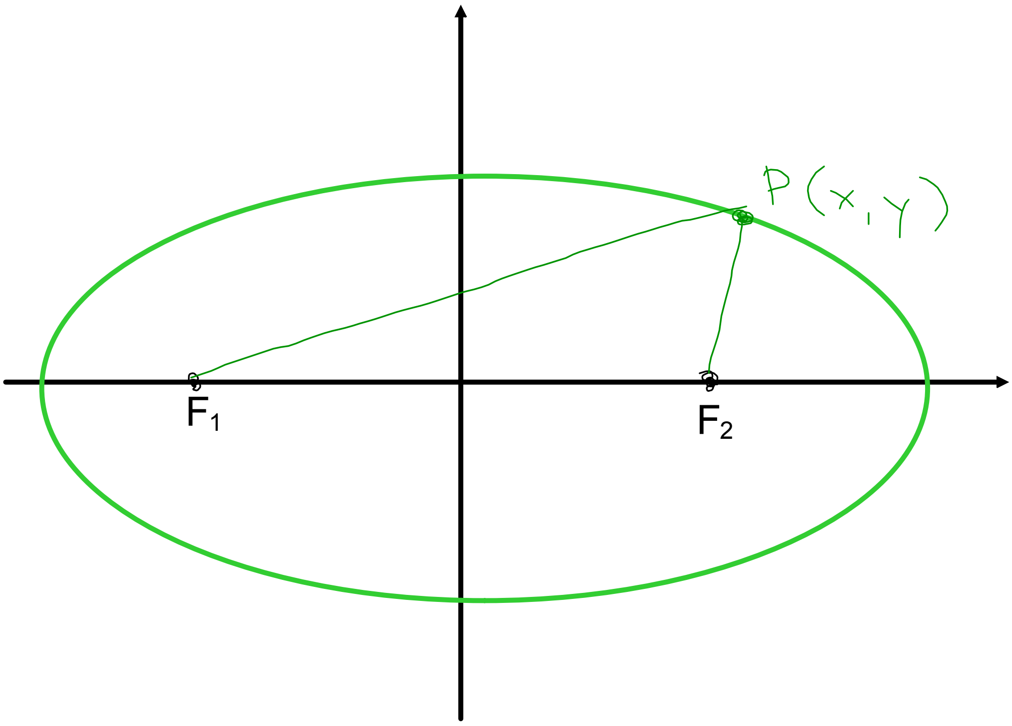


$\Delta y = y_2 - y_1 = \sqrt{51} - (-\sqrt{51})$

$= 2\sqrt{51}$

Ellipse definition: A set of points, where the sum of distances to two fixed points, foci, is constant.

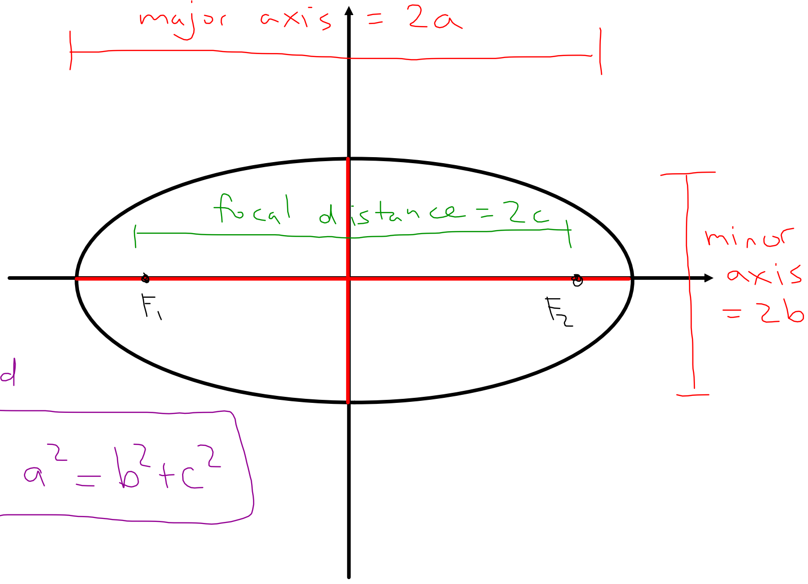
$$d(PF_1) + d(PF_2) = \text{constant}$$



The equation for any ellipse centered at the origin is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

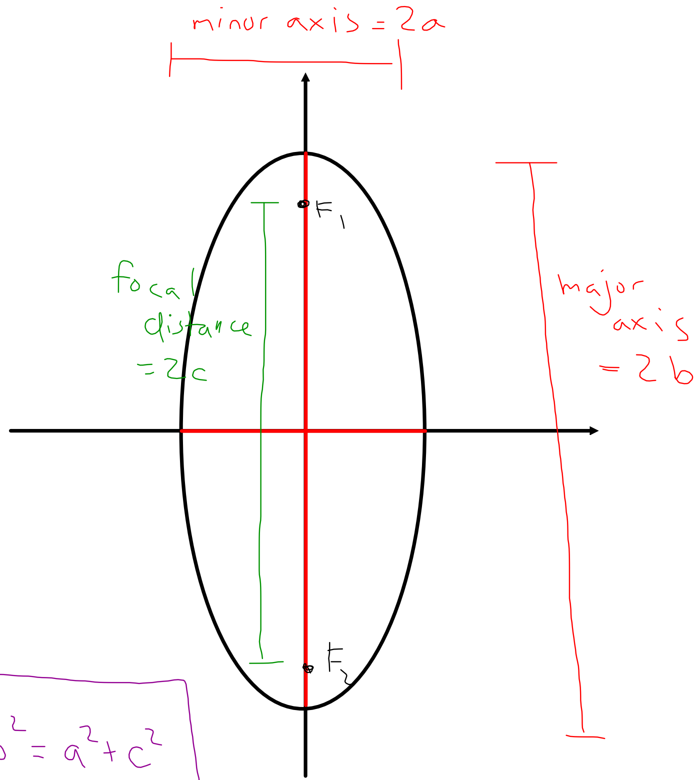
An ellipse can have a horizontal major axis:



and

$$\boxed{a^2 = b^2 + c^2}$$

Or it can have a vertical major axis:

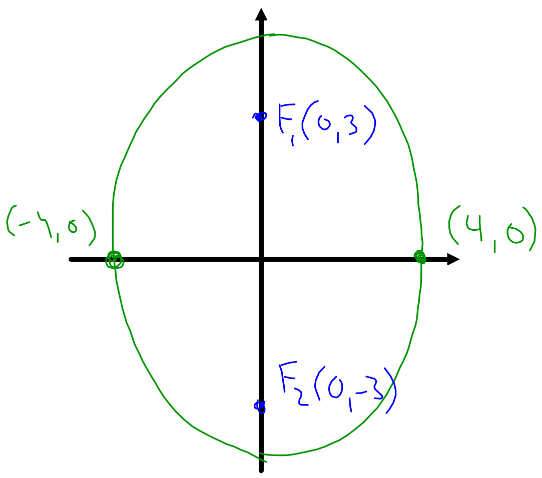


and

$$\boxed{b^2 = a^2 + c^2}$$

Write the equation of an ellipse centred at the origin:

a) with focus at (0,3) and co-vertex at (4,0)



$b^2 = a^2 + c^2$

$b^2 = 4^2 + 3^2$

$b^2 = 25$

$b = \pm 5$

$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$