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#7. a) $\sin x = \cos^2 x + 1$

$\sin x = (1 - \sin^2 x) + 1$

$\sin x = 2 - \sin^2 x$

$\sin^2 x + \sin x - 2 = 0$

$(\sin x + 2)(\sin x - 1) = 0$

$\sin x + 2 = 0$

$\sin x = -2$

$\varnothing$

$\sin x - 1 = 0$

$\sin x = 1$

$x = \sin^{-1}(1)$

$x = \frac{\pi}{2} + 2\pi n$

$x = \left\{ -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2} \right\}$

In order to solve this equation, there needs to be 1 trig ratio.

use pythag. identity

$\cos^2 x + \sin^2 x = 1$

Quadratic equation

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

7\. b) $1 = 2\sin^2 x + \cos x$

$1 = 2(1 - \cos^2 x) + \cos x$

$1 = 2 - 2\cos^2 x + \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

$x_1 = \frac{2\pi}{3} + 2\pi n$

$x_2 = \frac{4\pi}{3} + 2\pi n$

$\cos x - 1 = 0$

$\cos x = 1$

$x_3 = 0 + 2\pi n$

$x_3 = 2\pi n$

$x : \left\{ -\frac{8\pi}{3}, -2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3} \right\}$

7. c) $\tan^2 x + 3\sec x + 3 = 0$

$(\sec^2 x - 1) + 3\sec x + 3 = 0$

$\sec^2 x + 3\sec x + 2 = 0$

$(\sec x + 2)(\sec x + 1) = 0$

$\sec x + 2 = 0$ | $\sec x + 1 = 0$

$\sec x = -2$ | $\sec x = -1$

$\frac{1}{\cos x} = -2$ | $\frac{1}{\cos x} = -1$

$\cos x = -\frac{1}{2}$ | $\cos x = -1$

$x_3 = \pi + 2\pi n$

$x_1 = \frac{2\pi}{3} + 2\pi n$

$x_2 = \frac{4\pi}{3} + 2\pi n$

$x: \left\{ -3\pi, -\frac{8\pi}{3}, -\frac{4\pi}{3}, -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{8\pi}{3}, 3\pi \right\}$

The really fun part is that these identities can now be used to prove more complex identities.

You may only work with one side of the equation at a time.

- Use identities
- Use algebraic manipulations (factoring, simplifying, adding rational expressions...

Ex: Prove the following:

a) $\cos\theta\tan\theta = \sin\theta$

Left-side $= \cos\theta\tan\theta$

$= \cancel{\cos}\theta\left(\frac{\sin\theta}{\cancel{\cos}\theta}\right)$

$= \sin\theta$

$=$ Right-side

QED

b) $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \csc^2 x \sec^2 x$

$LS = \dfrac{1}{\sin^2 x}\cdot\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{1}{\cos^2 x}\cdot\dfrac{\sin^2 x}{\sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin^2 x\cos^2 x}$

$= \dfrac{1}{\sin^2 x\cos^2 x}$

$= \dfrac{1}{\sin^2 x}\cdot\dfrac{1}{\cos^2 x}$

$= \csc^2 x\sec^2 x$

$= RS$

QED