Expected Value

(Mathematical Expectation)

Goal:

- to find the expected value of a situation
- use expected value to determine if a game is fair

I will roll a 6-sided die.

If its a 6 you pay me $10, anything else I pay you $2. Are you interested in playing this game?

Expected value = $\$2\left(\frac{5}{6}\right) + (-\$10)\left(\frac{1}{6}\right)$

(For you)

$= \$1.67 + (-\$1.67)$

$= 0$

This is a fair game

The expected value of a situation is the equal to the sum of each outcome times its probability. In the end, this calculates the average outcome.

$$EV = \text{sum of}\left(\text{each outcome} \times \text{probability}\right)$$

Ex: Rolling a die, calculate the expected value of a roll.

$$EV = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \frac{21}{6} = 3.5$$

A game of chance is considered fair if the expected value is zero.

Ex: If you draw a club from a deck of cards you win $6, if not you lose $4.

Is the game fair? If not how can you make it fair?

$EV = \$6\left(\frac{1}{4}\right) + (-\$4)\left(\frac{3}{4}\right)$

$= \$1.50 - \$3.00$

$= -\$1.50$ ← player disadvantage

To make it fair you could:

a) win more or b) lose less

$EV = 0$

$0 = x\left(\frac{1}{4}\right) + (-\$4)\left(\frac{3}{4}\right)$

$0 = 0.25x - 3$

$3 = 0.25x$

$x = 12$

You could win $12

$EV = 0$

$0 = \$6\left(\frac{1}{4}\right) + x\left(\frac{3}{4}\right)$

$0 = 1.5 + 0.75x$

$-1.5 = 0.75x$

$x = -2$

or you could lose $2.