CONIC SECTIONS
IN GENERAL FORM

Goal:

- to convert a conic section equation from general form to standard form by completing the square

All the equations we have looked at have been in ***standard form***.

Each conic section has its own equation form.

Ex:

$$\frac{(x-1)^2}{16}+\frac{(y+4)^2}{25}=1$$

Ellipse
C(1,-4)
major axis = 10
minor axis = 8

However every conic section can be represented by an equation in ***general form (expanded)***.

(Bxy for rotated axes)

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

This forms works for every conic.

The values of parameter A and C determine which type.

Circle: $A = C$

Ellipse: $A \neq C$ and $AC > 0$

Hyperbola: $A \neq C$ and $AC < 0$

Parabola: $AC = 0$

How can we convert an equation from general form to standard form?

COMPLETE THE SQUARE → $(x-h)^2$ → $(y-k)^2$

$1x^2+bx+\left(\frac{b}{2}\right)^2$

$x^2 + 8x = 0$

$x^2+8x+16 = +16$

$(x+4)^2=16$

$y^2 - 3y = 1$

$y^2-3y+\frac{9}{4}=1+\frac{9}{4}$

$\left(y-\frac{3}{2}\right)^2=\frac{13}{4}$

$3x^2 + 18x = 4$

$3(x^2+6x)=4$

$3(x^2+6x+9)=4+3(9)$

$3(x+3)^2=31$

Write the following equations in standard form.

$x^2 + 2x + y^2 - 4y + 1 = 0$ circle: $A = C$

$(x^2+2x+1) + (y^2-4y+4) + 1 = +1+4$

$(x+1)^2 + (y-2)^2 + 1 = 5$

$(x+1)^2 + (y-2)^2 = 4$ $r = 2$

$C(-1,2)$

$x^2 - y^2 - 4x + 10y - 15 = 0$ hyperbola: $AC < 0$

$x^2-4x-y^2+10y = 15$

$x^2-4x+4-(y^2-10y+25) = 15+4-25$

$(x-2)^2-(y-5)^2 = -6$

$\frac{(x-2)^2}{6} - \frac{(y-5)^2}{6} = -1$ $C(2,5)$

vertical transverse

$2x^2+16x+y^2-12y+67=0$

Ellipse: $A \neq C$, $AC > 0$

$2(x^2+8x+16)+y^2-12y+36=-64+32+36$

$2(x+4)^2+(y-6)^2=4$

$\frac{2(x+4)^2}{4}+\frac{(y-6)^2}{4}=1$

$\frac{(x+4)^2}{2}+\frac{(y-6)^2}{4}=1$

$\frac{9(x-4)^2}{4}+\frac{3(y-1)^2}{4}$

$\frac{(x-4)^2}{\frac{4}{9}}+\frac{(y-1)^2}{\frac{4}{3}}$

$y^2-6y+4x+5=0$

$y^2-6y+9=-4x-5+9$

$(y-3)^2=-4x+4$

$(y-3)^2=-4(x-1)$

$V(1,3)$

opens left

$F(0,3)$

directrix: $x=2$