Intersection of Conics

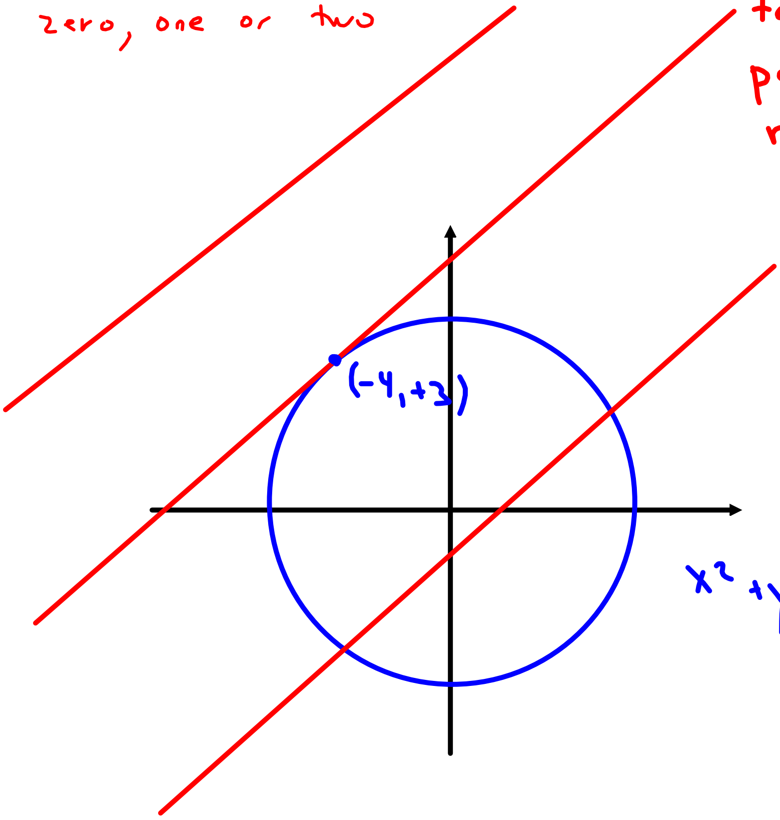
Goal:

- to determine the coordinates of the points of intersection between two conics

How many times can a line intersect a circle?

zero, one or two

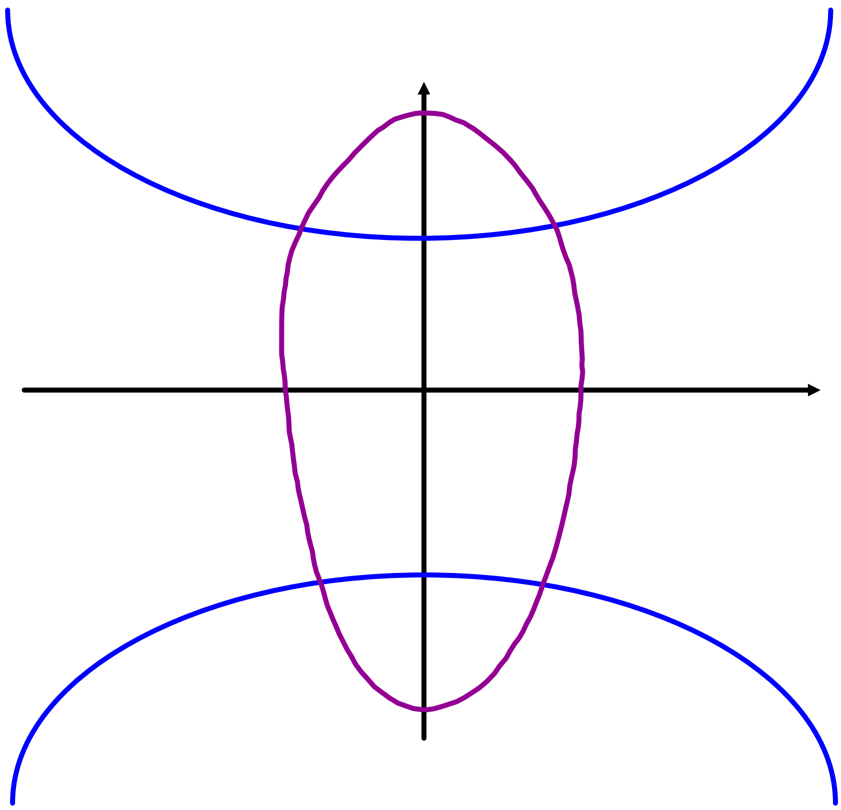
tangent line is perpendicular to radius at point of tangency



$x^2 + y^2 = 25$

How many times will an ellipse intersect a hyperbola?

up to 4 times (0, 1, 2, 4)



The coordinates of these points can be found using systems of equations (substitution and elimination).

Ex: Find the coordinates of the intersection points of
$y^2=8(x-8)$ and $x^2/100 + y^2/36 = 1$

$$\frac{x^2}{100} + \frac{8(x-8)}{36} = 1$$

$$\frac{x^2}{100} + \frac{2(x-8)}{9} = 1$$

$$9x^2 + 200(x-8) = 900$$

$$9x^2 + 200x - 1600 = 900$$

$$9x^2 + 200x - 2500 = 0$$

$$x = \frac{-200 \pm \sqrt{200^2 - 4(9)(-2500)}}{2(9)}$$

$$= \frac{-200 \pm \sqrt{130\,000}}{18}$$

$$= \frac{-200 \pm 100\sqrt{13}}{18}$$

$$= \frac{-100 \pm 50\sqrt{13}}{9}$$

$x_1 \simeq 8.92$ $\qquad$ $x_2 \simeq -31.14$

p.204
#1-3

1.c) $\frac{x^2}{16} - \frac{y^2}{4} = -1$

$a = \frac{4-1}{5-1} = \frac{3}{4}$

$y = \frac{3}{4}x + b$

$1 = \frac{3}{4}(1) + b$

$\frac{1}{4} = b$

$y = \frac{3}{4}x + \frac{1}{4}$

$\frac{x^2}{16} - \frac{\left(\frac{3}{4}x + \frac{1}{4}\right)^2}{4} = -1$

$\frac{x^2}{16} - \frac{\frac{9}{16}x^2 + \frac{3}{8}x + \frac{1}{16}}{4} = -1$

$x^2 - 4\left(\frac{9}{16}x^2 + \frac{3}{8}x + \frac{1}{16}\right) = -16$

$x^2 - \frac{9}{4}x^2 - \frac{3}{2}x - \frac{1}{4} = -16$

$4x^2 - 9x^2 - 6x - 1 = -64$

$-5x^2 - 6x - 1 = -64$

$0 = 5x^2 + 6x - 63$

$0 = 5x^2 - 15x + 21x - 63$

$0 = 5x(x-3) + 21(x-3)$

$0 = (5x+21)(x-3)$

$5x + 21 = 0$ $\quad$ $x - 3 = 0$

$x = -\frac{21}{5}$ $\quad$ $x = 3$

$y = \frac{3}{4}\left(-\frac{21}{5}\right) + \frac{1}{4}$

$= \frac{-63}{20} + \frac{5}{20}$

$= \frac{-58}{20} = \frac{-29}{10}$

$y = \frac{3}{4}(3) + \frac{1}{4}$

$= \frac{9}{4} + \frac{1}{4}$

$= \frac{10}{4} = \frac{5}{2}$

$\left(-\frac{21}{5}, -\frac{29}{10}\right)$ and $\left(-3, \frac{5}{2}\right)$