Kinematics Equations

Goal:

- to derive the four kinematics equations

If I drop a medicine ball and a tennis ball from the same height at the same time, which object will hit the ground first?

~~Medicine ball~~

They fall at the same rate

Why? Acceleration due to gravity is the same for both.

Is the force of gravity acting on the balls the same or different?

$F_{med.\ ball} > F_{tennis\ ball}$

↑

Ouch!

This is an example of uniform acceleration. The acceleration due to gravity is constant.

Uniform acceleration problems are a large part of physics, specifically *kinematics*. Kinematics is the study of how objects move.

To help with these calculations physicists have derived **four** uniform acceleration equations. Better known as the **kinematic equations.**

A car is traveling at 25 m/s along the highway. All of sudden the car in front slams on the brakes.

The average person has a reaction time of 0.2 seconds. The maximum deceleration that can be applied is 5.0 m/s^2.

How much room does the car need to safely come to a stop?

Equation 1:

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

No $\Delta\vec{d}$ in equation

Derivation:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}\Delta t = \Delta\vec{v}$$

$$\vec{a}\Delta t = \vec{v}_f - \vec{v}_i$$

$$\vec{v}_i + \vec{a}\Delta t = \vec{v}_f$$

Equation 2:

$$\vec{\Delta d} = \frac{(\vec{v}_i + \vec{v}_f)}{2}\Delta t$$

Don't know $\vec{a}$

Derivation:

$$\vec{v}_{av} = \frac{\vec{\Delta d}}{\Delta t}$$

$$\vec{v}_{av}\Delta t = \vec{\Delta d}$$

$$\frac{(\vec{v}_i + \vec{v}_f)}{2}\Delta t = \vec{\Delta d}$$

$$*v_{av} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

when $\vec{a}$ is constant

Equation 3:

$$\vec{\Delta d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

Don't know v_f

Derivation:

$$\vec{\Delta d} = \frac{(\vec{v}_i + \vec{v}_f)}{2}\Delta t \quad \text{and} \quad \vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

$$\vec{\Delta d} = \frac{(\vec{v}_i + (\vec{v}_i + \vec{a}\Delta t))}{2}\Delta t$$

$$\vec{\Delta d} = \frac{(2\vec{v}_i + \vec{a}\Delta t)}{2}\Delta t$$

$$= \frac{2\vec{v}_i\Delta t + \vec{a}(\Delta t)^2}{2}$$

$$= \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

Equation 3b:

$$\Delta d = v_f \Delta t - \frac{1}{2} a (\Delta t)^2$$

Don't know v_i

Derivation:

$$\vec{\Delta d} = \frac{(\vec{v}_i + \vec{v}_f)}{2} \Delta t$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

$$v_i = \vec{v}_f - \vec{a}\Delta t$$

$$\vec{\Delta d} = \frac{(\vec{v}_f - \vec{a}\Delta t + \vec{v}_f)\Delta t}{2}$$

$$= \frac{(2\vec{v}_f - \vec{a}\Delta t)\Delta t}{2}$$

$$\vec{\Delta d} = \frac{2\vec{v}_f \Delta t - \vec{a}(\Delta t)^2}{2}$$

$$\vec{\Delta d} = \vec{v}_f \Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

Equation 4:

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\Delta\vec{d}$$

Don't know Δt

Derivation:

$$\Delta\vec{d} = \frac{(\vec{V}_i + \vec{V}_f)\Delta t}{2}$$

$$\vec{V}_f = \vec{V}_i + \vec{a}\Delta t$$

$$\vec{V}_f - \vec{V}_i = \vec{a}\Delta t$$

$$\frac{\vec{V}_f - \vec{V}_i}{\vec{a}} = \Delta t$$

$$\Delta\vec{d} = \frac{(\vec{V}_i + \vec{V}_f)}{2} \cdot \frac{(\vec{V}_f - \vec{V}_i)}{\vec{a}}$$

$$\Delta\vec{d} = \frac{-\vec{V}_i^2 + \vec{V}_f^2}{2\vec{a}}$$

$$2\vec{a}\Delta\vec{d} = \vec{V}_f^2 - \vec{V}_i^2$$

$$\vec{V}_i^2 + 2\vec{a}\Delta\vec{d} = \vec{V}_f^2$$

Five Uniform Acceleration Equations

$$v_f = v_i + a\Delta t$$

$$\Delta d = \frac{(v_i + v_f)}{2}\Delta t$$

$$\Delta d = v_i\Delta t + \frac{1}{2}a\left(\Delta \mathrm{t}\right)^2$$

$$\Delta d = v_f\Delta t - \frac{1}{2}a\left(\Delta \mathrm{t}\right)^2$$

$${v_f}^2 = {v_i}^2 + 2a\Delta d$$

A car is traveling at 25 m/s along the highway. All of sudden the car in front slams on the brakes.

The average person has a reaction time of 0.2 seconds. The maximum deceleration that can be applied is 5.0 m/s².

How much room does the car need to safely come to a stop?

First 0.2sec:

uniform motion: $\vec{v} = \frac{\vec{\Delta d}}{\Delta t}$

$25\ m/s = \frac{\vec{\Delta d}}{0.2\ s}$

$\vec{\Delta d} = 25\ m/s\ (0.2\ s)$

$= 5\ m$

Uniform acceleration:

$\vec{v}_f = 0$

$\vec{a} = -5.0\ m/s^2$

$\vec{v}_i = 25\ m/s$

$\vec{\Delta d} = ?$

$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{\Delta d}$

$0 = (25\ m/s)^2 + 2(-5.0\ m/s^2)\vec{\Delta d}$

$0 = 625\ m^2/s^2 - 10\ m/s^2\ \vec{\Delta d}$

$\frac{-625\ m^2/s^2}{-10\ m/s^2} = \vec{\Delta d}$

$62.5\ m = \vec{\Delta d}$

Total room needed:

$5\ m + 62.5\ m = 67.5\ m$