

Properties of Logarithms

Goal:

- to understand the properties of logarithms
 - addition/subtraction of logs
 - power rule

- use the properties of logarithms to simplify and manipulate logarithmic expressions

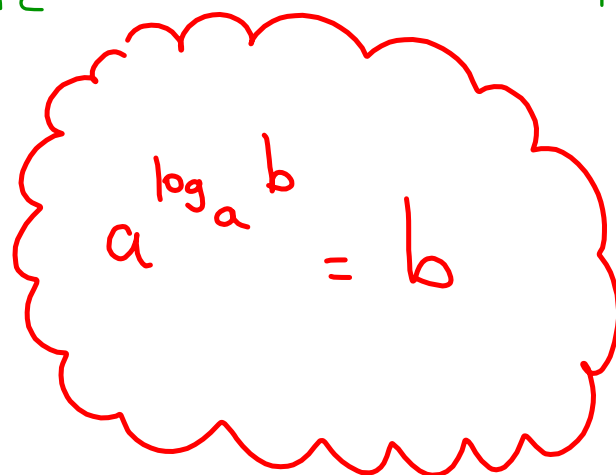
Fundamental law of logs:

$$2^{\log_2 8} = 2^3 \\ = 8$$

$$5^{\log_5 25} \\ = 25$$

$$10^{\log 12} \\ = 12$$

$$e^{\ln 9} \\ = 9$$


$$a^{\log_a b} = b$$

Laws of logarithms:

1. addition of logarithms

$$\begin{aligned} \log_3 9 + \log_3 3 &\stackrel{?}{=} \log_3 12 &&= \log_3 (9 \cdot 3) \\ 2 + 1 &&&= \log_3 27 \\ = 3 &\neq 2.27 &&= 3 \end{aligned}$$

$$\begin{aligned} \log_2 16 + \log_2 4 &= \log_2 (64) \\ = 4 + 2 &= 6 \\ = 6 &= 6 \end{aligned}$$

$$\log_c a + \log_c b = \log_c ab$$

2. subtraction of logarithms

$$\begin{aligned} \log_3 9 - \log_3 3 &= \log_3 \left(\frac{9}{3} \right) = \log_3 3 = 1 \\ = 2 - 1 & \\ = 1 & \end{aligned}$$

$$\begin{aligned} \log_2 16 - \log_2 4 &= \log_2 \left(\frac{16}{4} \right) = \log_2 4 \\ = 4 - 2 &= 2 \\ = 2 & \end{aligned}$$

$$\log_c a - \log_c b = \log_c \left(\frac{a}{b} \right)$$

Ex: Write the following as a single logarithm:

a) $\log_3 x + \log_3 y$

b) $\log_{1/2} a - \log_{1/2} b$

c) $\ln 25 + \ln 4$

3. power of a logarithm

$$\begin{aligned}\log_4 4^2 &= 2 \log_4 4 \\ &= \log_4 16 &= 2 \\ &= 2\end{aligned}$$

$$\begin{aligned}\log_5 25^3 &= 3 \log_5 25 \\ &= \log_5 15625 &= 3(2) \\ &= 6 &= 6\end{aligned}$$

$$\log_c a^b = b \log_c a$$

Ex: Write the following as a single logarithm:

$$\begin{aligned}\text{a) } 2 \log_a 4 - 3 \log_a 2 &= \log_a 4^2 - \log_a 2^3 \\ &= \cancel{\log_a \left(\frac{4}{2}\right)} &= \log_a 16 - \log_a 8 \\ & &= \log_a \left(\frac{16}{8}\right) \\ & &= \log_a 2\end{aligned}$$

$$\begin{aligned}\text{b) } \log x^5 + \log x^{-4} &= 5 \log x - 4 \log x \\ &= \log (x^5 \cdot x^{-4}) &= \log x \\ &= \log x\end{aligned}$$

$$\begin{aligned} 11. e) \quad & \log_a 360 \\ &= \log_a (36 \cdot 10) \\ &= \log_a (6^2 \cdot 2 \cdot 5) \\ &= \log_a ((2 \cdot 3)^2 \cdot 2 \cdot 5) \\ &= \log_a (2^3 \cdot 3^2 \cdot 5) \\ &= \log_a 2^3 + \log_a 3^2 + \log_a 5 \\ &= 3\log_a 2 + 2\log_a 3 + \log_a 5 \\ &= 3x + 2y + z \end{aligned}$$