Properties of Logarithms

Goal:

- to understand the properties of logarithms
    - addition/subtraction of logs
    - power rule

- use the properties of logarithms to simplify and manipulate logarithmic expressions

Fundamental law of logs:

$2^{\log_2 8}$ $= 2^3$

$= 8$

$5^{\log_5 25}$

$= 25$

$10^{\log 12}$

$= 12$

$e^{\ln 9}$

$= 9$

$$a^{\log_a b} = b$$

Laws of logarithms:

1. addition of logarithms

$\log_3 9 + \log_3 3$ $\overset{?}{=} \log_3 12$ $= \log_3 (9 \cdot 3)$

$2 + 1$ $\neq 2.27$ ✗ $= \log_3 27$

$= 3$ $= 3$

$\log_2 16 + \log_2 4$ $= \log_2 (64)$

$= 4 + 2$

$= 6$ $= 6$

$$\log_c a + \log_c b = \log_c ab$$

2. subtraction of logarithms

$\log_3 9 - \log_3 3$ $= \log_3 \left(\frac{9}{3}\right) = \log_3 3 = 1$

$= 2 - 1$

$= 1$

$\log_2 16 - \log_2 4$ $= \log_2 \left(\frac{16}{4}\right) = \log_2 4$

$= 4 - 2$ $= 2$

$= 2$

$$\log_c a - \log_c b = \log_c \left(\frac{a}{b}\right)$$

Ex: Write the following as a single logarithm:

a) $\log_3 x + \log_3 y$

b) $\log_{1/2} a - \log_{1/2} b$

c) $\ln 25 + \ln 4$

3. power of a logarithm

$\log_4 4^2$ $= 2\log_4 4$

$= \log_4 16$ $= 2$

$= 2$

$\log_5 25^3$ $= 3\log_5 25$

$= \log_5 15625$ $= 3(2)$

$= 6$ $= 6$

$$\log_c a^b = b\log_c a$$

Ex: Write the following as a single logarithm:

a) $2\log_a 4 - 3\log_a 2$ $= \log_a 4^2 - \log_a 2^3$

$= \log_a\left(\frac{4}{2}\right)$ $= \log_a 16 - \log_a 8$

$= \log_a\left(\frac{16}{8}\right)$

$= \log_a 2$

b) $\log x^5 + \log x^{-4}$ $= 5\log x - 4\log x$

$= \log\left(x^5 \cdot x^{-4}\right)$ $= \log x$

$= \log x$

11. e) $\log_a 360$

$= \log_a (36 \cdot 10)$

$= \log_a (6^2 \cdot 2 \cdot 5)$

$= \log_a ((2 \cdot 3)^2 \cdot 2 \cdot 5)$

$= \log_a (2^3 \cdot 3^2 \cdot 5)$

$= \log_a 2^3 + \log_a 3^2 + \log_a 5$

$= 3\log_a 2 + 2\log_a 3 + \log_a 5$

$= 3x + 2y + z$