Linear Combinations and the Dot Product

(Linear Algebra)

Goal:

- to be able to express a vector as the sum of two others
- to be able to calculate the dot product of two vectors

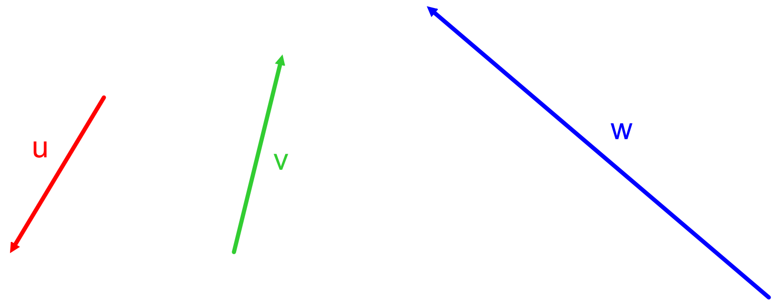
p.48 #1-3,5

A linear combination of vectors means taking one vector and representing it as a sum of two other vectors:

How can you combine u and v to equal w?

$$k_1\vec{u} + k_2\vec{v} = \vec{w}$$

Given $\vec{u}$=(-2,3) and $\vec{v}$=(1,4) and $\vec{w}$=(-7,6)



Using components:

Horizontal: $k_1(-2) + k_2(1) = -7$

$-2k_1 + k_2 = -7$ ①

Vertical: $k_1(3) + k_2(4) = 6$

$3k_1 + 4k_2 = 6$ ②

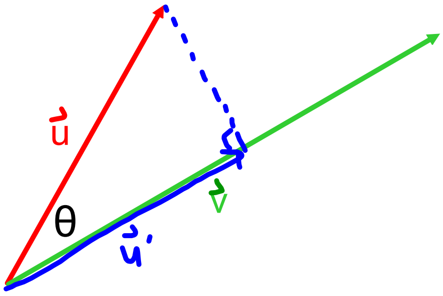
Solve system of equations:

①×4
$$-8k_1 + 4k_2 = -28$$
$$3k_1 + 4k_2 = 6$$
$$-11k_1 = -34$$
$$k_1 = \frac{34}{11}$$

$$-2\left(\frac{34}{11}\right) + k_2 = -7$$
$$-\frac{68}{11} + k_2 = -7$$
$$k_2 = -7 + \frac{68}{11}$$
$$= -\frac{9}{11}$$

$$\frac{34}{11}\vec{u} - \frac{9}{11}\vec{v} = \vec{w}$$

The dot product (scalar product) is an extension of the orthogonal projection. Instead of simply projecting a vector on a line. You are projecting a vector onto another vector and then multiplying the norms.



$$\vec{u} \cdot \vec{v} = \|\vec{u}'\| \|\vec{v}\|$$

dot product

$$= \|\vec{u}\| \cos\theta \, \|\vec{v}\|$$

$$\boxed{\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta}$$

if $[[\vec{u}]]=6$ and $[[\vec{v}]]=8$ and $\theta=20^0$

$$\vec{u} \cdot \vec{v} = 6(8)\cos 20^\circ$$

$$\approx 45.1$$

If $\vec{u}$ and $\vec{v}$ are expressed in terms of components, the dot product can still be found:

$\vec{u} = (a, b)$ $\vec{v} = (c, d)$

$$\boxed{\vec{u} \cdot \vec{v} = ac + bd}$$

If $\vec{u}=(2,6)$ and $\vec{v}=(4,8)$, find $\vec{u} \cdot \vec{v}$

$$\vec{u} \cdot \vec{v} = (2)(4) + (6)(8)$$
$$= 8 + 48$$
$$= 56$$

If $\vec{w}=(-1,5)$ and $\vec{z}=(10,2)$ find $\vec{w} \cdot \vec{z}$

$$\vec{w} \cdot \vec{z} = -1(10) + 5(2)$$
$$= 0 \;!$$

$\vec{w}$ and $\vec{z}$ are orthogonal

Perpendicular vectors have a dot product equal to zero!