

Operations on Functions and Composite Functions

Goal:

- to perform the following operations on functions:
 - addition
 - subtraction
 - multiplication
 - division

- to find the composite of two functions

Any of the basic four operations can be performed on given functions.

Ex: Given functions $f(x)=3x+2$ and $g(x)=x-4$ determine:

a) $(f+g)(x)$

b) $(f-g)(x)$

c) $(f \cdot g)(x)$

d) $(f \div g)(x)$

$$\begin{aligned} \text{a) } (f+g)(x) &= (3x+2) + (x-4) \\ &= 4x-2 \quad \leftarrow \text{a new linear function!} \end{aligned}$$

$$\begin{aligned} \text{b) } (f-g)(x) &= (3x+2) - (x-4) \\ &= 2x+6 \end{aligned}$$

$$\begin{aligned} \text{c) } (f \cdot g)(x) &= (3x+2)(x-4) \\ &= 3x^2 - 10x - 8 \quad \text{Quadratic function!} \end{aligned}$$

$$\text{d) } (f \div g)(x) = \frac{3x+2}{x-4} \quad \text{Rational function}$$

Another operation that can be performed on two functions is to find their composite. This means replacing the independent variable of one function with another function.

The notation to show a composite function is the following:

$$(f \circ g)(x) \quad \text{Read "f of g of x"}$$

↑
composite

when f depends on g and

$$(f \circ g)(x) = f(g(x))$$

if g depends on f

$$(g \circ f)(x) = g(f(x))$$

Given the functions $f(x) = 2x - 1$ and $g(x) = 2\sqrt{x+3} - 1$

dom g : $x \geq -3$

Find:

$$\begin{aligned} \text{a) } (f \circ g)(6) &= f(g(6)) & \text{OR } g(6) &= 2\sqrt{6+3} - 1 \\ &= f(5) & &= 2(3) - 1 \\ &= 2(2\sqrt{6+3} - 1) - 1 & &= 5 \\ &= 2(5) - 1 & f(g(6)) &= 2(5) - 1 \\ &= 9 & &= 9 \end{aligned}$$

$$\begin{aligned} \text{b) } (f \circ g)(x) &= f(g(x)) \\ &= 2(2\sqrt{x+3} - 1) - 1 \\ &= 4\sqrt{x+3} - 2 - 1 \\ &= 4\sqrt{x+3} - 3 \quad \text{dom } f \circ g: x \geq -3 \end{aligned}$$

$$\begin{aligned} \text{c) } (g \circ f)(x) &= g(f(x)) \\ &= 2\sqrt{(2x-1)+3} - 1 \\ &= 2\sqrt{2x+2} - 1 \\ &= 2\sqrt{2(x+1)} - 1 \quad \text{dom } g \circ f: x \geq -1 \end{aligned}$$

$$\begin{aligned} \text{d) } (f \circ f)(x) &= f(f(x)) \\ &= 2(2x-1) - 1 \\ &= 4x - 2 - 1 \\ &= 4x - 3 \end{aligned}$$

$$\begin{aligned} \text{e) } (f \circ f^{-1})(x) &= f(f^{-1}(x)) & y &= 2x - 1 \\ &= f\left(\frac{1}{2}x + \frac{1}{2}\right) - 1 & x &= 2y - 1 \\ &= 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1 & x + 1 &= 2y \\ &= x + 1 - 1 & y &= \frac{1}{2}x + \frac{1}{2} \\ &= x & f^{-1}(x) &= \frac{1}{2}x + \frac{1}{2} \end{aligned}$$