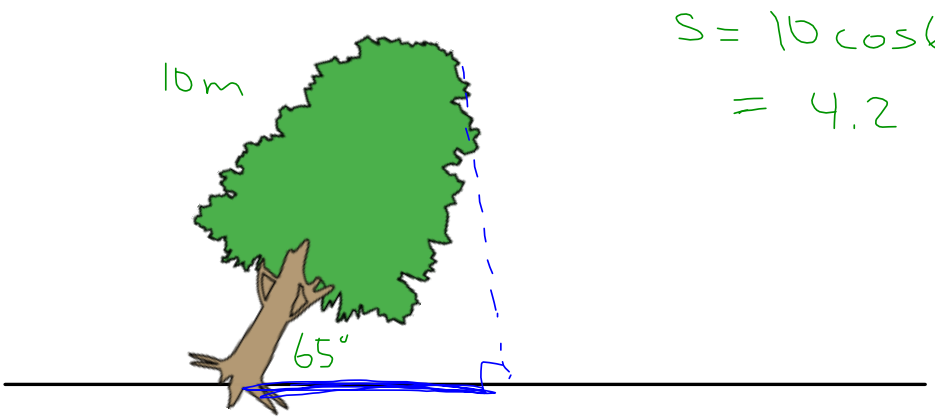
OPERATIONS ON VECTORS

Goal:

- to find the projection of a vector
- to add and subtract vectors
- to become familiar with Chasles relation
- to multiply vectors by scalars

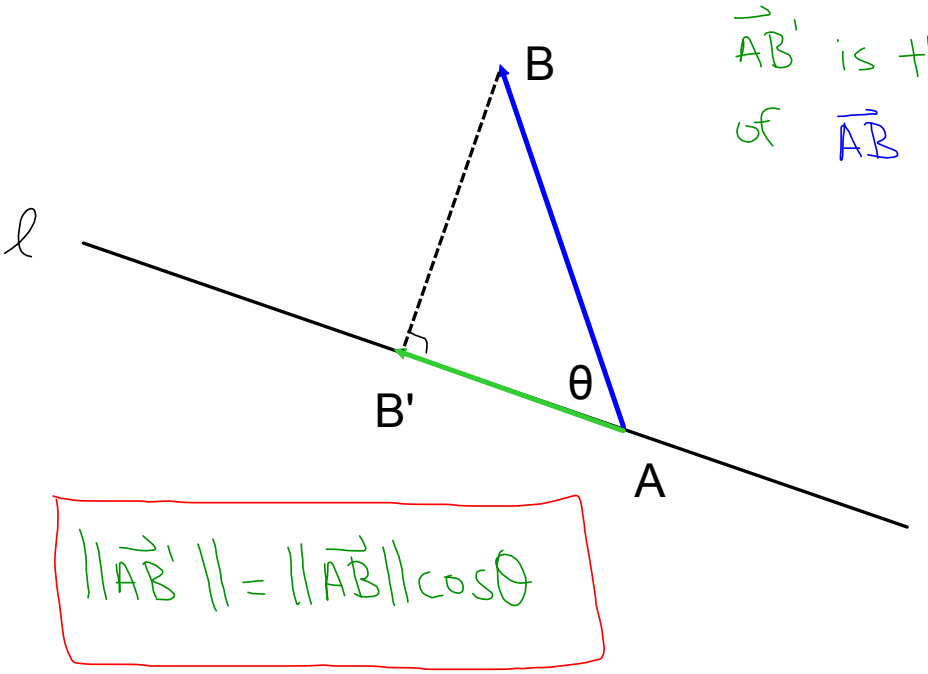
If the sun is directly overhead how long is the shadow of the tree?



$s = 10\cos 65^\circ$

$= 4.2 \text{ m}$

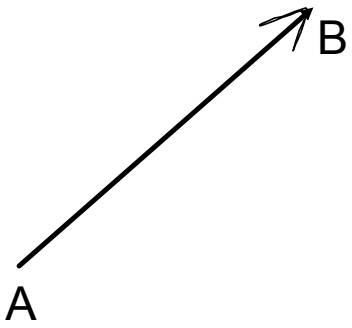
This concept is one of projection. We have taken a vector (the tree) and projected it onto a line (the ground).



$\vec{AB'}$ is the projection of $\vec{AB}$ onto line ℓ.

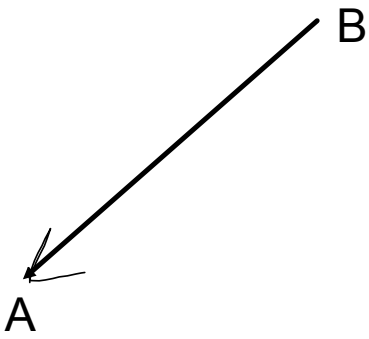
$$\|\vec{AB'}\| = \|\vec{AB}\|\cos\theta$$

Vectors can be described in terms of norm and orientation or in terms of components. They can also be described in terms of points.



$\vec{AB} \neq \vec{BA}$

order matters!

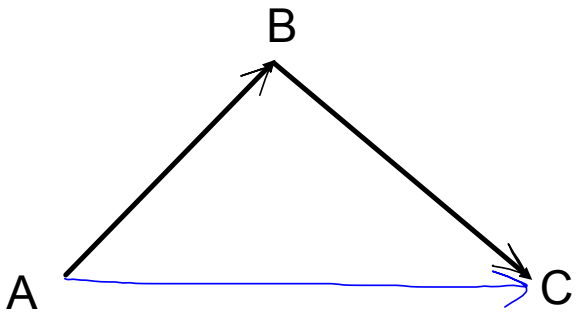


$\vec{BA} = -\vec{AB}$

opposite

Chasles' relation states that if you have three points A,B and C then:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



What about $\overrightarrow{AC}$ - $\overrightarrow{BC}$?

$\overrightarrow{BC} = -\overrightarrow{CB}$

$\overrightarrow{AC} - \overrightarrow{BC}$

$= \overrightarrow{AC} - (-\overrightarrow{CB})$

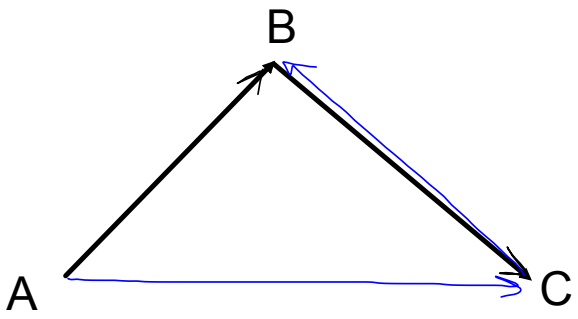
$= \overrightarrow{AC} + \overrightarrow{CB}$

$= \overrightarrow{AB}$

$\overrightarrow{AC} - \overrightarrow{CB}$

$= \overrightarrow{AC} + \overrightarrow{BC}$

$= ?$



The combination of vectors can be thought of with use of notation or graphically but is easiest when done using components.

Given vectors $\vec{v}=(1,4)$ and $\vec{u}=(-2,5)$, we can add, subtract and multiply.

a) $\vec{v} + \vec{u} = (1+(-2), 4+5)$ add components

$= (-1, 9)$

b) $\vec{v} - \vec{u} = (1-(-2), 4-5)$ subtract components

$= (3, -1)$

c) $3\vec{v} = (3(1), 3(4))$ multiply components by scalar

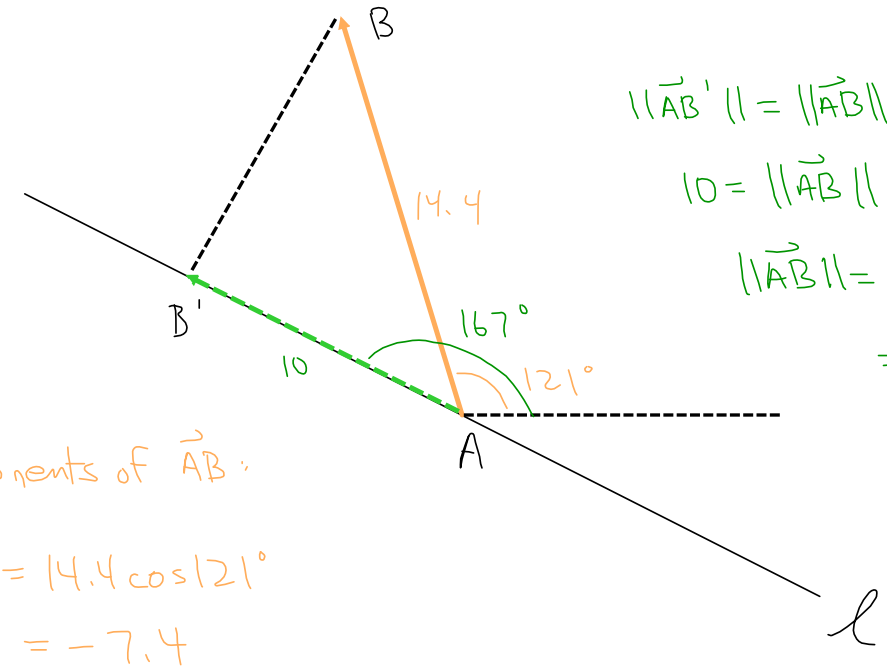
$= (3, 12)$

d) $-2\vec{u} = (-2(-2), -2(5))$

$= (4, -10)$

Homework: p.26 #15 and p.38 #2,4,9

15.



$\|\vec{AB'}\| = \|\vec{AB}\| \cos\theta$

$10 = \|\vec{AB}\| \cos 46°$

$\|\vec{AB}\| = \frac{10}{\cos 46°}$

$= 14.4$

Components of $\vec{AB}$:

$\Delta x = 14.4 \cos 121°$

$= -7.4$

$\Delta y = 14.4 \sin 121°$

$= 12.3$

$\vec{AB} = (-7.4, 12.3)$