

Radians

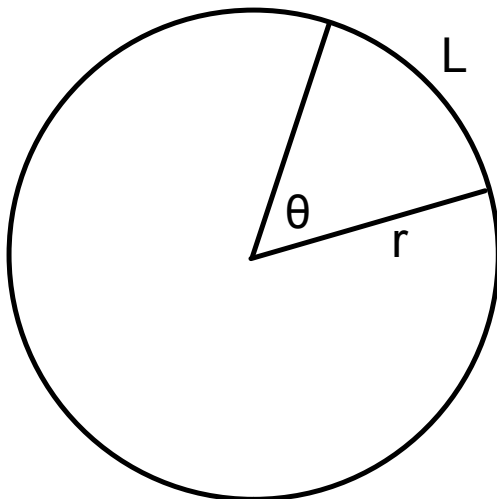
Goal:

- to become familiar with the use of radians to measure angles
- to convert between degrees and radians
- to connect the use of radians with arcs

What is one degree?

$\frac{1}{360}$ of a circle

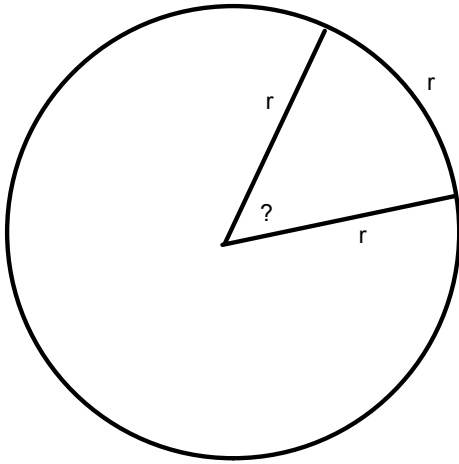
How is the central angle, θ , in a circle related to the arc, L , it defines (subtends)?



$$\frac{\theta}{360} = \frac{L}{2\pi r}$$

What is the measure of a central angle when the arc it subtends is equal to the radius of the circle?

$$L = r$$



$$\frac{\theta}{360^\circ} = \frac{r}{2\pi r}$$

$$\frac{\theta}{360^\circ} = \frac{1}{2\pi}$$

$$\theta = \frac{360^\circ}{2\pi}$$

$$\theta = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

The central angle that produces an arc equal to the radius is the definition of one radian.

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

therefore

$$\pi \text{ rad} = 180^\circ$$

and $2\pi \text{ rad} = 360^\circ$

$$\frac{\pi}{2} \text{ rad} = 90^\circ$$

⋮

Using radians then allows very quick calculations of arcs.

In degrees:

$$\frac{\theta}{360^\circ} = \frac{L}{2\pi r}$$

In radians:

$$\frac{\theta}{2\pi} = \frac{L}{2\pi r}$$

$$\theta = \frac{L}{r}$$

$$L = \theta r$$

θ : angle (rad)
 r : radius
 L : arc length

Find:

a) the arc subtended by an angle of 3 rad in a circle with a radius of 10 cm.

$$\begin{aligned} L &= \theta r \\ &= 3(10 \text{ cm}) \\ &= 30 \text{ cm} \end{aligned}$$

b) the radius of a circle when a central angle of 1.5 radians defines an arc of 9 m.

$$\begin{aligned} L &= \theta r \\ 9 &= 1.5r \\ r &= 6 \text{ m} \end{aligned}$$

Convert the following into radians:

a) 30°

$$\frac{x}{30^\circ} = \frac{\pi \text{ rad}}{180^\circ}$$

$$x = \frac{\pi}{6} \text{ rad}$$

b) 60°

$$\frac{\pi}{3} \text{ rad}$$

c) 45°

$$\frac{\pi}{4} \text{ rad}$$

d) 135°

$$\frac{3\pi}{4}$$

Convert the following into degrees:

a) 4 rad

$$\frac{x}{4 \text{ rad}} = \frac{180^\circ}{\pi \text{ rad}}$$
$$x = \frac{720^\circ}{\pi} \approx 229^\circ$$

b) $2 \pi/3$ rad

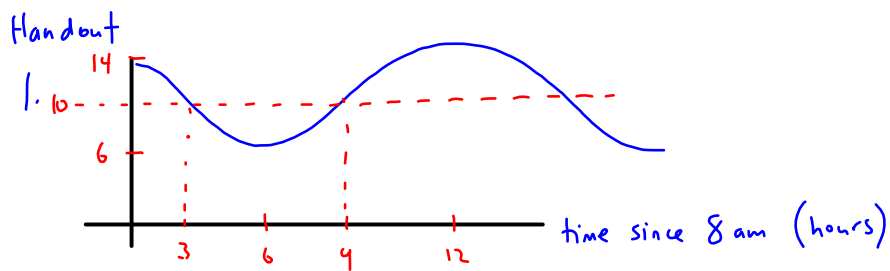
$$\frac{2(180^\circ)}{3} = 120^\circ$$

c) $11 \pi/6$ rad

$$\frac{11(180^\circ)}{6} =$$

$$330^\circ$$

goal



$$k=10$$

$$A=4$$

$$b = \frac{360}{12} = 30$$

$$h=3 \quad a=-4$$

or

$$h=9 \quad a=+4$$

$$f(x) = -4 \sin 30(x-3) + 10$$

$$a) f(2) = -4 \sin 30(2-3) + 10$$

$$= -4 \sin(-30) + 10$$

$$= -4\left(-\frac{1}{2}\right) + 10$$

$$= 12 \text{ m}$$

b) 2 m below mean = 8 m

$$8 = -4 \sin 30(x-3) + 10$$

$$-2 = -4 \sin 30(x-3)$$

$$\frac{1}{2} = \sin 30(x-3)$$

$$\sin^{-1}\left(\frac{1}{2}\right) = 30(x-3)$$

$$30 = 30(x-3) \quad \text{and} \quad 150 = 30(x-3)$$

$$1 = x-3$$

$$x=4$$

$$8 \text{ am} + 4 \text{ h} = 12 \text{ pm}$$

$$5 = x-3$$

$$x=8$$

$$8 \text{ am} + 8 \text{ h} = 4 \text{ pm}$$

Height is 2 m below at 12 pm and 4 pm and every 12 hours after.

Homework: p.194 #1-6
p. 196 #7,8