

RATIONAL EQUATIONS AND INEQUALITIES

Goal:

- to solve rational equations and inequalities

Solving rational equations is relatively straight-forward (compared to quadratic, square root and absolute value equations). *One solution unless $f(x)=k$*

Ex: a) $\frac{-3}{2(x+2)} - 1 = 4$

$$\frac{-3}{2(x+2)} = 5$$

$$-3 = 5(2(x+2))$$

$$-3 = 10(x+2)$$

$$-3 = 10x + 20$$

$$-23 = 10x$$

$$x = \frac{-23}{10}$$

b) $\frac{x-6}{2x+1} = -3$

Rational inequalities on the other hand require some thoughtfulness.

Ex: a) $\frac{1}{x-5} + 1 \geq 2$

$$\frac{1}{x-5} + 1 = 2$$

$$\frac{1}{x-5} = 1$$

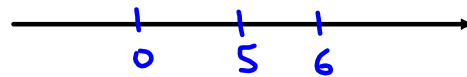
$$1 = x - 5$$

$$x = 6$$

Critical points:

$$x = 6 \text{ and } x = 5$$

"h"



test $x = 0$

$$\begin{aligned} f(0) &= \frac{1}{0-5} + 1 \\ &= -\frac{1}{5} + 1 \\ &= \frac{4}{5} \times \end{aligned}$$

test $x = 5.5$

$$\begin{aligned} f(5.5) &= \frac{1}{5.5-5} + 1 \\ &= \frac{1}{0.5} + 1 \\ &= 3 \checkmark \end{aligned}$$

test $x = 10$

$$\begin{aligned} f(10) &= \frac{1}{10-5} + 1 \\ &= \frac{1}{5} + 1 \\ &= \frac{6}{5} \times \end{aligned}$$

$$x :]5, 6]$$

$$b) \frac{x+1}{x-1} > 2 \quad \text{V.A. } x=1$$

$$\frac{x+1}{x-1} = 2$$

$$x+1 = 2(x-1)$$

$$x+1 = 2x-2$$

$$3 = x$$

Critical points:

$$x=1$$

$$x=3$$

$$x:]-\infty, 1[$$

$$x:]1, 3[$$

$$x:]3, \infty[$$

$$f(0) = \frac{0+1}{0-1}$$

$$= -1 \quad \times$$

$$f(2) = \frac{2+1}{2-1}$$

$$= 3 \quad \checkmark$$

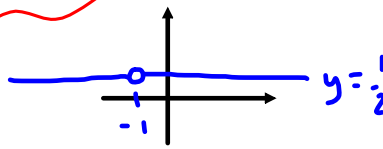
$$f(4) = \frac{4+1}{4-1}$$

$$= \frac{5}{3} \quad \times$$

$$x:]1, 3[$$

HK: p. 68 #6
p. 69 #10

p. 67



1. d) $f(x) = \frac{x+1}{2x+2}$

$$2x+2 \overline{) \begin{array}{r} x+1 \\ -(x+1) \\ \hline 0 \end{array}}$$

$$= \frac{\cancel{x+1}}{2(\cancel{x+1})}$$

$$f(x) = \frac{1}{2}$$

Not actually a rational function

1. c) $f(x) = \frac{4x-5}{3-x}$

$$-x+3 \overline{) \begin{array}{r} -4 \\ 4x-5 \\ -(4x-12) \\ \hline 7 \end{array}}$$

$$= \frac{7}{3-x} - 4$$

$$= \frac{7}{-x+3} - 4$$

$$= \frac{7}{-(x-3)} - 4$$

$$= \frac{-7}{x-3} - 4$$

$$1.e) f(x) = \frac{4x+9}{9-4x}$$

$$\begin{array}{r} -1 \\ -4x+9 \overline{) 4x+9} \\ \underline{-(4x-9)} \\ 18 \end{array}$$

$$= \frac{18}{9-4x} - 1$$

$$= \frac{18}{-4x+9} - 1$$

$$= \frac{\cancel{18}^9}{\frac{-4}{2}(x-\frac{9}{4})} - 1$$

$$= \frac{-\frac{9}{2}}{x-\frac{9}{4}} - 1$$