# Trigonometric Identities

Goal:

- to learn the basic identities (reciprocal, quotient and pythagorean)

- to use the basic identities to prove other identities

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Everyone loves puzzles.

Proving mathematical identities can be thought of as mathematical puzzles.

Have you ever wondered why the trig ratios are the way they are and not the reverse (reciprocal)?

They exist and are known as the reciprocal ratios (identities):

$\sin\theta = \frac{1}{\csc\theta} \Rightarrow \csc\theta = \frac{1}{\sin\theta} \Rightarrow \boxed{\csc\theta = \frac{hyp}{opp}}$

"cosecant"

$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta} \quad \boxed{\sec\theta = \frac{hyp}{adj}}$

"secant"

$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta} \Rightarrow \boxed{\cot\theta = \frac{adj}{opp}}$

"cotangent"

What is the exact value of:

a) sec(7π/6)

$$= \frac{1}{\cos\left(\frac{7\pi}{6}\right)}$$

$$= \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$$

b) sec(-5π/3)

$$= \frac{1}{\cos\left(-\frac{5\pi}{3}\right)}$$

$$= \frac{1}{\cos\left(\frac{\pi}{3}\right)}$$

$$= \frac{1}{\frac{1}{2}} = 2$$

c) csc(7π/6)

$$= \frac{1}{\sin\left(\frac{7\pi}{6}\right)}$$

$$= \frac{1}{-\frac{1}{2}} = -2$$

d) csc(-3π/4)

$$= \frac{1}{\sin\left(-\frac{3\pi}{4}\right)}$$

$$= \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

e) cot(7π/6)

$$= \frac{1}{\tan\left(\frac{7\pi}{6}\right)}$$

$$= \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

f) cot(3π/2)

$$= \frac{1}{\tan\left(\frac{3\pi}{2}\right)} = \frac{0}{-1} = 0$$

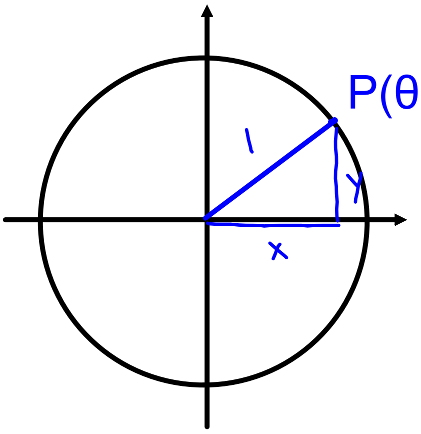
undefined ?

An identity is a statement that is true for all values of the unknown.

We have also seen the quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

In addition to the reciprocal identities, there are Pythagorean identities. These can be found from the unit circle:




$x^2 + y^2 = 1$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

and $\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

Combining this identity with the reciprocal identities we can find two other pythagorean identities:

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

Find the value of:

a) cosx, given sinx=11/12

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x + \left(\frac{11}{12}\right)^2 = 1$

$\cos^2 x = 1 - \frac{121}{144}$

$\cos^2 x = \frac{23}{144}$

$\cos x = \pm\sqrt{\frac{23}{144}}$

$= \pm\frac{\sqrt{23}}{12}$

b) sinx, given cosx=3/5 and π<x<2π

↳ 3rd + 4th quad sine is negative

$\sin^2 x + \left(\frac{3}{5}\right)^2 = 1$

$\sin^2 x = 1 - \frac{9}{25}$

$\sin^2 x = \frac{16}{25}$

$\sin x = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

$\sin x = -\frac{4}{5}$

# Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta$

$\cos^2\theta=1-\sin^2\theta$

$\sec^2\theta=1+\tan^2\theta$

$\sec^2\theta-1=\tan^2\theta$

$\sec^2\theta-\tan^2\theta=1$

$\csc^2\theta=1+\cot^2\theta$

$\csc^2\theta-1=\cot^2\theta$

$\csc^2\theta-\cot^2\theta=1$