**2- Correction key**

1

Example of an appropriate method

• Maximum profit before the new constraint was added

|  |  |  |
| --- | --- | --- |
| Vertex | Profit: 60*x* + 50*y* |  |
| P(1, 17) | $910 |  |
| Q(17, 1) | $1070 | ← Usual maximum profit |
| R(5, 9) | $750 |  |

• Graph showing the additional constraint

|  |  |
| --- | --- |
| *x* ≤ 5  The coordinates of the new vertex  are (5, 13). |  |

• Maximum profit after the new constraint was added

|  |  |  |
| --- | --- | --- |
| Vertex | Profit: 60*x* + 50*y* |  |
| P(1, 17) | $910 |  |
| (5, 13) | $950 | ← Today’s maximum profit |
| R(5, 9) | $750 |  |

• Difference between the maximum profits

$1070 − $950 = $120

Answer: The difference between the usual maximum profit and today’s maximum profit is $**120**.

2

Example of an appropriate method

Maximum profit

|  |  |  |
| --- | --- | --- |
| Vertex | Profit | |
| Supplier A:  0.50*x* + 1.50*y* | Supplier B:  0.90*x* + 1.30*y* |
| P(25, 25) | $50 | $55 |
| Q(40, 10) | $35 | $49 |
| R(12, 0) | $6 | $10.80 |

The maximum profit with Supplier A is $50.

The maximum profit with Supplier B is $55.

Answer: In order to maximize her profit, Julie should choose Supplier **B**.

**Note:** Students who used an appropriate method in order to determine the maximum profit Julie earns with **either** supplier have shown that they have a partial understanding of the problem.

Students who added the profits associated with the vertices of the polygon and then compared the resulting totals have not used an appropriate method.

3

Of these four points, points **R** and **S** represent solutions for this system of inequalities.

4

The minimum weekly cost of running this business is $**462**.

5

B

6

The coordinates of vertex P of this polygon of constraints are P(**2, 5**).

7

Example of an appropriate method

Minimum revenue for each crossing before the new constraint was added

|  |  |  |
| --- | --- | --- |
| Vertex | Revenue: 4*x* + 10*y* |  |
| A(50, 50) | $700 | ← Minimum |
| B(350, 350) | $4900 |  |
| C(450, 50) | $2300 |  |

|  |  |
| --- | --- |
| New constraint  *x* + *y* ≥ 300  The coordinates of the 2 new vertices are  (150, 150) and (250, 50). |  |

Minimum revenue for each crossing after the new constraint was added

|  |  |  |
| --- | --- | --- |
| Vertex | Revenue: 4*x* + 10*y* |  |
| (150, 150) | $2100 |  |
| B(350, 350) | $4900 |  |
| C(450, 50) | $2300 |  |
| (250, 50) | $1500 | ← Minimum |

Difference between the minimum revenues for each crossing

$1500 − $700 = $800

Answer: This new constraint increases the minimum revenue for each crossing by $**800**.

8

Example of an appropriate method

Maximum possible weekly income without the additional constraint

|  |  |  |
| --- | --- | --- |
| Vertex | Income: 10*x* + 8*y* |  |
| P(5, 35)  Q(20, 5)  R(15, 5)  S(5, 15) | $330  $240  $190  $170 | ← Maximum |

Additional constraint

|  |  |
| --- | --- |
| *y* ≤ *x*  The coordinates of the 2 new vertices are (10, 10) and (15, 15) |  |

Maximum possible weekly income with the additional constraint

|  |  |  |
| --- | --- | --- |
| Vertex | Income: 10*x* + 8*y* |  |
| (10, 10)  (15, 15)  Q(20, 5)  R(15, 5) | $180  $270  $240  $190 | ← Maximum |

Difference between the maximum possible weekly incomes

$330 − $270 = $60

Answer: This constraint decreases Vincent's maximum possible weekly income by $**60**.

**Note:** Students who use an appropriate method in order to determine the maximum possible weekly income with **or** without the additional constraint have shown that they have a partial understanding of the problem.

9

B

10

Example of an appropriate method

Each member's maximum profit

|  |  |  |
| --- | --- | --- |
| Vertex | Profit: 1.50*x* + 2.50*y* |  |
| A(10, 14)  B(10, 4)  C(20, 4) | $50  $25  $40 | ← Maximum |

Each member made a maximum profit of $50.

Number of members

Total amount raised ÷ Each member's maximum profit

$800 ÷ $50 per member

16 members

Answer: This club has **16** members

**Note:** Students who use an appropriate method in order to determine each member's maximum profit have shown that they have a partial understanding of the problem.

11

|  |  |  |
| --- | --- | --- |
| There are | 2 🞏  3 🞏  4 ◼ | different combinations of child and adult passengers that will enable the guide to maximize his revenue for one trip. |
|

12

The coordinates of vertex R of this polygon of constraints are R**(9, 18)**.

13

C

14

C

15

D

16

A

17

C

18

Example of an appropriate method

Coordinates of the vertices of the polygon

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *y* = 2*x* and *x* + *y* = 27 | | |  | | *x* = 5 and *x* + *y* = 27 | | |
| *x* + 2*x* = | 27 | |  | | 5 + *y* = | 27 | |
| 3*x* = | 27 | |  | | *y* = | 22 | |
| *x* = | 9 | |  | |  |  | |

If *x* = 9 then *y* = 18

Q(9, 18) P(5, 22)

Jerry's income

|  |  |
| --- | --- |
| Vertex | Income: 12*x* + 20*y* |
| P(5, 22) | $500 |
| Q(9, 18) | $468 |
| R(5, 10) | $260 |

Jerry's minimum income is $260.

Answer: Jerry can always be sure that these computer lessons will pay him enough money every month to cover his rent.

Yes ✓ No

Explanation: His minimum income is less than his rent.

**Note**: Students who use an appropriate method in order to determine the coordinates of the vertices of the polygon have shown that they have a partial understanding of the problem.

19

Example of an appropriate method

Maximum revenue before the runway is built

|  |  |
| --- | --- |
| Vertex | Revenue: 4*x* + 6*y* |
| P(60, 60)  Q(150, 150)  R(240, 60) | $600  $1500  $1320 |

The maximum revenue is $1500.

Additional constraint

|  |  |
| --- | --- |
| *x* + *y* ≤ 240 |  |

Maximum revenue after the runway is built

|  |  |
| --- | --- |
| Vertex | Revenue: 4*x* + 6*y* |
| (60, 60)  (120, 120)  (180, 60) | $600  $1200  $1080 |

The maximum revenue is $1200.

Decrease in maximum revenue

$1500 − $1200 = $300

Answer: The maximum possible revenue will decrease by $300 once the runway is built.

20

A

21

Work : (example)

a) The linear inequalities that express the constraints

(*x* ≥ 0 and *y* ≥ 0)

*x* + *y* ≥ 15

*x* + *y* ≤ 30

*y* > *x*

or equivalent linear inequalities

b) The polygon of constraints



Result : Yes \_\_\_\_\_ No X

22

B

23

The polygon of constraints associated with the system of inequalities is the following :



24



25

B

26

Luke can score a maximum of 152 points.

27

D

28

D

29

D

30

B

31

D

Name : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Group : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Review Optimization**

1

Two industrial robots are used in an auto production plant. There are certain constraints that limit the amount of time these robots can be used.

Polygon of constraints PQR represents this situation.

|  |  |
| --- | --- |
|  |  |
| Coordinates of the vertices of the polygon of constraints |
| P(1, 17) |
| Q(17, 1) |
| R(17, 1) |
|  |

*x*: number of hours Robot A is used, per day

*y*: number of hours Robot B is used, per day

The expression 60*x* + 50*y* is used to calculate the daily profits generated by the use of these robots.

Robot A cannot be used for more than 5 hours today because it must undergo routine maintenance. This additional constraint changes the minimum profit that can be earned.

What is the difference between the usual maximum profit and today’s maximum profit?

Show all your work.

2

To earn money for a trip, Julie buys chocolate bars from a supplier and then resells them for a profit. Polygon of constraints PQR below represents her potential sales figures.



*x*: number of bars of dark chocolate sold

*y*: number of bars of white chocolate sold

The following table shows the profits that Julie could make with each supplier.

|  |  |  |
| --- | --- | --- |
|  | Profit per bar of  dark chocolate sold | Profit per bar of  white chocolate sold |
| Supplier A | $0.50 | $1.50 |
| Supplier B | $0.90 | $1.30 |

Julie must buy all her chocolate bars from the same supplier.

Which supplier should Julie choose in order to maximize her profit?

Show all your work.

3

The polygon of constraints below is associated with a system of inequalities. Points P, Q, R and S are shown on the graph.



Which of these four points represent solutions for this system of inequalities?

4

Polygon of constraints QRS below takes into account the operating constraints of a housekeeping business.



The weekly costs involved in running this business are calculated using the expression 9*x* + 7*y* + 90.

What is the minimum weekly cost of running this business?

5

The system of inequalities below represents the constraints associated with an optimization situation:



Which one of the following polygons of constraints represents this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

6

The constraints associated with an optimization situation are represented by the system of inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

|  |  |
| --- | --- |
| ➀ *y* ≤ 2*x* + 1  ➁ 2*x* + 3*y* ≤ 35  ➂ *y* ≤ -5*x* + 42  ➃ *x* + 2*y* ≥ 12 |  |

What are the coordinates of vertex P of this polygon of constraints?

7

A ferry transports people and vehicles. Polygon of constraints ABC below is based on the constraints the transport company must take into account in order to make each crossing profitable.

|  |  |
| --- | --- |
|  |  |
| Coordinates of the  vertices of the  polygon of constraints |
| A(50, 50)  B(350, 350)  C(450, 50) |
|  |

*x*: number of people aboard for each crossing

*y*: number of vehicles aboard for each crossing

The fare is $4 per person and $10 per vehicle.

To ensure profitability, the company must take into account a new constraint represented by the rule *x* + *y* ≥ 300.

By how much does this new constraint increase the minimum revenue for each crossing?

Show all your work.

8

Vincent works for a company that makes storage racks for compact disks. Each week, he divides his time between assembly work and finishing work.

The polygon of constraints below represents the different constraints that Vincent faces.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Coordinates of the vertices of the polygon of constraints |  |
|  | P(5, 35)  Q(20, 5)  R(15, 5)  S(5, 15) |  |
|  |  |  |

*x*: number of hours spent on assembly work each week

*y*: number of hours spent on finishing work each week

Vincent is told that from now on he faces the following additional constraint: the number of hours spent on finishing work must be less than or equal to the number of hours spent on assembly work.

He makes $10 an hour for assembly work and $8 an hour for finishing work.

By how many dollars does this constraint decrease Vincent's maximum possible weekly income?

Show all your work.

9

The following system of inequalities represents the constraints associated with an optimization situation.



Which one of the following polygons of constraints represents the solutions for this system of inequalities?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

10

The members of a sports club participated in a fundraising campaign. They sold chocolate bars and packets of coffee. They made a profit of $1.50 for each chocolate bar sold and a profit of $2.50 for every packet of coffee sold.

The polygon of constraints below represents the different constraints faced by each member.



*x*: number of chocolate bars sold by each member

*y*: number of packets of coffee sold by each member

Each club member made the maximum possible profit. This campaign enabled them to raise a total of $800.

How many members does this club have?

Show all your work.

11

There are constraints on how many children and how many adults can board a boat to go whitewater rafting. The polygon of constraints below represents the different possible combinations of children and adults the boat can hold.



*x*: number of children in the boat

*y*: number of adults in the boat

To determine the maximum possible revenue that can be earned from one trip, the guide calculated the values indicated in the table below.

|  |  |
| --- | --- |
| Vertex of the polygon of constraints | Revenue |
| O(0, 0) | $0 |
| P(0, 9) | $180 |
| Q(6, 6) | $180 |

In your answer booklet, fill in the appropriate box to indicate how many different combinations of child and adult passengers enable the guide to maximize his revenue for one trip.

12

The system of inequalities and the polygon of constraints below are associated with an optimization situation. Each side of the polygon and its related inequality are identified by the same number.

|  |  |
| --- | --- |
| ➀ *y* ≤ 2*x*  ➁ *x* ≤ 16  ➂ *y* ≥ -2*x* + 36 |  |

What are the coordinates of vertex R of this polygon of constraints?

13

The following system of inequalities represents the constraints associated with an optimization situation.



Which one of the following polygons of constraints could represent this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

14

The solutions for a system of inequalities are represented in the Cartesian plane below. Points P, Q and R are shown on the graph.



Which of points P, Q and R represent solutions for this system of inequalities?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | P and Q only | C) | Q and R only |
| B) | P and R only | D) | P, Q and R |

15

The system of inequalities below represents the constraints of an optimization situation.

|  |  |
| --- | --- |
| *x* ≥ | 0 |
| *y* ≥ | 0 |
| *x* ≤ | 6 |
| 5*x* + 4*y* ≤ | 40 |

Which of the following polygons of constraints represents the solution set of this system of inequalities?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

16

The following three statements refer to optimization situations represented in a Cartesian plane.

• In a polygon of constraints, the coordinates of the vertex closest to the *x*‑axis always minimize the function to be optimized.

• In a polygon of constraints, the coordinates of the vertex closest to the *y*‑axis always minimize the function to be optimized.

• In a polygon of constraints, the coordinates of the vertex closest to the origin of the Cartesian plane always minimize the function to be optimized.

How many of these 3 statements are true?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | None of the statements | C) | Only 2 statements |
| B) | Only 1 statement | D) | All 3 statements |

17

The constraints related to a situation are represented by the following system of inequalities.

|  |  |
| --- | --- |
| *x* ≥ | 0 |
| *y* ≥ | 0 |
| *y* ≥ | -2*x* + 14 |
| *2*y + 2 ≥ | *x* |
| *x + y* ≤ | 14 |

Which of the following polygons of constraints represents this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

18

Jerry is a university student. He rents an apartment for $360 a month. Every month, he spends some of his free time giving computer lessons. He earns $12 for each lesson given to a child and $20 for each lesson given to an adult.

His availability and the demand for lessons create certain constraints, which are represented by the inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

|  |  |
| --- | --- |
| *x* ≥ 0  *y* ≥ 0  ➀ *y* ≥ 2*x*  ➁ *x* ≥ 5  ➂ *x* + *y* ≤ 27 |  |

*x*: the number of lessons given to children per month

*y*: the number of lessons given to adults per month

Can Jerry always be sure that these computer lessons will pay him enough money every month to cover his rent?

Show all your work and explain your final answer.

Show all your work.

19

The student council of a school is organizing a fashion show, which will be held in the school auditorium. The tickets will cost $4 for students at the school and $6 for the general public.

The following polygon of constraints reflects the different constraints faced by the student council.



*x*: number of tickets for students at the school

*y*: number of tickets for the general public

A runway will be built so that the models can walk down the middle of the auditorium. The runway will reduce the seating capacity. As a result, only a maximum of 240 tickets can be sold.

By how much will the maximum possible revenue decrease once the runway is built?

Show all your work.

20

The three statements below concern optimization problems represented by polygons of constraints drawn in the Cartesian plane.

– Any optimization problem has at least one optimal solution.

– The point (0, 0) always represents a solution to the system of inequalities associated with the constraints in a problem.

– The coordinates of the highest point of the polygon of constraints always maximize the value of the function to be optimized.

How many of the above statements are true?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | None of these statements | C) | 2 statements |
| B) | 1 statement | D) | All 3 statements |

21

A town decides to hire some high school and CEGEP students for the summer. The following constraints must be observed :

‑ a minimum of 15 students are needed;

‑ a maximum of 30 students can be hired;

‑ more CEGEP students than high school students must be hired.

Draw the polygon of constraints and verify if the town can hire 10 high school students and 7 CEGEP students.

*x* : number of high school students

*y* : number of CEGEP students

22

To reduce its operating costs, a farming co-operative must respect the following constraints :

|  |  |
| --- | --- |
| *x* ≥ | 0 |
| *y* ≥ | 0 |
| *y* ≤ | 50 |
| *x* ≤ | 50 |
| *x* + *y* ≥ | 30 |
| *x* ≤ | 2*y* |

*x* : number of hectares for growing potatoes

*y* : number of hectares for growing cabbage

Which of the graphs below represents the polygon of constraints obtained from this system of inequalities?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

23

Participants at a 10-day music camp are allowed to devote part of their time to sports activities. However, they must respect the constraints expressed by the following linear inequalities :

|  |  |
| --- | --- |
| *x* ≥  *y* ≥  *x* + *y* ≥  *x* ≤  *x* − *y* ≥  *y* ≥ | 0  0  80  120  50  10 |

*x* : number of hours of music activities during the stay at camp

*y* : number of hours of sports activities during the stay at camp

Graph the polygon of constraints associated with this system of linear inequalities.

24

The recreation department of a certain town is planning to hire secondary and college students as monitors at the playgrounds this summer.

Given *x* : the number of college students who will be hired

*y* : the number of secondary students who will be hired

The person in charge must respect the following constraints :

*x* + *y* ≥ 12

*x* ≥ 2*y*

*x* ≤ 16

*y* ≥ 3

In your Answer Booklet, draw the polygon of constraints representing this situation.

25

The constraints related to a given situation are represented by the following system of inequalities:

|  |  |
| --- | --- |
| *x* ≥ | 0 |
| *y* ≥ | 0 |
| *x* + 2*y* ≥ | 26 |
| *y* ≤ | 2*x* − 7 |

Which one of the following polygons of constraints could represent this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

26

Luke has entered a mathematics competition. His question booklet is divided into two parts. He must choose ***x*** questions from part A and ***y*** questions from part B, but is faced with certain constraints.

The following polygon represents the different possible combinations of questions that Luke can choose.



Each correct answer in part A is worth 6 points, and each correct answer in part B is worth 8 points.

What is the maximum number of points that Luke can score?

27

An optimization situation is represented by the polygon below.

The rule of the function *z* to be optimized is *z* = 6*x* + 3*y* where *x* and *y* are integers.



How many ordered pairs minimize the value of function *z?*

|  |  |  |  |
| --- | --- | --- | --- |
| A) | 1 | C) | 4 |
| B) | 2 | D) | 5 |

28

The constraints for a situation are represented by the following system of inequalities.

|  |  |
| --- | --- |
| *x* | ≥ 0 |
| *y* | ≥ 0 |
| *y* | ≤ 2*x* |
| 5*x* + 12*y* | ≥ 60 |
| *x* | ≤ 8 |

Which polygon of constraints represents this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

29

The following inequalities represent the constraints for an optimization situation.

|  |  |
| --- | --- |
| *x* ≥ | 0 |
| *y* ≥ | 0 |
| 2*x* + *y* ≥ | 16 |
| *x* ≥ | 3*y* |

Which graph represents this situation?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

30

|  |  |
| --- | --- |
| The solutions for a system of inequalities are represented on the right. Points A, B, C, D, E, F and G are shown on the graph. |  |

How many of these points represent a solution for this system of inequalities?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | 1 | C) | 6 |
| B) | 3 | D) | 7 |

31

The following polygon of constraints represents the solution for an optimization situation that involves minimizing the cost of an order of seafood and steaks.

|  |  |
| --- | --- |
| *x*: mass of seafood in kg  *y*: mass of steaks in kg |  |

The values in the following table were calculated in order to determine the minimum cost.

|  |  |
| --- | --- |
| Vertex | Cost |
| A | $64 |
| B | $64 |
| C | $128 |
| D | $184 |

In this situation, how many solutions minimize the cost?

|  |  |  |  |
| --- | --- | --- | --- |
| A) | No solution | C) | 2 solutions |
| B) | 1 solution | D) | More than 2 solutions |