RULE OF RATIONAL FUNCTION AND INVERSE

Goal:

- to find the rule for a rational function

- to find the rule of the inverse of a rational function in standard or general form

Finding the rule for a rational function is similar to other functions. Some parameters must be given and others are solved for.

For rational functions, the asymptotes are typically given.

Ex: A rational function has asymptotes at x=3 and y=-1. The function also passes through the point (4,-0.5). Find the rule that describes this rational function.

$$f(x) = \frac{a}{b(x-h)} + k = \frac{\frac{a}{b}}{x-h} + k$$

if $a_1 = \frac{a}{b}$

$$f(x) = \frac{a_1}{x-h} + k$$

V.A. $x = 3$

H.A. $y = -1$

$$f(x) = \frac{a_1}{x-3} - 1$$

$P(4, -0.5)$

$$-0.5 = \frac{a_1}{4-3} - 1$$

$$0.5 = a_1$$

$$a_1 = \frac{1}{2} \qquad \frac{a}{b} = \frac{1}{2}$$

$$f(x) = \frac{1}{2(x-3)} - 1$$

Also, as with other functions the inverse of a rational function can be determined.

Ex: Determine the inverse of the function $f(x)=\frac{3}{4(x-1)}+2$

$y=\frac{3}{4(x-1)}+2$

$x=\frac{3}{4(y-1)}+2$

$x-2=\frac{3}{4(y-1)}$

$(y-1)(x-2)=\frac{3}{4}$

$y-1=\frac{3}{4(x-2)}$

$y=\frac{3}{4(x-2)}+1$

$f^{-1}(x)=\frac{3}{4(x-2)}+1$

The inverse of the standard form $f(x)=\frac{a}{b(x-h)}+k$

is in general... $f^{-1}(x)=\frac{a}{b(x-k)}+h$

Finally, the inverse of a rational function in general form can be determined. This form requires the most attention and care.

Ex: Find the inverse for the function $f(x)=\frac{2x-1}{x+3}$

$y = \frac{2x-1}{x+3}$

$x = \frac{2y-1}{y+3}$

$x(y+3) = 2y-1$

$xy+3x = 2y-1$

$xy-2y = -3x-1$

$y(x-2) = -3x-1$

$y = \frac{-3x-1}{x-2}$

$f^{-1}(x) = \frac{-3x-1}{x-2}$