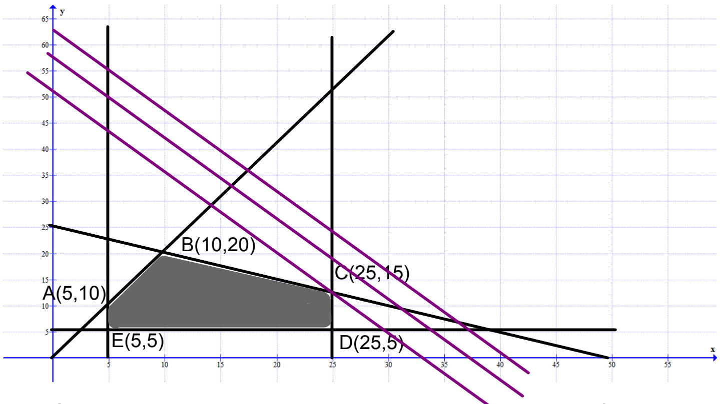
SCANNING LINE AND NON-UNIQUE SOLUTIONS

Goal:

- to use a scanning line to determine the minimum and maximum ordered pairs

- to analyze situations where there is more than one solution

An optimization problem has constraints that produce the polygon of constraints shown below (not to scale):



Once the polygon is drawn and the vertices are found, the optimal solution is found using an objective function.

For example, in this case we want to maximize:

R=6x+4y

To find the ordered pair you must test the points.

Intuitively, we can see it must be B or C:

Point B (10,20) $R=6(10)+4(20)=140$

Point C (25,15) $R=6(25)+4(15)=210$

Alternatively, a scanning line can be used.

This method is helpful if many points must be tested. It is a graphical method.

- Graph any line with the same slope as objective function

$$R=6x+4y$$

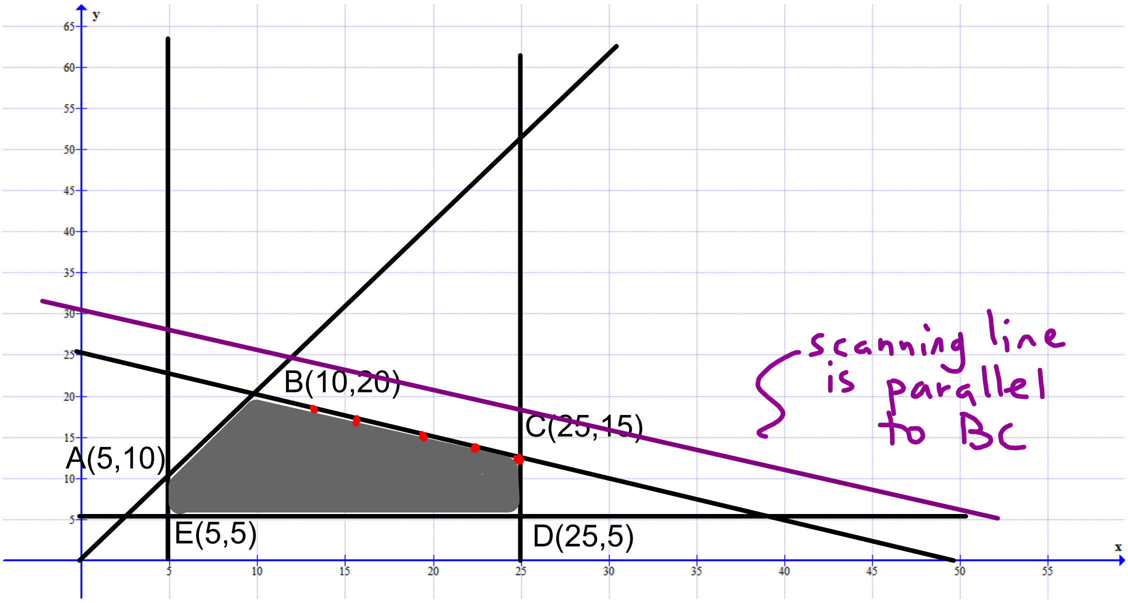
$$4y=-6x+R$$

$$y=-\frac{6}{4}x+\frac{R}{4}$$

$$y=-\frac{3}{2}x+\frac{R}{4}$$

As the scanning line passes through the Polygon of constraints it will touch the Vertices that are min/max first/last.

Here is the same polygon of constraints:



This time the objective function is

R=2x+6y $\Rightarrow$ slope $= -\frac{1}{3}$

Now, which ordered pair produces a maximum?

Point B $R=2(10)+6(20)=140$

Point C $R=2(25)+6(15)=140$

This means the entire line segment $\overline{BC}$ produces max solutions.

* For this example there are 6 integer ordered pairs.