student book volume 1

Science

VISIONS

MATHEMATICS

Secondary
Cycle Two, Year Three

ANSWER KEY

Vision 1

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PRELIMINARY VERSION

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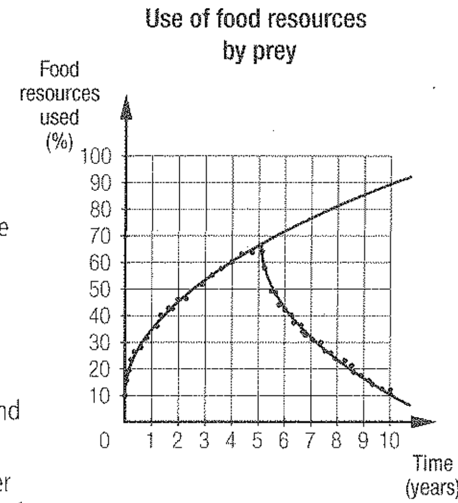
The following is an example of an approach that will help produce the expected document, as well as a sample of the document.

A) Approach

- Determine the rule of the function that represents the evolution of food resources as a function of time.

  The scatter plot shows the trend of an increasing square root function over [0, 5], and the trend of a decreasing square root function over [5, 10]. The adjacent graph shows the curves that best fits these points.



  By substituting the coordinates of the vertex and a point belonging to the curve for each curve, the following rules are found:

$$r(t) = \begin{cases} 25\sqrt{t} + 10 & \text{if } t \in [0, 5] \\ -25\sqrt{t-5} + 66 & \text{if } t \in \,]5, 10] \end{cases}$$

  These rules are valid for a predator-prey cycle and the graphical representations of these functions repeat themselves during each cycle and undergo a translation of 10 units to the right each time.

- Determine the rule of the function that states the evolution of the number of prey as a function of time. Since the relationship between the number of prey and food resources is known, as well as the relationship between the quantity of food resources and time, one can deduce that function p, which describes the evolution of the number of prey as a function of time, corresponds to the composition of functions $f(r(t))$.

$$p: f(r(t)) = \begin{cases} -\frac{8}{5}\sqrt{25\sqrt{t} + 10} + 20 & \text{if } t \in [0, 5] \\ -\frac{8}{5}\sqrt{-25\sqrt{t-5} + 67} + 20 & \text{if } t \in \,]5, 10] \end{cases}$$

  These rules are valid for a predator-prey cycle and the graphical representations of these functions repeat themselves during each cycle and undergo a translation of 10 units to the right each time.

- Determine the rule of the function that states the evolution of the number of predator as a function of time. According to the graph provided, the function that describes the evolution of the number of predator based on the number of prey is a square root function.

  By using the coordinates of the vertex and a point belonging to the curve, the following rule is found:

$$g(x) = \frac{4\sqrt{7}}{7}\sqrt{x-5} + 3$$

  Since the relationship between the number of predator and prey is known, as well as the relationship between the number of prey and time, it can be deduced that function P which describes the evolution of the number of predator in function of time corresponds to the composition of functions $g(p(t))$.

$$P: g(p(t)) = \begin{cases} \frac{4\sqrt{7}}{7}\sqrt{-\frac{8}{5}\sqrt{25\sqrt{t} + 10} + 15} + 3 & \text{if } t \in [0, 5] \\ \frac{4\sqrt{7}}{7}\sqrt{-\frac{8}{5}\sqrt{-25\sqrt{t-5} + 67} + 15} + 3 & \text{if } t \in \,]5, 10] \end{cases}$$

  These rules are valid for a predator-prey cycle and the graphical representations of these functions repeat themselves during each cycle and undergo a translation of 10 units to the right each time.

B) The document intended for the ecologist

The graphs on the following page help in visualizing the evolution of food resources used, number of prey and number of predator present in the ecosystem over a period of 50 years. It is possible to estimate these quantities by reading the graph.

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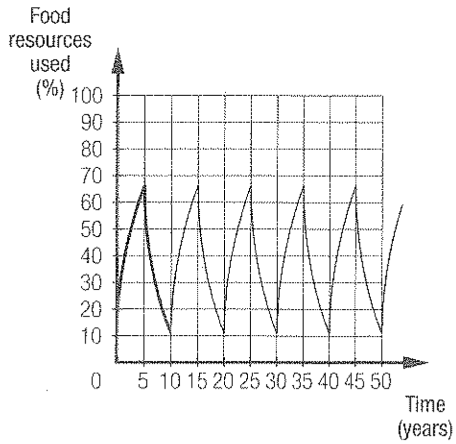
To obtain a more precise estimation, one must determine the moment of the cycle desired for making an estimate and refer to the table of values below representing the predator-prey cycle.

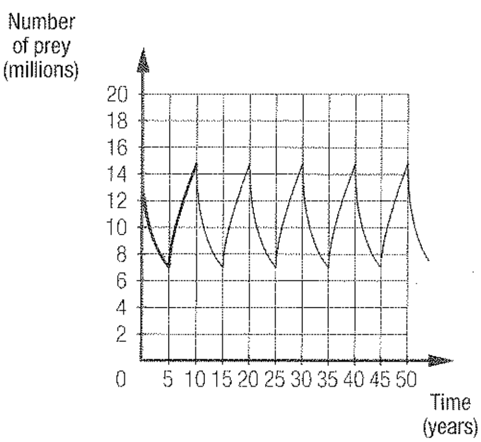
Evolution of food resources used by prey



Time (years)	% of used resources
0	10
0.5	27.7
1	35.0
1.5	40.6
2	45.3
2.5	49.5
3	53.3
3.5	56.8
4	60.0
4.5	63.0
5	65.9

Time (years)	% of used resources
5.5	49.3
6	42.0
6.5	36.4
7	31.6
7.5	27.4
8	23.7
8.5	20.2
9	17.0
9.5	14.0
10	11.0

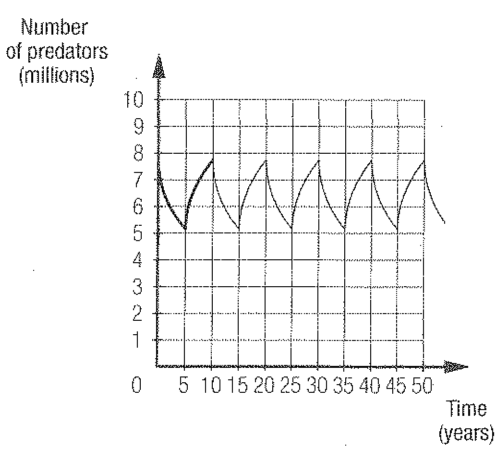
Evolution of number of prey



Time (years)	Number of prey (millions)
0	14.9
0.5	11.6
1	10.5
1.5	9.8
2	9.2
2.5	8.7
3	8.3
3.5	7.9
4	7.6
4.5	7.3
5	7.0

Time (years)	Number of prey (millions)
5.5	8.7
6	9.6
6.5	10.3
7	11.0
7.5	11.6
8	12.2
8.5	12.8
9	13.4
9.5	14.0
10	14.7

Evolution of number of predators



Time (months)	Number of predators (millions)
0	7.8
0.5	6.9
1	6.6
1.5	6.3
2	6.1
2.5	5.9
3	5.7
3.5	5.6
4	5.4
4.5	5.3
5	5.1

Time (months)	Number of predators (millions)
5.5	5.9
6	6.2
6.5	6.5
7	6.7
7.5	6.9
8	7.1
8.5	7.2
9	7.4
9.5	7.5
10	7.7

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The following is an example of an approach that will help determine when the capacity to store energy (H) is at its maximum.

- Determine the rule of the function that calculates the quantity of energy (S).
  The function associated with this energy is a rational function whose horizontal asymptote is $y = 80$ and passes through points (0, 0) and (16, 64.85).
  $S = \frac{a}{x - h} + 80$
  By using the coordinates of both points, the following two equations are obtained:
  ① $0 = -\frac{a}{h} + 80 \Rightarrow a = 80h$
  ② $64.85 = \frac{a}{16 - h} + 80$
  It is possible to solve the equation obtained by substituting 80h by parameter **a** in Equation ②.
  $$64.85 = \frac{80h}{16 - h} + 80$$
  $$h \approx -3.74$$
  $$\text{and } a \approx 80 \times -3.74 \approx -299.2$$
  The rule of the function is therefore $S = \frac{-299.2}{x + 3.74} + 80$
- Determine the rule of the function that calculates the quantity of energy (R).
  The function associated with this energy is a square root function whose vertex is (0, 0) and passes through point (36, 48). By substituting the coordinates of the vertex and point into the standard equation of a square root function, the following is obtained:
  $$R = a\sqrt{x - h} + k$$
  $$48 = a\sqrt{36}$$
  $$8 = a$$
  The rule of the square root function is therefore $R = 8\sqrt{x}$
- Determine the rule of the function that calculates the quantity of energy (C).
  The function associated with this energy is the absolute value function whose vertex is (25, 15) and passes through point (8, 4.8). By substituting the coordinates of the vertex and point into the standard equation of an absolute function, the following is obtained:
  $$C = a|x - h| + k$$
  $$4.8 = a|8 - 25| + 15$$
  $$-0.6 = a$$
  The rule of the absolute value function is therefore $C = -0.6|x - 25| + 15$

- Represent the evolution of energy (H).

Since $S = R + C + H$, it can be deduced that the quantity of energy H is constantly given by $H = S - (R + C)$. The following table of values can also be created.

Time (weeks)	Quantity of energy (% of solar energy)			
	S	R	C	**H**
0	0.00	0.00	0.00	**0**
1	16.88	8.00	0.60	**8.28**
2	27.87	11.31	1.20	**15.36**
3	35.61	13.86	1.80	**19.95**
4	41.34	16.00	2.40	**22.94**
5	45.77	17.89	3.00	**24.88**
6	49.28	19.6	3.60	**26.08**
7	52.14	21.17	4.20	**26.77**
8	54.51	22.63	4.80	**27.08**
9	56.51	24.00	5.40	**27.11**
10	58.22	25.30	6.00	**26.92**
11	59.7	26.53	6.60	**26.57**
12	60.99	27.71	7.20	**26.08**
13	62.13	28.84	7.80	**25.49**
14	63.13	29.93	8.40	**24.8**
15	64.03	30.98	9.00	**24.05**
16	64.84	32.00	9.60	**23.24**
17	65.57	32.98	10.20	**22.39**
18	66.24	33.94	10.80	**21.5**
19	66.84	34.87	11.40	**20.57**
20	67.4	35.78	12.00	**19.62**
21	67.91	36.66	12.60	**18.65**
22	68.38	37.52	13.20	**17.66**
23	68.81	38.37	13.80	**16.64**
24	69.21	39.19	14.40	**15.62**
25	69.59	40.00	15.00	**14.59**

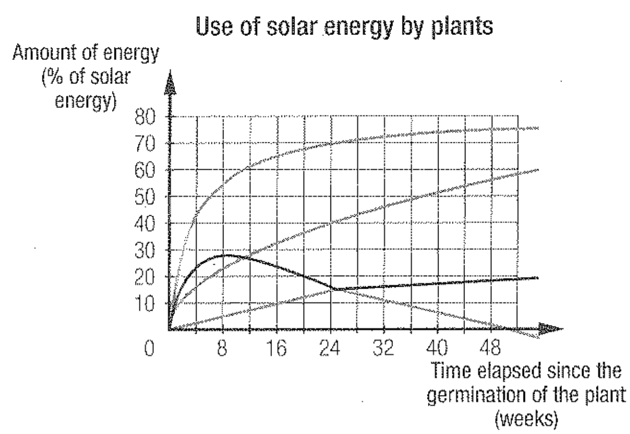
Time (weeks)	Quantity of energy (% of solar energy)			
	S	R	C	**H**
26	69.94	40.79	14.40	**14.75**
27	70.27	41.57	13.80	**14.9**
28	70.57	42.33	13.20	**15.04**
29	70.86	43.08	12.60	**15.18**
30	71.13	43.82	12.00	**15.31**
31	71.39	44.54	11.40	**15.45**
32	71.63	45.25	10.80	**15.58**
33	71.86	45.96	10.20	**15.7**
34	72.07	46.65	9.60	**15.82**
35	72.28	47.33	9.00	**15.95**
36	72.47	48.00	8.40	**16.07**
37	72.66	48.66	7.80	**16.2**
38	72.83	49.32	7.20	**16.31**
39	73.00	49.96	6.60	**16.44**
40	73.16	50.60	6.00	**16.56**
41	73.31	51.22	5.40	**16.69**
42	73.46	51.85	4.80	**16.81**
43	73.60	52.46	4.20	**16.94**
44	73.73	53.07	3.60	**17.06**
45	73.86	53.67	3.00	**17.19**
46	73.98	54.26	2.40	**17.32**
47	74.10	54.85	1.80	**17.45**
48	74.22	55.43	1.20	**17.59**
49	74.33	56.00	0.60	**17.73**
50	74.43	56.57	0.00	**17.86**

You can also establish the rule of the function associated to energy H and represent it graphically by using a graphing tool or a scatter plot. The following equations are found:

$$H = \frac{^-299.2}{x + 3.74} + 80 - (8\sqrt{x} + {^-0.6}|x - 25| + 15) \text{ or}$$

$$H = \frac{^-299.2}{x + 3.74} - 8\sqrt{x} + 0.6|x - 25| + 60$$

The graphical representation of the function associated with energy (H) is the following:



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By analyzing the table of values, it can be established that the maximum point associated to energy (H) is situated between 8 and 10 weeks after germination. The graphical representation of the function confirms this interval. To obtain even more accuracy, one can create a table of values representing the evolution of energy between 8 to 10 weeks in which the increments of the scale are 0.1.

Time (weeks)	Quantity of energy (% of solar energy)			
	S	*R*	*C*	**H**
8	54.51	22.63	4.80	**27.08**
8.1	54.73	22.77	4.86	**27.10**
8.2	54.94	22.91	4.92	**27.11**
8.3	55.15	23.05	4.98	**27.12**
8.4	55.35	23.19	5.04	**27.12**
8.5	55.56	23.32	5.10	**27.14**
8.6	55.75	23.46	5.16	**27.13**
8.7	55.95	23.60	5.22	**27.13**
8.8	56.14	23.73	5.28	**27.13**
8.9	56.33	23.87	5.34	**27.12**
9	56.51	24.00	5.40	**27.11**

Time (weeks)	Quantity of energy (% of solar energy)			
	S	*R*	*C*	**H**
9	56.51	24.00	5.40	**27.11**
9.1	56.70	24.13	5.46	**27.11**
9.2	56.88	24.27	5.52	**27.09**
9.3	57.06	24.40	5.58	**27.08**
9.4	57.23	24.53	5.64	**27.06**
9.5	57.40	24.66	5.70	**27.04**
9.6	57.57	24.79	5.76	**27.02**
9.7	57.74	24.92	5.82	**27.00**
9.8	57.90	25.04	5.88	**26.98**
9.9	58.06	25.17	5.94	**26.95**
10	58.22	25.30	6.00	**26.92**

The capacity of the storage of energy (*H*) is highest at 8.5 weeks, in other words midway through the 9th week of life. At that moment, in order to feed herbivores, plants absorb 27.14% of the sun's rays.

REVISION 1

Prior learning 1

Page 4

a. Graph for 1998: domain [0, 26]; range [0, 22].
Graph for 2008: domain [0, 26]; range [2, 30].

b. The zeros of the function are 16 and 26. They correspond to the moment the hole in the ozone layer was completely closed.

c. The initial value is 6×10^6 km^2 and it corresponds to the area of the hole in the ozone layer at the start of the observations.

d.

Period	Maximum area of the hole	Minimum area of the hole	Periods during which the area of the hole increases	Periods during which the area of the hole decreases
Summer and fall 1998	**22×10^6 km^2**	**0 km^2**	**[0, 8] and [16, 22]**	**[8, 16] and [22, 26]**
Summer and fall 2088	**30×10^6 km^2**	**2×10^6 km^2**	**[0, 1] and [2, 7]**	**[1, 2] and [7, 26]**

e. No.
The maximum area of the hole was greater in 2008 than in 1998.
During the reported period in 2008, the hole in the ozone layer was never completely closed.
Since at the end of the observations in 2008 the hole was not closed, it can be deduced that the period during which there was a hole in the ozone layer was longer in 2008 than in 1998.

Prior learning 2

Page 5

a. A second-degree polynomial function.

b. No, because an *x*-coordinate is associated with more than one *y*-coordinate for the inverse of this function.

c. The function is positive over intervals [0, 1.1] and [4.4, +∞[. These intervals represent the periods during which the bird was above the water level. The function is negative over interval [1.1, 4.4]. This interval represents the period during which the bird was underwater.

d. Approximately 3 m.

Knowledge in action

Page 9

1. a) 1) 1800 people.

2) 500 people.

b) At 1 h, 3 h, 10 h and 14 h after the opening.

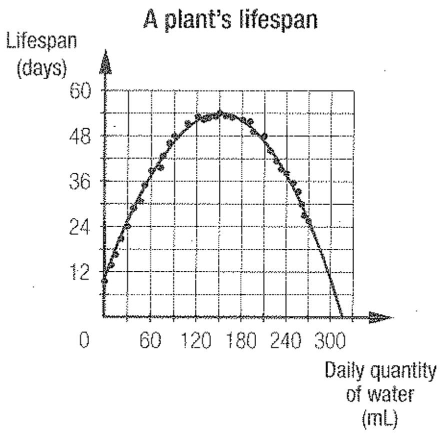
c) Between 1 h and 3 h and between 10 h and 14 h after the opening.

d) 1) Increasing.

2) Decreasing between 2 h and 5 h and between 6 h and 9 h after the opening. Increasing between 5 h and 6 h after the opening.

2. a) A quadratic function.

b)



c) 1) Approximately 233.67 mL or 66.33 mL of water.

2) 150 mL of water.

Knowledge in action (*cont'd*)

Page 10

3. a) The domain is [0, 4] s and includes the moments used to define the diving stage. The range is [-1, 2] m and corresponds to the set of possible heights of the diver's hands during the diving stage.

b) The minimum is -1.5 m and the maximum is 2 m. These extremas correspond respectively to the minimum height and maximum height of the diver's hands during this stage.

c) The initial value is 0.5 m and corresponds to the height of the diver's hands at the very beginning of the diving stage.

d) The zeros are 1.25 s and 4 s and correspond to the moments when the diver's hands touch the surface of the water.

e) The function is increasing over intervals [0, 0.5] s and [2, 4] s. These intervals correspond to the periods where the height of the hands increases. The function is decreasing over interval [0.5, 2] s. This interval corresponds to the moment when the height of the hands decreases.

f) The function is positive over interval [0, 1.25] s and negative over interval [1.25, 4] s. These intervals correspond to the moments when the diver's hands are respectively above and under the water.

4. a) 1) Function f: Increasing:]-∞, 1]; decreasing: [1, +∞[.
Function g: Increasing: ℝ
Function h: Increasing: ℝ
Function i: Increasing: [-4, -2] ∪ [2, 4]; decreasing: [-2, 2]

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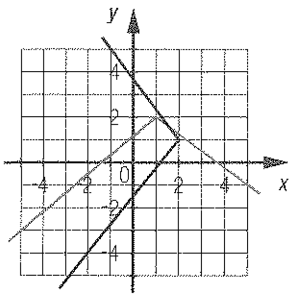
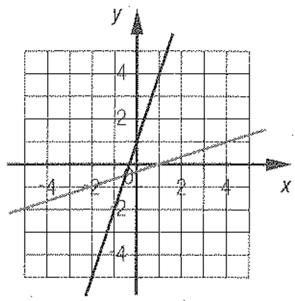
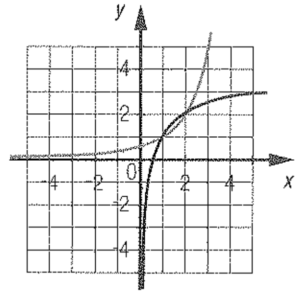
2) Function f



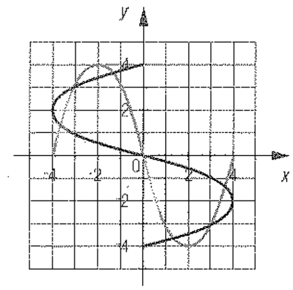
Function g



Function h



Function i



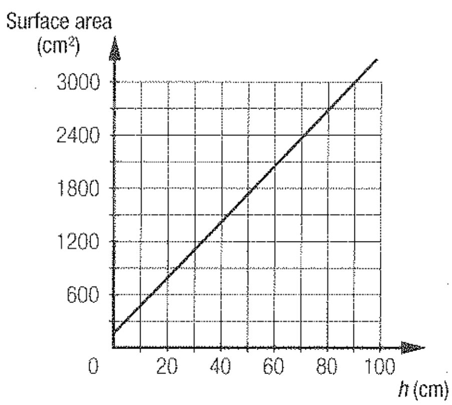
3) Function f: No.
Function g: Yes.
Function h: Yes.
Function i: No.

b) The inverse of the function will be a function only if there is one type of variation in the initial function.

Knowledge in action (*cont'd*)

5. a) 1) $f(h) = 10\pi h + 50\pi$

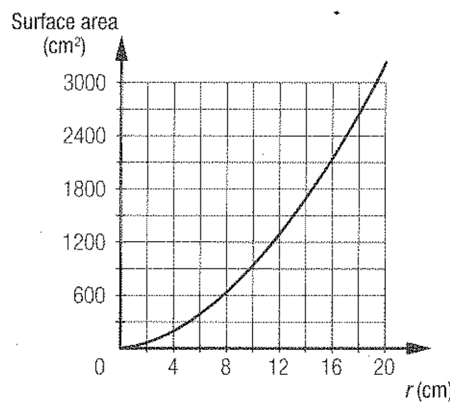
2)



3) First-degree polynomial function.

b) 1) $f(r) = 2\pi r^2 + 10\pi r$

2)



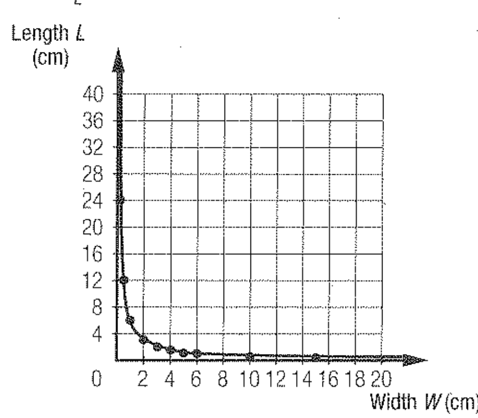
3) Second-degree polynomial function.

6. a)

Length L (cm)	Width W (cm)
0.25	**24**
0.5	**12**
1	**6**
2	**3**
3	**2**
4	**1.5**
5	**1.2**
6	**1**
10	**0.6**
15	**0.4**

b) $W = \frac{6}{L}$

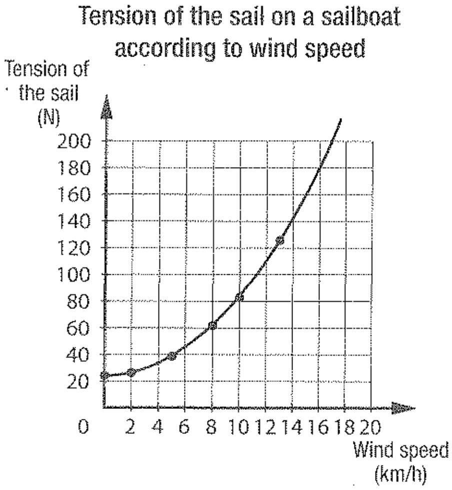
c)



d) An exponential function.

e) 1) Towards 0.
2) Towards infinity.

7. a) 1) and 2)

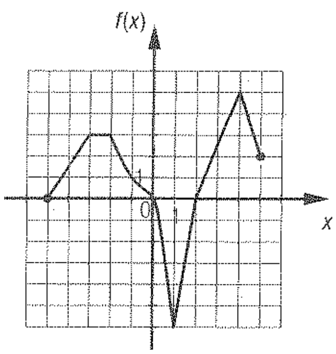


b) A second-degree polynomial function.

c) It represents the initial tension of the sail when there is no wind.

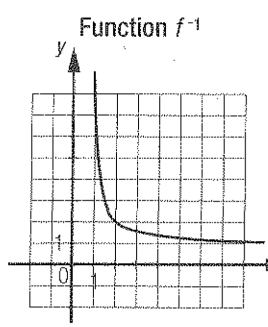
d) Approximately 17.3 km/h.

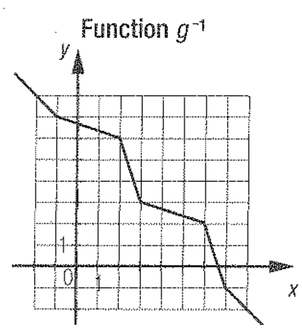
8. *Several possible answers. For example:*



9. a) Both graphical representations are symmetric in relation to the bisector.

b)





In each example, the graphical representation of the function is identical to the graphical representation of the inverse.

c) The curve of the inverse of a function whose graphical representation is symmetric in relation to the bisector of the 1st and 3rd quadrant is congruent to the curve of the function itself.

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Problem

Page 13

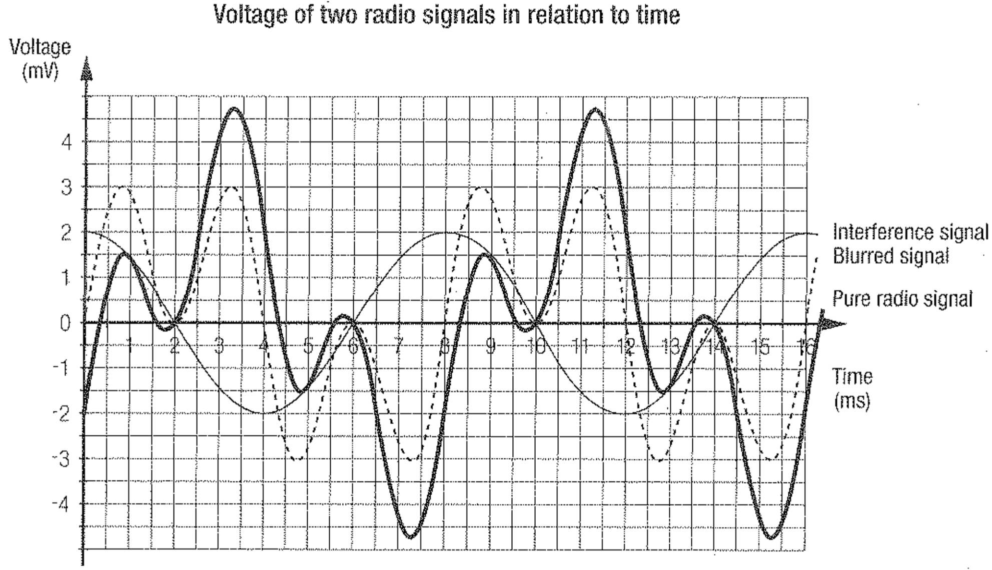
The voltage associated with a pure radio signal at a given moment is obtained by subtracting, at that moment, the voltage of the interference signal from that of the voltage of the blurred signal.

Time (ms)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Voltage of interference signal (mV)	2	1.85	1.41	0.77	0	-0.77	-1.41	-1.85	-2	-1.85	-1.41
Voltage of blurred signal (mV)	0	2.62	2.83	1.08	0	1.08	2.83	2.62	0	-2.62	-2.83
Voltage of pure radio signal (mV)	-2	0.77	1.42	0.31	0	1.85	4.24	4.47	2	-0.77	-1.42

Time (ms)	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5	11
Voltage of interference signal (mV)	-0.77	0	0.77	1.41	1.85	2	1.85	1.41	0.77	0	-0.77	-1.41
Voltage of blurred signal (mV)	-1.08	0	-1.08	-2.83	-2.62	0	2.62	2.83	1.08	0	1.08	2.83
Voltage of pure radio signal (mV)	-0.31	0	-1.85	-4.24	-4.47	-2	0.77	1.42	0.31	0	1.85	4.24

By placing the points on the graph and joining them by following the trend, you obtain the curve shown below in bold.

Voltage of two radio signals in relation to time



Activity 1

Page 14

a. 1) The dependent variable was multiplied by 2.
2) The dependent variable was divided by 2 (multiplied by 0.5).

b. 1) The curve was vertically stretched with a scale factor of 2 (the extremas are two times further from the *x*-axis).
2) The curve was vertically compressed with a scale factor of $\frac{1}{2}$ (the extremas are two times closer to the *x*-axis).

c. When multiplying an expression associated to the dependent variable of a function by a real number, the curve associated with the function undergoes a vertical stretch if this number is greater than 1 or a vertical compression if this number is included between 0 and 1.

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d. 1) The independent variable was multiplied by 2.
2) The independent variable was divided by 2 (multiplied by 0.5).

e. 1) The curve was horizontally compressed with a scale factor of 2 (the zeros are two times closer to the origin).
2) The curve was horizontally stretched with a scale factor of $\frac{1}{2}$ (the zeros are two times further from the origin).

f. When multiplying the real number by the independent variable of a function, the curve associated with the function undergoes a horizontal compression if this number is greater than 1 or a horizontal stretch if this number is between 0 and 1.

g. 1) The dependent variable was multiplied by -1.
2) The independent variable was multiplied by -1.

h. 1) The curve underwent a reflection over the x-axis.
2) The curve underwent a reflection over the y-axis.

i. When multiplying the expression associated to the dependent variable of a function by -1, the curve associated with the function undergoes a reflection over the x-axis. When multiplying the independent variable of a function by -1, the curve associated with the function undergoes a reflection over the y-axis.

Activity 1 *(cont'd)*

Page 15

j. 1) 3 units were added to the independent variable.
2) 2 units were subtracted from the independent variable.

k. 1) The curve underwent a translation of 3 units to the left.
2) The curve underwent a translation of 2 units to the right.

l. When adding a real number to the independent variable of a function, the curve associated with the function undergoes a translation to the left if this number is positive and to the right if this number is negative.

m. 1) 3 units were added to the expression that corresponds to the dependent variable.
2) 1 unit was subtracted from the expression that corresponds to the dependent variable.

n. 1) The curve underwent a translation of 3 units upward.
2) The curve underwent a translation of 1 units downward.

o. When adding a real number to the expression that corresponds to the dependent variable of a function, the curve associated with the function undergoes an upward translation if this number is positive and a downward translation if this number is negative.

Activity 2

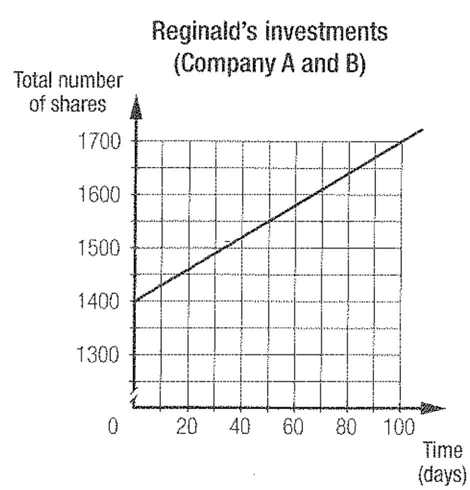
Page 16

a.

Reginald's investments (Company A and B)

Time (days)	Number of shares in Company A	Number of shares in Company B	Total number of shares
0	400	1000	1400
25	300	1175	1475
50	200	1350	1550
75	100	1525	1625
100	0	1700	1700

b. 1)



2) Total number of shares: $1400 + 3t$

c. $(400 - 4t) + (1000 + 7t) = (1000 + 400) + (7t - 4t)$
$= 1400 + 3t$

d. The rules are identical.

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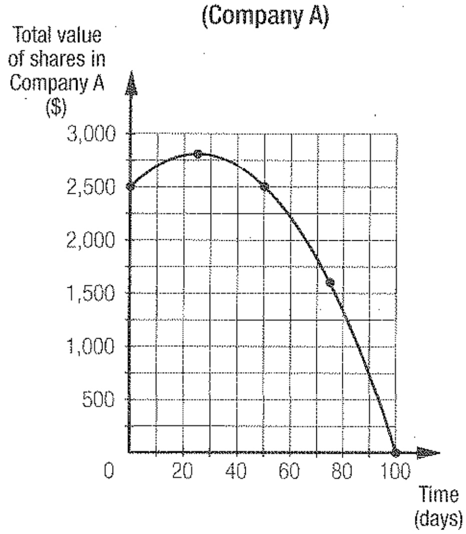
e. **Reginald's investments (Company A)**

Time (jours)	Number of shares in Company A	Value of a share in Company A ($)	Total value of shares in Company A ($)
0	400	6.25	2,500.00
25	300	9.375	2,812.50
50	200	12.50	2,500.00
75	100	15.625	1,562.50
100	0	18.75	0.00

f. **1)** **Reginald's investments (Company A)**



2) The rule is a quadratic function found in the form of $y = a(x - h)^2 + k$ whose vertex is (25, 2812.5). By replacing **h** and **k** by the coordinates of the vertex and by substituting the coordinates of point (0, 2500) into variables x and y, it is clear that parameter **a** equals -0.5. The rule is therefore:
Total value of the shares in Company **A**: $-0.5(t - 25)^2 + 2812.5$; or
Total value of the shares in Company **A**: $-0.5t^2 + 25t + 2500$

g. $(400 - 4t)(0.125t + 6.25) = 50t - 0,5t^2 + 2500 - 25t$
$= -0.5t^2 + 25t + 2500$

h. They are identical.

i. **1)** A first-degree polynomial function. **2)** A piecewise function.

j. $n = 50[0.028(t + 18)]$

k. 130 shares.

Activity 3

a. **1)** The height. **2)** The mass.

b. $m = \frac{3}{4}t - 40$

$\frac{4}{3}(m + 40) = t$

c. **1)** The rule of Situation ②. **2)** The rule of Situation ①.

d. $m = \frac{3}{4}t - 62.5$

$\frac{4}{3}(m + 62.5) = t$

The rule is $t = \frac{4}{3}(m + 62.5)$.

Technomath

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a. A second-degree polynomial function.

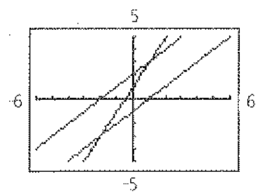
b. A rational function.

c. 1) The x-coordinates can be obtained by multiplying the y-coordinates associated with Y1 and Y2.
2) The x-coordinates can be obtained by dividing the y-coordinates associated with Y1 by the y-coordinates associated with Y2.

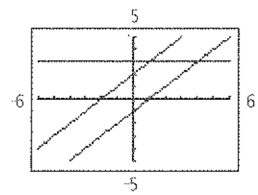
d. A zero of either functions associated with Y1 and Y2 generates a zero for the function associated with Y3.

e. A division by 0 is undefined and cannot be calculated using a calculator.

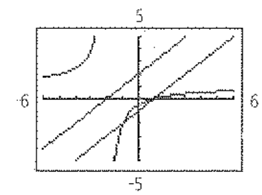
f. 1)



2)



3)



Practice 1.1

Page 24

1. Parameters **a**, **b**, **h** and **k** respectively equal:

a) 3, 1, 2 and 4
b) -2, 4, 0 and -5
c) -1, 0.5, -1 and 6
d) 7, 1, 9 and 11
e) 0,7, -2, -4 and 0
f) -2, -1, -13 and -1
g) 0.5, -3, 2.5 and 0
h) 3, 1, 9 and 6
i) 1, 3, -2 and 0 or $\sqrt{3}$, 1, -2 and 0

2. 1 C, 2 B, 3 D, 4 A

Practice 1.1 *(cont'd)*

Page 25

3. a) $g(x) = 4 \sin (2x)$
b) $g(x) = \sqrt{x + 3} + 2$
c) $g(x) = -|x - 3| + 3$
d) $g(x) = \cos (x - 90) - 0.5$
e) $g(x) = 0.5(x - 5)^2 - 4$
f) $g(x) = -1.5^{-x}$

Practice 1.1 *(cont'd)*

Page 26

4. a) 19
b) 54
c) 4.5
d) $\frac{8}{3}$
e) 79
f) 119
g) 2
h) The answer is not a real number.

5. a) Reflection over the x-axis.
Vertical stretch with a scale factor of 3.
Horizontal stretch with a scale factor of 4.
Translation of 2 units to the left, a translation of 5 units downward.

b) Reflection over the x-axis.
Reflection over the y-axis.
Vertical compression with a scale factor of $\frac{2}{5}$.
Horizontal stretch with a scale factor of 5.
Translation of 6 units upward.

c) Reflection over the y-axis.
Vertical compression with a scale factor of $\frac{1}{3}$.
Translation of 0.5 units to the right, a translation of 2.4 units upward.

d) Vertical compression with a scale factor of 0.5.
Horizontal compression with a scale factor of 0.5.
Translation of 3 units to the right and 4 units upward.

6. 1 D, 2 B, 3 A, 4 C

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Practice *(cont'd)*

7. **a)** $f^{-1}(x) = \frac{x - 5}{3}$ **b)** $h^{-1}(x) = \frac{2}{x - 7}$ **c)** $i^{-1}(x) = -\frac{x^2 - 9}{2}$

8. **a)** (3, 1), (6, 5), (8, 0), (-5, 10), (-12, 22) **b)** (0, 0), (1.5 16), (2.5, -4), (-4, 36), (-7.5, 84)

c) (-6, -4), (0, -16), (4, -1), (-22, -31), (-36, -67) **d)** (0.5, 2.3), (-2.5, 3.9), (-4.5, 2.7), (8.5, 5.9), (15.5, 10.7)

e) (6, 11), (2, 10.2), $\left(-\frac{2}{3}, 16\right)$, $\left(\frac{50}{3}, -34\right)$, (26, -94) **f)** $\left(\frac{4}{7}, -\frac{11}{7}\right)$, $\left(-\frac{11}{7}, \frac{1709}{35}\right)$, $\left(-3, -\frac{496}{35}\right)$, $\left(\frac{44}{7}, \frac{3914}{35}\right)$, $\left(\frac{79}{7}, \frac{9206}{35}\right)$

9. **a)** $g(x) = -3 \sin x$ **b)** $h(x) = \sin 2x$ **c)** $i(x) = -\sin x + 2$

10. **a)** **1)** $3x^2 - 2x + 3$ **2)** $3x^2 - 2x + 3$ **3)** $3x^2 - 1.5x + 3$
4) $3x^2 - 1.5x + 3$ **5)** $-6x^3 - 3x^2 - 8x - 4$ **6)** $-6x^3 - 3x^2 - 8x - 4$
7) $-3x^4 - 1.5x^3 - 4x^2 - 2x$ **8)** $-3x^4 - 1.5x^3 - 4x^2 - 2x$ **9)** $12x^2 + 12x + 7$ **10)** $-6x^2 - 9$

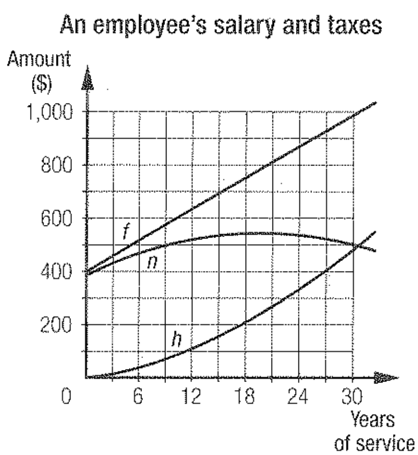
b) **1)** Yes, because the rules in **a) 1)** and **a) 2)** are identical.
2) Yes, because the rules in **a) 3)** and **a) 4)** are identical.
3) Yes, because the rules in **a) 5)** and **a) 6)** are identical.
4) Yes, because the rules in **a) 7)** and **a) 8)** are identical.
5) Yes, because the rules in **a) 9)** and **a) 10)** are different.

Practice 1.1 *(cont'd)*

11. **a)** $h(t) = 0.001(20t + 100)^2$; $n(t) = -0.4t^2 + 16t + 390$

b) **1)** Function *h* calculates the amount of monthly taxes to pay based on the employee's number of years of service.
2) Function *n* calculates the net salary of this employee based on the number of years of service.

c)



d) No, because the curve associated with the growth of the employee's net salary is a parabola facing downward whose vertex is at \$550. The maximum net salary that this employee can expect to receive is therefore \$550.

12. 1 C, 2 A, 3 B, 4 D

Practice 1.1 *(cont'd)*

13. **a)** **1)** The parameter **a**.
2) Since the curve is vertically stretched, the absolute value of parameter **a** has increased. Since **a** was negative from the start, this indicates that the value of parameter **a** has decreased.

b) **1)** The parameter **a**.
2) Since the curve is vertically compressed, the absolute value of parameter **a** has decreased. Since **a** was negative from the start, this indicates that the value of parameter **a** has increased.

c) Parameter **k**, as well as parameter **a**.

d) **1)** The parameter **h**.
2) The value of parameter **h** has increased because the curve underwent a translation to the right.

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SECTION 1.2 | Square root functions

Problem

Page 30

The curve of this model passes through points (3, 0), (4, 50) and (12, 150). The curve associated with the inverse of this representation would be a quadratic function passing through points (0, 3) (the initial value), (50, 4) and (150, 12), and the equation of the inverse would be in the form:

$i = af^2 + bf + c$, where f represents the electric impulse frequency and i represents the electrical intensity, and where c equals 3.

By substituting the coordinates of points (50, 4) and (150, 12) by variables f and i, the following system is obtained:

$4 = a(50)^2 + b(50) + 3$

$12 = a(150)^2 + b(150) + 3$

By solving this system, the following is obtained: a = 0.0004 and b = 0.

The rule of the quadratic function is therefore $i = 0.0004f^2 + 3$.

The rule of the inverse is $f = \sqrt{\dfrac{i - 3}{0.0004}}$.

By substituting the variable i by 30 in this rule, the following is obtained:

$f_{max} = \sqrt{\dfrac{30 - 3}{0.0004}} = \sqrt{67\,500} \approx 259.81$ Hz.

The maximum electric impulse frequencies emitted by this neuron is approximately 259.8 Hz.

Activity 1

Page 31

a. 1) i) $\sqrt[3]{5^2}$ ii) $\sqrt[9]{7^5}$ iii) $\sqrt[4]{11^3}$ iv) $\sqrt[n]{a^m}$

2) The even root of a negative radicand is not a real number.

b. 1) i) $\sqrt{18}$ ii) $\sqrt{143}$ iii) $\sqrt{3}$ iv) $\sqrt{ab}$

2) The law of exponents associated with this reasoning only applies if the exponents affected have bases that are the same. The property only applies in this case if the radicals are the same.

Activity 1 *(cont'd)*

Page 32

c. 1) i) $\sqrt{5}$ ii) $\sqrt{5}$ iii) $\sqrt{5}$ iv) $\sqrt{\dfrac{a}{b}}$

2) The law of exponents associated with this reasoning only applies if the exponents affected have bases that are the same. The property only applies in this case if the radicals are the same.

d. 1) It has been multiplied by a unit fraction. The unit is the neutral element of the multiplication.

2) i) $\dfrac{11\sqrt{3}}{3}$ ii) $\dfrac{\sqrt{2}}{2}$ iii) $\dfrac{5\sqrt{7}}{7}$ iv) $\dfrac{a\sqrt{b}}{b}$

e. 1) It has been multiplied by a unit fraction. The unit is the neutral element of the multiplication.

2) Finding the product of the sum and the difference of two of the same terms gives the difference of the squares of these terms. The square of the square root of a number is equal to this number.

3) i) $\dfrac{\sqrt{12} - \sqrt{7}}{5}$ ii) $\sqrt{3} + \sqrt{2}$ iii) $\dfrac{\sqrt{26} - \sqrt{32}}{-6}$ or $\dfrac{\sqrt{32} - \sqrt{26}}{6}$

iv) $\dfrac{\sqrt{11} + \sqrt{5}}{6}$ v) $\dfrac{\sqrt{a} - \sqrt{b}}{a - b}$ vi) $\dfrac{\sqrt{a} + \sqrt{b}}{a - b}$

Activity 2

Page 33

a. $m = \dfrac{1}{11\,025}f^2$

b. 1) 1 g/m 2) $\approx$ 22.68 g/m

c. $f = \sqrt{11\,025m}$ or $105\sqrt{m}$

d. 1) 157.5 Hz 2) $\approx$ 565.44 Hz

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Activity 3

a. Solving the equation helps to determine when the gondola is at a height of 8 m.

b. **1)** 28 units is subtracted from each side of the equation. Each side of the equation is then divided by -10.
2) 4
3) $t - 0.5 = 4$
4) $t = 4.5$. At 4.5 s, the gondola is at a height of 8 m.

c. Solving the inequality helps to determine the interval of time at which the gondola is at a height of at least 12 m.

d. **1)** 28 units is subtracted from each side of the inequality and the inequality sign remains the same. Each side of the inequality is divided by -10 and the inequality sign is reversed.
2) **i)** No. **ii)** Yes. **iii)** No.
3) The following must occur: $t - 0.5 \geq 0$. The minimum value for t is therefore 0.5.
4) 2.56
5) $t - 0.5 \leq 2.56$
6) $0.5 \leq t \leq 3.06$. From 0.5 s to 3.06 s inclusive, the gondola is at a height of at least 12 m.

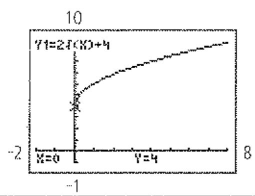
Technomath

a. For Y_1, h = 1 and k = 2. For Y_2, h = 4 and k = -3. For Y_3, h = -3 and k = 4. For Y_4, h = -2 and k = -1.

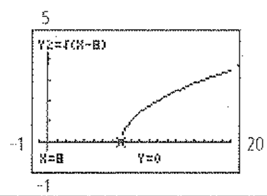
b. In the case of a function whose rule is in the form of $y = a\sqrt{\pm(x - h)} + k$, parameters **h** and **k** correspond respectively to the x-coordinate and y-coordinate of the vertex of the curve associated with this function.

c. You must choose $x = -2$.

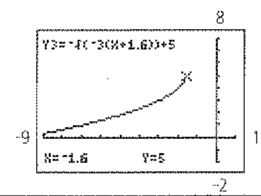
d. **1)**



2)



3)



Practice 1.2

1. a) $\sqrt{42}$ **b)** $\sqrt{10}$ **c)** $\sqrt{165}$ **d)** $\sqrt{14}$
e) $\sqrt{1}$ **f)** $\sqrt{\frac{23}{2}}$ **g)** $\sqrt{128}$ **h)** $\sqrt{\frac{1}{30}}$

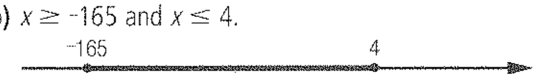
2. a) $\frac{5\sqrt{7}}{7}$ **b)** $-2\sqrt{3}$ **c)** $\frac{\sqrt{13} - \sqrt{2}}{11}$ **d)** $\frac{-(\sqrt{19} + \sqrt{42})}{23}$
e) $\frac{\sqrt{23} + \sqrt{14}}{9}$ **f)** $2(\sqrt{5} + \sqrt{11})$ **g)** $\frac{\sqrt{ab}(\sqrt{a} + \sqrt{b})}{a - b}$ **h)** $\frac{(\sqrt{a} - \sqrt{b})^2}{a - b}$

3. a) $4\sqrt{3}$ **b)** $10\sqrt{5}$ **c)** $5\sqrt{7}$ **d)** $3\sqrt{6}$
e) $10\sqrt{a}$ **f)** $-18\sqrt{b}$ **g)** $2c\sqrt{7}$ **h)** $4\sqrt{a + 2b}$

4. a) $x = 79$ **b)** $x = 53$ **c)** $x = -16$ **d)** $x = -\frac{22}{3}$
e) $x = -17.25$ **f)** $x = 49$ **g)** $x = 266$ **h)** $x = \frac{32}{29}$

5. a) $x \geq 9$

b) $x \geq -165$ and $x \leq 4$.



c) $x \leq -3$

d) There are no solutions.

e) $x \leq 9$

f) $x \leq -12$

Practice 1.2 *(cont'd)* **Page 40**

6. ① Ⓖ, ② Ⓕ, ③ Ⓗ, ④ Ⓒ, ⑤ Ⓓ, ⑥ Ⓔ, ⑦ Ⓐ, ⑧ Ⓑ

Practice 1.2 *(cont'd)* **Page 41**

7. a) f_2 b) f_4 c) f_3 d) f_1

8. a) $x = 3i$ b) $x = \sqrt{8}i$ c) $x = 6i$
   d) $x = \sqrt{23}i$ e) $x = -1$ f) $x = \frac{-1 + 2i}{2.5}$ and $x = \frac{-1 - 2i}{2.5}$.

9.

	Table Ⓐ	Table Ⓑ	Table Ⓒ	Table Ⓓ
a)	$y = -2\sqrt{x - 3} + 5$	$y = 3\sqrt{-(x + 2)} - 3$	$y = -\sqrt{-(x - 4)} + 2$	$y = 5\sqrt{x + 8}$
b)	≈ -0.29	Non real answer.	Non real answer.	≈ 21.21
c)	$\left[3, \frac{37}{4}\right]$	$]-\infty, -3]$	$[0, 4]$	$[-8, +\infty[$

10. a) $i(x)$ b) $g(x)$ c) $h(x)$ d) $f(x)$

Practice 1.2 *(cont'd)* **Page 42**

11. a) $y = 10\sqrt{x - 1} + 3$ b) $y = -12\sqrt{-(x + 7)} - 6$ c) $y = 4\sqrt{x - 3}$
    d) $y = -2\sqrt{x - 4} + 31$ e) $y = -3\sqrt{-(x - 10)} + 0{,}5$ f) $y = \frac{3}{20}\sqrt{x + \frac{1}{2}} - \frac{1}{5}$

12. a) $y = 2\sqrt{x + 3} - 2$ b) $y = -0.5\sqrt{-(x - 5)} + 3$ c) $y = -1.25\sqrt{x + 4} + 3$

13. a) $x = 4.81$ b) $x = -18$ c) Non real answer. d) $x = -\frac{383}{3}$

14. a) $]16, +\infty[$ b) $]-\infty, -0.32]$, which is the entire domain of the function. c) $[103, 255.5225[$ d) $[2.5, 514.5[$

15. a) Product **A**: $y = \frac{145}{3}\sqrt{x} - 30$ Product **B**: $y = 51\sqrt{x - 3} - 40$
    b) The minimum investment required is approximately $385 for Product **A** and approximately $3,615 for Product **B**.
    c) Product **A** would generate a greater profit than Product **B**.
    d) The amount of the advertising investment must be greater than approximately $9,631 for Product **A** and greater than approximately $12,842 for Product **B**.

Practice 1.2 *(cont'd)* **Page 43**

16. a) $(-6, -5)$
    b) Domain: $[-6, +\infty[$; range: $[-5, +\infty[$.
    c) This function's minimum is at the vertex because it increases starting from the vertex until infinity and there does not exist any *x*-coordinates lower than the *x*-coordinate of the vertex.
    d) f is increasing over its entire domain.
    e) $-\frac{71}{16}$
    f) $f(x)$ is negative over $\left[-6, -\frac{71}{16}\right]$ and positive over $\left[-\frac{71}{16}, +\infty\right[$.

17. a) Since $D_{basin} = 120$ s and $D_{actual} = 350$ s when $x = 49$ m, it can be deduced that the length L of the reduced model is 5.76 m.
    b) Since $L = 10$ m and $D_{actual} = 350$ s when $x = 49$ m, it can be deduced that the duration of the manoeuvre in the basin D_{basin} is approximately 158 s.
    c) $D_{actual} = 50\sqrt{x}$
    d) This manoeuvre would last approximately 403.11 s, that is almost 7 min.

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Practice 1.2 *(cont'd)* **Page 44**

18. a) Metal Ⓒ. **b)** Metal Ⓒ. **c)** h = 125

19. a) Approximately 9262 sick people.

b) One must solve the equation $23\ 000 = {-200\ 000}\sqrt{30 - h} + 200\ 000\sqrt{h}$.
The treatment was administered during the 16th day, because $h \approx 15.45$.

Practice 1.2 *(cont'd)* **Page 45**

20. a) $10,000

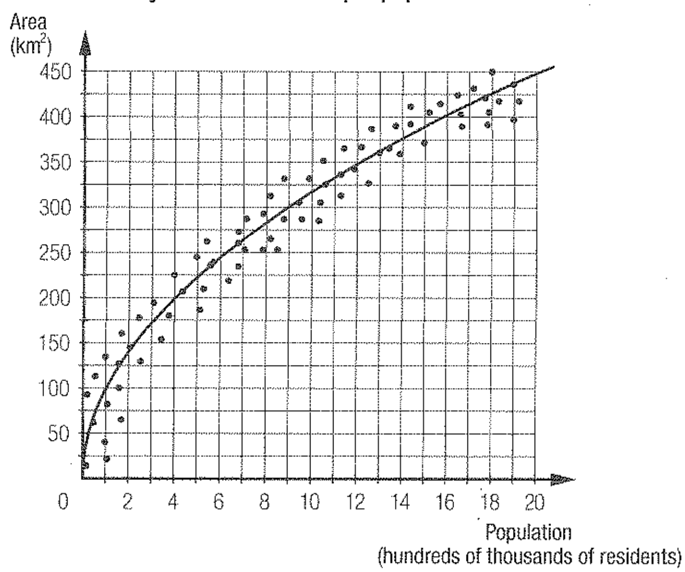
b) 1) After approximately 28.44 months. **2)** After 16 months.

c) During approximately 11.11 months.

d) The zero is approximately 44.44 months and represents the moment when the value of the investment is zero.

21. a) Yes, because the points are not placed randomly in the plane and seem to follow a certain trend.

b) Density of various municipal populations



c) $y = 100\sqrt{x}$

d) Approximately 894.43 km^2.

SECTION 1.3 | Absolute value functions

Problem **Page 46**

Several possible answers. For example:

This statement is not valid. Even if in this particular case, the result were the same, there can be cases where the result would not be. For example, $|5 - 8| - |-14 \times 2|$ does not give the same result if the method proposed by the student is used.

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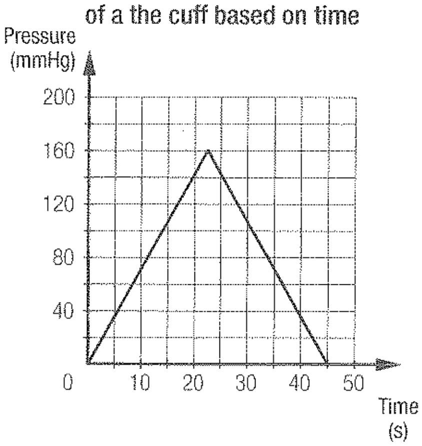
Activity 1

Page 47

a. Evolution of the pressure of a the cuff based on time



b. $x = 22.5$

c. 160 mmHg

d. $y = \frac{64}{9}x$ if $x \in [0, 22.5]$; $y = -\frac{64}{9}x + 320$ if $x \in [22.5, 45]$.

e. At approximately 14.77 s and 30.23 s.

Activity 2

Page 48

a. 77 K

b. At 30 s

c. 1) 197 K 2) 131 K 3) 317 K

d. 1) Two solutions.
 2) Subtract 77 and divide the result by 6: $107 = 6|x - 30| + 77 \Leftrightarrow 30 = 6|x - 30| \Leftrightarrow 5 = |x - 30|$
 3) 5 and -5: $|5| = 5$ and $|-5| = 5$.
 4) At 25 s and 35 s.

e. 1) The time interval during which the temperature is lower than or equal to 158 K.
 2) Subtract 77 and divide the result by 6: $6|x - 30| + 77 \leq 158 \Leftrightarrow 6|x - 30| \leq 81 \Leftrightarrow |x - 30| \leq 13.5$
 3) 13.5 and -13.5: $|13.5| = 13.5$ and $|-13.5| = 13.5$.
 4) The temperature is lower than or equal to 158 K in the time interval [16.5, 43.5] s.

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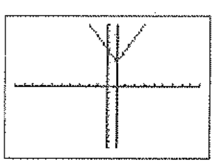
a. For Y_1, a = 2, h = -6 and k = -4. For Y_2, a = -1, h = 3 and k = -2. For Y_3, a = 4, h = 7 and k = 0.

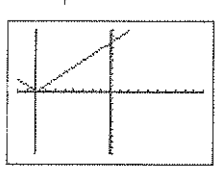
b. 1) $x = -6$ 2) $x = 3$ 3) $x = 7$

c. *Several possible answers. For example:* The equation of the axis of symmetry of a curve associated with an absolute value function whose rule is written as $y = a|x - h| + k$ is $x = h$.

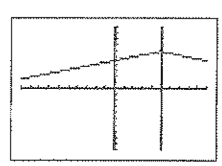
d. 1)



2)



3)



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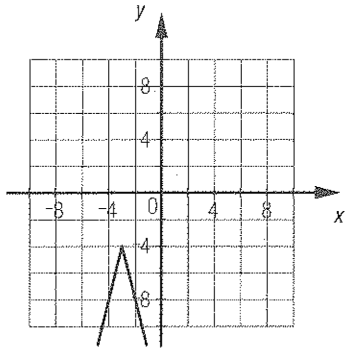
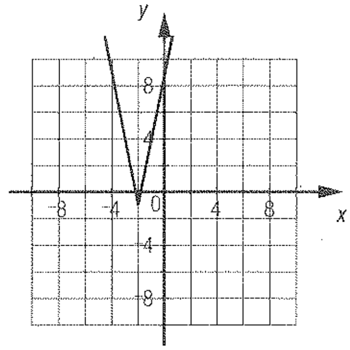
Practice 1.3

1. a) 1) 0.5 and −0.5. 2) (7, 2)
   b) 1) −3 and 3. 2) (−4, −5)
   c) 1) 1 and −1. 2) (−2, −1)
   d) 1) −4 and 4. 2) (−3, −4)
   e) 1) 5 and −5. 2) (−2, −1)
   f) 1) −6 and 6. 2) (1.5, 7)

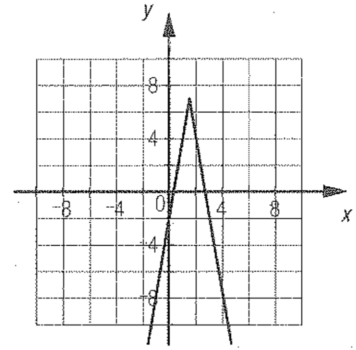
2. a)



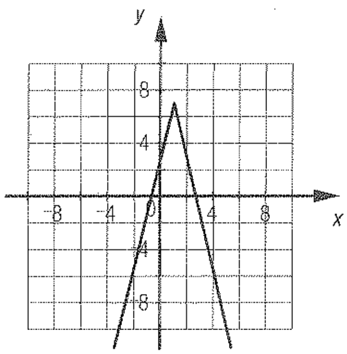
b)



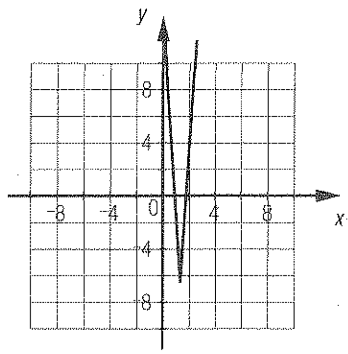
c)



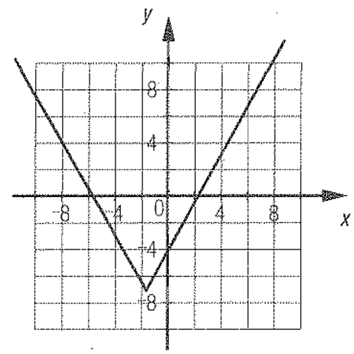
d)



e)



f)



3. a) $f(x) = -2.5|x - 8.5| - 2$
   b) $f(x) = 6|x| - 8$
   c) $f(x) = 2|x + 3| - 5$
   d) $f(x) = -3.5|x + 9.4| + 2$
   e) $f(x) = 10|x - 2| - 8.5$
   f) $f(x) = -9|x - 7| - 6.5$

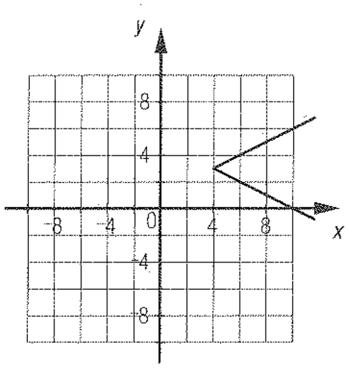
Practice 1.3 *(cont'd)*

4. a) 1) 12 2) 48 3) 10 4) 36 5) 9
   6) 42 7) 5 8) 2 9) $\frac{-1}{2}$
   b) $|a|^2 = |a| \cdot |a| = |a \cdot a| = |a^2|$

5. a) $f(x) = 2|x - 3|$
   b) $f(x) = 4|x + 1|$
   c) $f(x) = -3|x + 3| - 2$
   d) $f(x) = 2|x - 4| + 5$
   e) $f(x) = -4|x - 2| + 1$
   f) $f(x) = 6\left|x - \frac{2}{3}\right| + 3$

6. a) $x = 0$ or $x = -12$.
   b) No possible solutions.
   c) $x = -4$
   d) $x = 0$
   e) $x = 1.5$ or $x = -1.5$.
   f) No possible solutions.
   g) No possible solutions.
   h) $x = -17$ or $x = -27$.
   i) $x = \frac{5}{7}$ or $x = \frac{13}{7}$.

7. a)



b) No. There are several values of x that are associated with more than one value of y.

Practice 1.3 (*cont'd*)

8. a) 1) $\mathbb{R}$ **2)** $[4, +\infty[$ **3)** This function does not have any zeros.
4) This function is increasing over $[2, +\infty[$ and decreasing over $]-\infty, 2]$.
5) This function is positive over the entire domain.

b) 1) $\mathbb{R}$ **2)** $]-\infty, 6]$ **3)** The zeros are $x = -14$ and $x = 22$.
4) This function is increasing over $]-\infty, 4]$ and decreasing $[4, +\infty[$.
5) This function is negative over $]-\infty, -14]$ and $[22, +\infty[$ and positive over $[-14, 22]$.

c) 1) $\mathbb{R}$ **2)** $[5, +\infty[$ **3)** This function does not have any zeros.
4) This function is increasing over $[4, +\infty[$ and decreasing $]-\infty, 4]$.
5) This function is positive over the set of its domain.

d) 1) $\mathbb{R}$ **2)** $[-3, +\infty[$ **3)** The zeros are $x = 1.625$ and $x = 2.375$.
4) This function is increasing over $[2, +\infty[$ and decreasing over $]-\infty, 2]$.
5) This function is positive over $]-\infty, 1.625]$ and $[2.375, +\infty[$ and negative over $[1.625, 2.375]$.

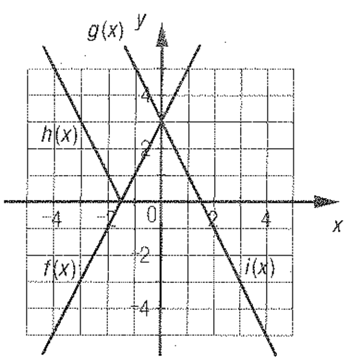
e) 1) $\mathbb{R}$ **2)** $]-\infty, 1]$ **3)** The zeros are $x = 1.75$ and $x = 2.25$.
4) This function is increasing over $]-\infty, 2]$ and decreasing over $[2, +\infty[$.
5) This function is negative over $]-\infty, 1.75]$ and $[2.25, +\infty[$ and positive over $[1.75, 2.25]$.

f) 1) $\mathbb{R}$ **2)** $\mathbb{R}_+$ **3)** The zero is $x = 2$.
4) This function is decreasing over $[2, +\infty[$ and decreasing over $]-\infty, 2]$.
5) This function is positive over the set of its domain and negative when $x = 2$.

9. a) $f(x) = 2|x + 8| - 8$ **b)** $f(x) = -2|x - 1| + 10$ **c)** $f(x) = 2|x - 10| - 8$

10. a) $]-1, 5[$ **b)** $]-\infty, -2.6] \cup [1.4, +\infty[$ **c)** $]-\infty, -10[\cup]-8, +\infty[$
d) $]-\infty, -4] \cup [14, +\infty[$ **e)** $[-3, 2]$ **f)** $\{\}$
g) $]-\infty, -4.8] \cup [6.4, +\infty[$ **h)** $]-\infty, 5[\cup]31, +\infty[$ **i)** $]-\infty, -1.4] \cup [3, +\infty[$

11. a)



b) Each of these curves is partially superimposed onto the curve of function *f*.

12. a) $f(x) = 2|x + 1| + 2$ **b)** $f(x) = -5|x + 12|$ **c)** $f(x) = 3|x - 2| - 2$

Practice 1.3 (*cont'd*)

13. $y = 0.25|x + 26| - 0.5$ and $y = -0.25|x + 26| - 0.5$.

14. No. The intersection of these two lines cannot result in a pair of symmetric half-lines in relation to a vertical axis.

15. a) 1) $f + g = 2|x - 4| - 4$ **2)** $f - g = 8$ **3)** $g - f = -8$
b) 1) Domain: $\mathbb{R}$; range: $[-4, +\infty[$. **2)** Domain: $\mathbb{R}$; range: 8. **3)** Domain: $\mathbb{R}$; range: -8.

16. A(0.625, 1); B(1.875, 3); C(3.125, 5); D(4.375, 7); E(6.25, 6); F(7.5, 4); G(8.75, 2)

17. a) $x = -7$ or $x \approx -1.83$, or $x \approx 3.83$, or $x = 9$. **b)** $x = 1$
c) $x = -7$ or $x = 5$. **d)** $x = -10$ or $x = 2$.

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18. **a)** $f(x) = |x - 2| - 7$ **b)** $f(x) = -2|x + 1| + 6$ **c)** $f(x) = 3|x - 5| + 1$
d) $f(x) = 1.5|x + 4| - 9$ **e)** $f(x) = 4|x - 6| - 3$ **f)** $f(x) = -6|x + 2| + 14$

Practice 1.3 *(cont'd)* **Page 57**

19. **a)** $f(x) = \begin{cases} -0.75x + 9.5;\ x \leq 10 \\ 0.75x - 5.5;\ x \geq 10 \end{cases}$ **b)** $f(x) = \begin{cases} 1.8x;\ x \leq 10 \\ -1.8x + 36;\ x \geq 10 \end{cases}$

c) $f(x) = \begin{cases} -1.5x + 18;\ x \leq 12 \\ 1.5x - 18;\ x \geq 12 \end{cases}$ **d)** $f(x) = \begin{cases} -2.6x + 15.7;\ x \geq -5.5 \\ 2.6x + 44.3;\ x \leq -5.5 \end{cases}$

20. **a)** $y = 2.5|x - 6| - 1$ **b)** $y = -2.5|x - 4| - 2$ **c)** $y = 2.5|x + 4| + 2$ **d)** $y = 2.5|x - 4| + 2$

21. $y = 2|x - 1.75| + 2.5$; $y = -2|x - 1.75| + 2.5$; $y = 2|x + 0.75| - 2.5$; $y = -2|x + 0.75| - 2.5$;
$y = 1.5\left|x - \frac{2}{3}\right| + 2$; $y = -1.5\left|x - \frac{2}{3}\right| + 2$

Practice 1.3 *(cont'd)* **Page 58**

22. For 7.5 days.

23. **a)** 21.66 units^2. **b)** Approximately 98.28 units^2.

24. For 8 days.

Practice 1.3 *(cont'd)* **Page 59**

25. For 8 h.

26.
- Determine how many days the humidity rate is less than or equal to 25%.
  $25 \geq 1.2|x - 6| + 20$
  $25 = 1.2|x - 6| + 20$
  $10.1\overline{6} = x$ or $1.8\overline{3} = x$
  The humidity rate had been less than or equal to 25% for $10.1\overline{6} - 1.8\overline{3} = 8.\overline{3}$ h.
  The sprinkler had been in use for 8 h and 20 min.
- Determine the amount of water used for the watering.
  The system consumed 12 L/h for $8.\overline{3}$ h, in other words 100 L.

27. **a)** 15 000 turns/min.
b) 30 s
c) **1)** $1000 < -500|t - 30| + 15\,000$
$t = 2$ or $t = 58$
The speed is greater than 1000 turns/min when $2 < x < 58$.
The speed is greater than 1000 turns/min during $58 - 2 = 56$ s.

2) $10\,000 < -500|t - 30| + 15{,}000$
$t = 20$ or $t = 40$
The speed is greater than 10 000 turns/min when $20 < x < 40$.
The speed is greater than 10 000 turns/min during $40 - 20 = 20$ s.

3) $12\,000 < -500|t - 30| + 15\,000$
$t = 24$ or $t = 36$
The speed is greater than 12 000 turns/min when $24 < x < 36$.
The speed is greater than 12 000 turns/min during $36 - 24 = 12$ s.

28. $14.3 \leq {}^-1.5|x - 12| + 17$

$13.8 = x$ or $10.2 = x$

The voltage is greater than or equal to 14.3 V if $10.2 \leq x \leq 13.8$.

The voltage is greater than or equal to 14.3 during $13.8 - 10.2 = 3.6$ h.

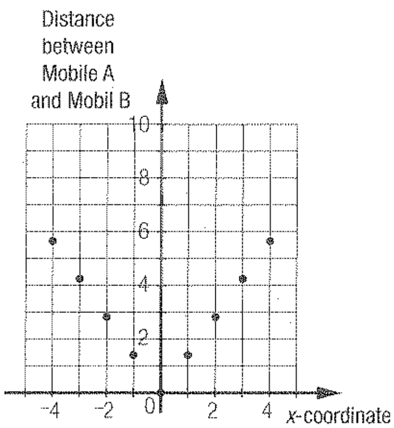
The circuit is cut for 3.6 h.

Practice 1.3 (*cont'd*)

29. a)

x-coordinate of point A	⁻4	⁻3	⁻2	⁻1	0	1	2	3	4
Distance between Mobile A and B (cm)	$4\sqrt{2}$	$3\sqrt{2}$	$2\sqrt{2}$	$\sqrt{2}$	0	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}$	$4\sqrt{2}$

b)



c) An absolute value function.

d) Yes, because these two expressions eliminate the negative value of *n*. In addition, the previous questions show that the graphical representation obtained by completing the square root of the square of a number is similar to that of an absolute value function.

30. a) The absolute uncertainty is 0.9°.

b) 1) ≈ 7.7%; ≈ 5.4%; ≈ 9.4%; ≈ 7.8%; ≈ 9.9%; ≈ 4.2%; ≈ 9.4%; ≈ 5.0%; ≈ 7.0%

2) ≈ 9.9%; ≈ 9.8%; ≈ 9.4%; ≈ 11.7%; ≈ 11.1%; ≈ 7.6%; ≈ 10.6%; ≈ 9.0%; ≈ 10.5%

c) ≈ 7.3%

d) The scientist is right, the absolute value is equal to the quotient of the absolute values.

SECTION 1.4 Rational functions

Problem

$\frac{9}{100}$

Activity 1

a. 1

b. 1)

A (volts)	4	7.6	9.5	9.7	9.9	9.99	10
R	**≈ 2.33**	**≈ 7.33**	**39**	**≈ 65.67**	**199**	**1999**	**undefined**

2) The value of R increasingly greater, it approaches infinity.

3) $x = 10$

c. 1) The amplitude of the standing wave so that the ratio is equal to 30.

2) The amplitude of the standing wave must be equal to ≈ 9.35 V so that the ratio of the standing wave is 30.

d. 1) The amplitude of the standing wave so that the ratio is greater than 35.

2) The amplitude of the standing wave must be situated between ≈ 9.44 and 10 V so that the ratio is greater than 35.

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e. $(10 + A) \div (10 - A) = (A + 10) \div (^{-}A + 10)$

$A + 10 \;\lfloor\; {}^{-}A + 10$

$\Leftrightarrow$

$A + 10 \;\lfloor\; {}^{-}A + 10$
$\quad\quad\quad\quad {}^{-}1$

$\Leftrightarrow$

$\quad A + 10 \;\lfloor\; {}^{-}A + 10$
$- A - 10 \quad {}^{-}1$
$\quad\quad 20$

$\Leftrightarrow$

$(10 + A) \div (10 - A) = {}^{-}1 + \frac{20}{10 - A}$

The division $(10 + A) \div (10 - A)$ being equal to ${}^{-}1 + \frac{20}{10 - A}$, the rule $R = \frac{10 + A}{10 - A}$ can also be written as $R = {}^{-}1 + \frac{20}{10 - A}$

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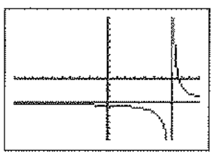
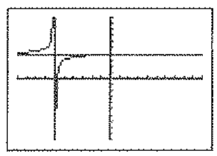
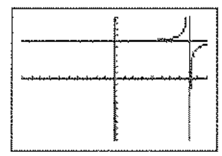
a. For Y_1: a = ⁻2, h = ⁻5 and k = ⁻6. For Y_2: a = 5, h = 2 and k = 3.

b. 1) $y = {}^{-}6$ 2) $x = {}^{-}5$ 3) $y = 3$ 4) $x = 2$

c. *Several possible answers. For example:* The equations of the asymptotes belonging to a curve that is associated with a rational function whose rule is written in the form $y = \frac{a}{x - h} + k$ are $x = h$ and $y = k$.

d. 1)  2)  3) 

Practice 1.4

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1. a) $f(x) = \frac{14}{x - 2} + 3$ b) $f(x) = \frac{^{-}2}{x - 1} + 2$ c) $f(x) = \frac{^{-}7}{x - 3} - 4$ d) $f(x) = \frac{1}{2}$ e) $f(x) = \frac{^{-}9}{2\left(x - \frac{9}{4}\right)} - 1$

f) $f(x) = \frac{21}{16\left(x + \frac{5}{4}\right)} - \frac{1}{4}$. g) $f(x) = \frac{14}{x - 7} + 2$ h) $f(x) = \frac{13}{4\left(x + \frac{3}{4}\right)} - 2$ i) $f(x) = \frac{7}{(x - 2)} - 1$

2. a) 1) $y = 5$ 2) $x = 3$ 3) $(3, 5)$ b) 1) $y = {}^{-}5$ 2) $x = {}^{-}2$ 3) $({}^{-}2, {}^{-}5)$

c) 1) $y = {}^{-}2$ 2) $x = 3$ 3) $(3, {}^{-}2)$

3. a) $f(x) = \frac{3}{x}$ b) $f(x) = \frac{^{-}3}{x}$ c) $f(x) = \frac{5}{x - 2} + 3$

d) $f(x) = \frac{3}{2(x + 4)} - 4$ e) $f(x) = \frac{^{-}1}{4(x - 5)}$ f) $f(x) = \frac{^{-}2}{x - 2} - 1$

Practice 1.4 (*cont'd*)

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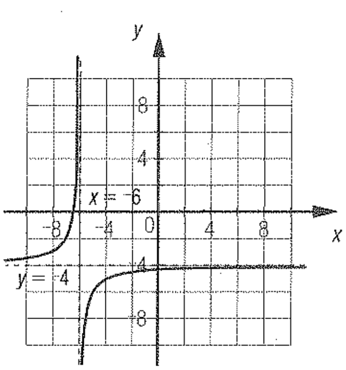
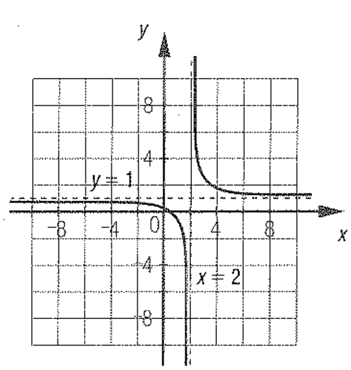
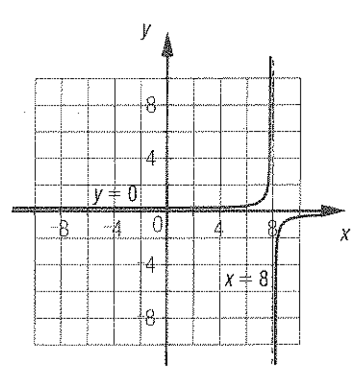
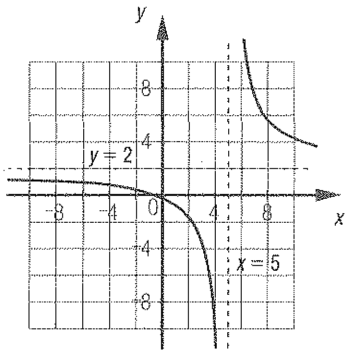
4. a) 

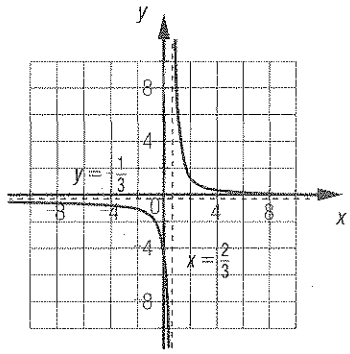
b) 

c) 

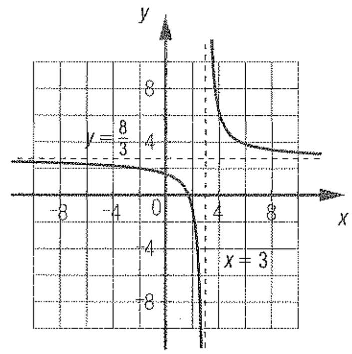
d)



e)



f)



5. **a)** **1)** $f^{-1}(x) = \frac{-5}{3(x-3)} + 4$ **2)** $f^{-1}(x) = \frac{1}{3(x+2)} + 2$ **3)** $f^{-1}(x) = \frac{-1}{x-2} + 1$

4) $f^{-1}(x) = \frac{1}{x-1} + 5$ **5)** $f^{-1}(x) = \frac{-1}{x}$ **6)** $f^{-1}(x) = \frac{-25}{x-16} - 9$

b) The inverse of a rational function is also a rational function.

6. **a)** $x = \frac{37}{11}$ **b)** $x = \frac{-57}{23}$ **c)** $x = \frac{-39}{14}$ **d)** $x = \frac{58}{7}$ **e)** { }

f) $x = \frac{2}{3}$ **g)** $x = 7$ **h)** $x = 3$ **i)** $x = \frac{-112}{109}$

7. **a)** $f + g = \frac{3x-8}{2x+4}$ **b)** $f \times g = \frac{6x}{x^2+4x+4}$ **c)** $f \div g = \frac{-8}{3x}$ **d)** $g - f = \frac{3x+8}{2x+4}$ **e)** $g \circ f = \frac{-3}{x}$

8. **a)** A first-degree polynomial function. **b)** A rational function.

Practice 1.4 (*cont'd*)

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9. **a)** **1)** $\mathbb{R} \setminus \{7\}$ **2)** $\mathbb{R}^*$ **3)** This function does not have any zeros. **4)** This function is increasing.

5) This function is positive if $x < 7$ and negative if $x > 7$.

b) **1)** $\mathbb{R} \setminus \{3\}$ **2)** $\mathbb{R} \setminus \{5\}$ **3)** $x = 1$ **4)** This function is decreasing.

5) This function is positive if $x \le 1$ or $x > 3$, and negative if $x \ge 1$ and $x < 3$.

c) **1)** $\mathbb{R} \setminus \{4\}$ **2)** $\mathbb{R} \setminus \{3\}$ **3)** $x = 2.5$ **4)** This function is decreasing.

5) This function is positive if $x \le 2{,}5$ or $x > 4$, and negative if $x \ge 2.5$ and $x < 4$.

d) **1)** $\mathbb{R} \setminus \{-1\}$ **2)** $\mathbb{R} \setminus \{5\}$ **3)** $x = -0.6$ **4)** This function is increasing.

5) This function is positive if $x < -1$ or $x \ge -0.6$, and negative if $x > -1$ and $x \le -0.6$.

e) **1)** $\mathbb{R} \setminus \{-3\}$ **2)** $\mathbb{R} \setminus \{1\}$ **3)** $x = 8$ **4)** This function is increasing.

5) This function is positive if $x \ge 8$ or $x < -3$, and negative if $x > -3$ and $x \le 8$.

f) **1)** $\mathbb{R} \setminus \{7\}$ **2)** $\mathbb{R} \setminus \{8\}$ **3)** $x = \frac{55}{8}$ **4)** This function is decreasing.

5) This function is positive if $x \le \frac{55}{8}$ or $x > 7$, and negative if $x \ge \frac{55}{8}$ and $x < 7$.

10. **a)** $x < \frac{43}{6}$ or $x > 8$. **b)** $x < \frac{-1}{4}$ or $x > \frac{-9}{40}$. **c)** $-\frac{2}{9} < x \le -\frac{13}{81}$. **d)** $x \le \frac{13}{7}$ or $x > \frac{8}{3}$. **e)** $x < 0$ or $x \ge \frac{2}{5}$.

f) $\frac{3}{2} < x \le \frac{11}{5}$ **g)** $x > -\frac{1}{3}$ or $x < -\frac{7}{36}$. **h)** $x \le \frac{1}{7}$ or $x \ge \frac{35}{13}$. **i)** $x \ge -3$ or $x \le -\frac{47}{21}$.

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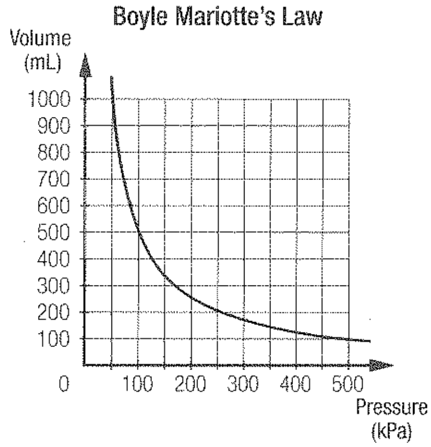
11. a)



b) Approximately 588.24 mL.

c) $P = \frac{50\,000}{V}$

d) 250 kPa

Practice 1.4 *(cont'd)* **Page 70**

12. a) 1) $V = 15 + 2x$ **2)** $V = 50 + x$ **3)** $V = 65 + 3x$ **4)** $C = \frac{2x + 15}{3x + 65} \times 100$

b) When the time surpasses 35 min.

13. a) 1) It consists of two first-degree polynomial functions.

2) The initial value of F is 14 MHz and the initial value of B is 10 KHz.

3) The zero of F is −14 000 MHz, which is impossible in the current context. The zero of B is 100 KHz.

4) Function f is increasing and function B is decreasing.

b) 1) A rational function.

2) Its initial value is 1.4.

3) Its zero is −14 000 MHz, which is impossible in the current context.

4) This function is decreasing.

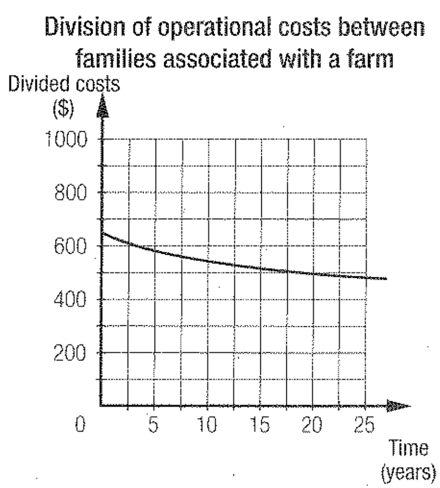
c) 1) At 40 s. **2)** When $t < 40$ or $t > 100$. **3)** When $t > 64.84$ and $t < 100$. **4)** When $40 < t < 64.84$.

14. $D = \frac{\frac{169}{12}}{x - \frac{1439}{12}} + 2$ or $D = \frac{169}{12x - 1439} + 2$.

Practice 1.4 *(cont'd)* **Page 71**

15. a) $R = \frac{16\,250 + 800x}{25 + 2x}$

b)



c) The operating costs decrease.

d) $400 per family. It is the value associated with the horizontal asymptote, which the curve increasingly approaches.

16. a)

	f	g	$\frac{f}{g}$	$\frac{g}{f}$
Initial value	5	8	$\frac{5}{8}$	$\frac{8}{5}$
Zero	-5	16	-5	16
Equation of the horizontal asymptote			$y = -2$	$y = -\frac{1}{2}$
Equation of the vertical asymptote			$y = 16$	$y = -5$

b) 1) The quotient of the initial values of both first-degree polynomial functions gives the initial value of the function that corresponds to the quotient of these two functions.

2) There are no evident links between these two values.

3) There are no evident links between these two values.

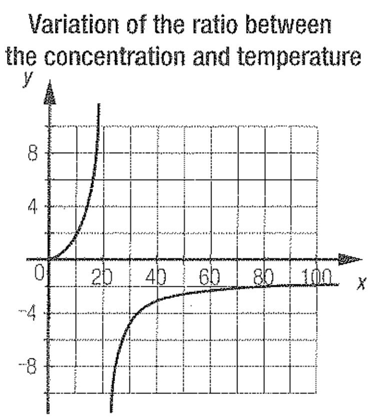
c) 1) There are no evident links between these two values.

2) The zero of the first-degree polynomial function found at the numerator of the rational expression corresponds to the zero of the function that corresponds to the quotient of these two functions.

3) The zero of the first-degree polynomial function found at the denominator of the rational expression gives the vertical asymptote of the function that corresponds to the quotient of these two functions.

Practice 1.4 (*cont'd*) **Page 72**

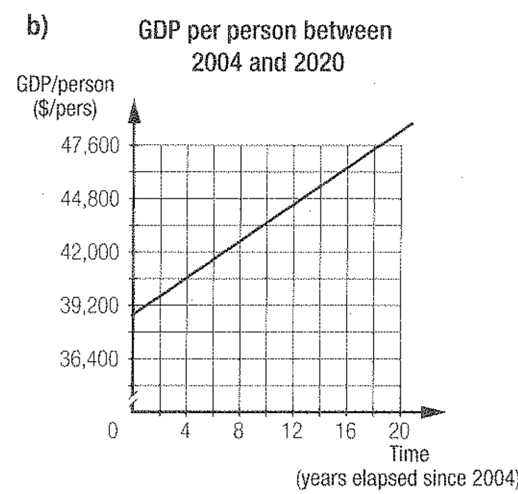
17. a)



b) Before 9 min 40 s and after 20 min.

18. a) $GDP_H = \dfrac{3.125 \times 10^{10}x + 1.275 \times 10^{12}}{3.25 \times 10^5 + 3.3 \times 10^7}$ in dollars per person

b)



c) The GDP per person is increasing.

d) It tends to stabilize on a long term basis, at approximately $91,153.85/person.

19. a) $L = \dfrac{3 \times 10^8}{F}$, where L represents the length of the wave in metres and F represents the frequency in Hz.

b) 1) $\approx$ 507.48 m 2) $\approx$ 705.03 m 3) $\approx$ 48.57 m

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Practice 1.4 (*cont'd*)

Page 73

20. a) **1)** $U = 5 + 3x$ **2)** $P = 1 + x$ **3)** $R = \frac{5 + 3x}{1 + x}$

b) $\mathbb{R}^*_+$ **c)** 5 **d)** $]3, 5]$ **e)** The value of R gradually approaches 3.

f) After 9 months.

21. a) $C = \frac{500 + 2.35n}{n}$ **b)** C gradually approaches \$2.35.

c) $\approx$ 765.61, therefore 766 chips.

22. a) $T = \frac{\frac{1625}{3}}{x + \frac{65}{3}}$ or $T = \frac{1625}{3(x + 65)}$ **b)** **1)** $\approx 43.\bar{3}$ °C **2)** More than $\approx -10.8\bar{3}$°C. **3)** Less than $\approx -46.9\bar{4}$°C.

SPECIAL FEATURES 1

Chronicle of the past

Page 75

1. $x^2 - 2x + \frac{11}{12}$

2. **1)** Approximately 6.57 m/s. **2)** Approximately 6.24 m/s. **3)** Approximately 4.83 m/s.

3. **1)** Approximately 91.36 kPa. **2)** Approximately 166.47 kPa. **3)** Approximately 280.25 kPa.

4. **1)** Approximately 444.89 Hz. **2)** Approximately 278.57 Hz. **3)** 812.5 Hz.

5. $F = \frac{3V}{2L}$

$L = \frac{3V}{2F}$

$0.5L = 0.5\frac{3V}{2F}$

$2F = 0.5\frac{3V}{L}$

$2F = \frac{3V}{\frac{1}{2}(2L)}$

In the workplace

Page 77

1. For 4.8 min.

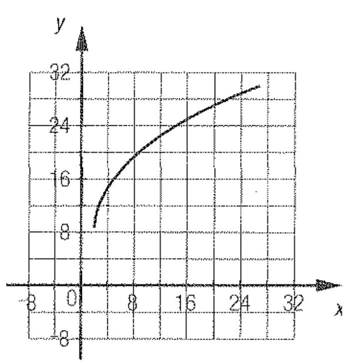
2. $\approx$ \$15,481.31

3. a) $y = \frac{10 + 3x}{4 + x}$ **b)** 16 new wells.

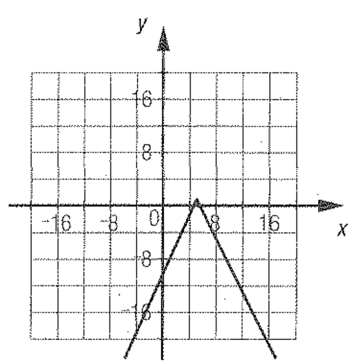
Overview

Page 78

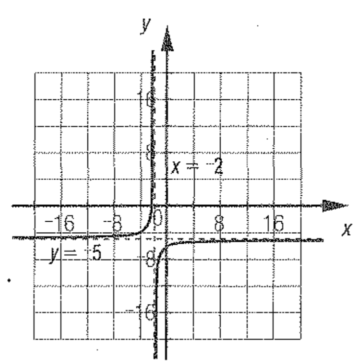
1. a)



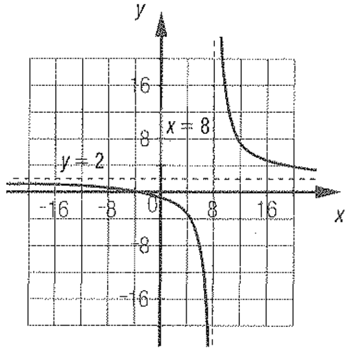
b)



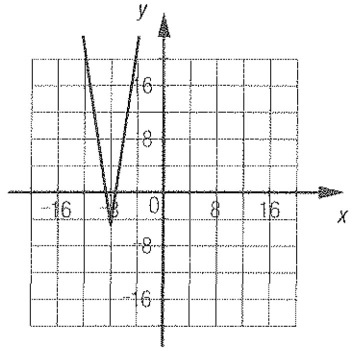
c)



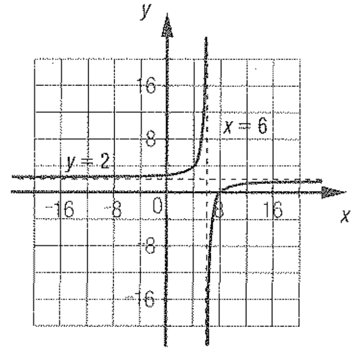
d)



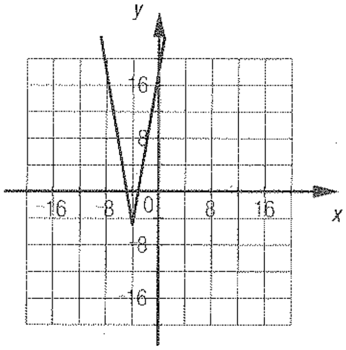
e)



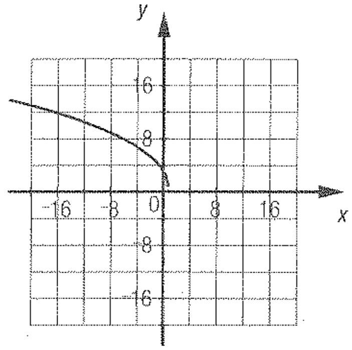
f)



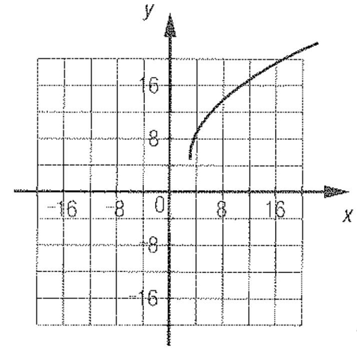
g)



h)



i)



2. a) $x = 4$ **b)** { } **c)** $x = 78$ **d)** { } **e)** $x = 0.56$ **f)** $x = \frac{11}{19}$ **g)** { }
h) $x = \frac{1}{3}$ or $x = \frac{11}{3}$. **i)** $x = -33$ **j)** $x = -\frac{19}{2}$ **k)** $x = 3$ **l)** $x = 10$

3. a) $-1 < x < 5$ **b)** $3 < x < 4$ **c)** $-\frac{29}{7} < x \leq 1$ **d)** { } **e)** $x \leq -6$ **f)** $x < 1$ or $x > 1.5$
g) $-1 \leq x < 2$ **h)** $x < -2$ **i)** $x \leq -6.5$ or $x \geq 2.5$. **j)** $\mathbb{R}^-$ **k)** { } **l)** { }

4. a) 60 m/s **b)** At 5 s. **c)** After more than 1.25 s.
d) 1) $k = 120$ **2)** $a = -6$

Overview *(cont'd)* **Page 79**

5. a) $f(x) = 2\sqrt{x - 7} + 9$ **b)** $f(x) = -3|x - 2| + 5$ **c)** $f(x) = \frac{-5}{x + 2} + 2$
d) $f(x) = \sqrt{-(x - 3)} + 1$ **e)** $f(x) = 6|x - 1| - 1$ **f)** $f(x) = 0.75|x - 2| + 5$
g) $f(x) = \frac{8}{x + 2} + 3$ **h)** $f(x) = \frac{1}{x + 6} + 9$

Overview *(cont'd)* **Page 80**

6. The maximum temperature is just over 1000°C; since there is an asymptote when $T = 1000$, the curve approaches this value without reaching it.

7. a) $\approx 5.97 \times 10^{24}$ kg **b) 1)** ≈ 7690.92 m/s **2)** ≈ 3.60 days

8. a) *Several possible answers. For example:* $T \approx \frac{0.06}{C}$.
b) Approximately 0.0096 mol/L.
c) For concentrations greater than 0.016 mol/L.

Overview *(cont'd)* **Page 81**

9. a) 1) The resistance is greater than 22 500 ohms. **2)** The power is approximately 1.82 watts
b) The intensity approaches infinity.

10. a) $A = \frac{3 - \rho_2}{3 + \rho_2}$ **b)** 3 g/cm³ **c)** 1

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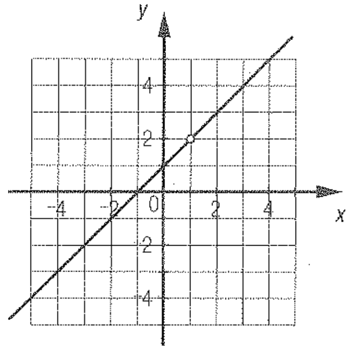
11. a) $\frac{f}{g} = x + 1, x \neq 1$

b)



Overview (*cont'd*) **Page 82**

12. – $f \times g = \frac{\sqrt{x}}{x}$

– $g \circ f = \frac{1}{\sqrt{x}}$

$= \frac{1}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$

$= \frac{\sqrt{x}}{x}$

13. a) $\mathbb{R} \setminus \{-5\}$ b) $\mathbb{R}$ c) -1 d) $\{-4.5, -3, 6.\overline{3}\}$

e) The function is positive over the intervals $]-\infty, -5[\cup [-4.5, -3] \cup [6.\overline{3}, +\infty[$ and negative over intervals $]-5, -4.5] \cup [-3, 6.\overline{3}]$.

f) The function is increasing over interval $]-\infty, -5[\cup]-5, -4] \cup [5, +\infty[$ and decreasing over interval $[-4, 5]$.

14. a) $f + g = \frac{6x - 4}{6x - 9}$ or $f + g = \frac{5}{6(x - 1.5)} + 1$. b) $y = 1$ and $x = 1.5$.

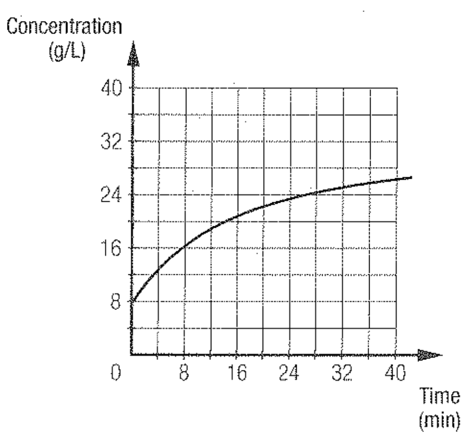
15. a) 1) Approximately 318.94 m/s. 2) Approximately 325.18 m/s. 3) 331.3 m/s. 4) Approximately 346.13 m/s.

b) 1) Approximately 18.04°C. 2) Approximately -17.22°C.

Overview (*cont'd*) **Page 83**

16. a)

Concentration of solution substance according to time



b) To a rational function.

c) 1) Yes, the concentration is 8.5 g/L at approximately 0.63 minutes.

2) Yes, the concentrationis 17g/L at approximately 8.94 minutes.

3) No, the asymptote of this function corresponds to a 34 g/L concentration.

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d) At the beginning of the experiment, 0 min.

17. a) 1) At 7.5 min. **2)** At 45 min. **b)** 5 g

18. a) 1) 750 mL/min **2)** 1000 mL/min **3)** 1312.25 mL/min **4)** The concentration must be zero.

Overview (*cont'd*)

Page 84

19. a) $t = \sqrt{\frac{3 - H}{4.91}}$ **b)** At approximately 0.78 s.

20. a) $g(x) = {}^-a\sqrt{b(x - h)} + k$ **b)** $g(x) = a\sqrt{{}^-b(x - h)} + k$

21. a) $P = \frac{350 + 250x}{x + 1}$

b) The asymptote is the value that the mean production of electricity approaches based on the number of turbines once the number of turbines is increased.

c) 1) $P = \frac{315 + 250x}{x + 1}$ **2)** $P = \frac{250x}{x + 1}$

22. a) 100 cm **b) 1)** 12 days after March 21, that is April 2. **2)** 16 days after March 21, on April 6.

Overview (*cont'd*)

Page 85

23. a) 1) For approximately 95.15 days. **2)** For approximately 84.85 days. **3)** For approximately 63.03 days.

b) 1) For approximately 85.6 days. **2)** For 47.2 days. **3)** For 18.4 days.

24. a) For approximately 44.5 min. **b)** For approximately 16.46 min. **c)** For approximately 3.14 min.

Bank of problems

Page 86

1.
- Determine the quantity of Solution **A** to add each time, so that no more than 0.5 mol of Substance **X** is added at one time.

  $C = \frac{n}{V}$

  $5 = \frac{0.5}{V} \Leftrightarrow V = 0.1$

- Determine the way in which the concentration of Substance **X** increases in the mixture of Solution **A** and Solution **B**.

  $C = \frac{0.2 + 0.5x}{0.1 + 0.1x}$, where C represents the concentration in mol/L of Mixtures **A** and **B** and x represents the number of additions of 100 mL of Solution **A**.

- Determine the number of times it is necessary to add 100 mL of Solution **A** to obtain a concentration of 4.8 mol/L.

  $4.8 = \frac{0.2 + 0.5x}{0.1 + 0.1x} \Leftrightarrow 14 = x$

  Solution **A** must be added 14 times to the mixture.

- Verify that the desired concentration is obtained at least 30 minutes and at most 35 minutes after the start of the first addition.

$30 \leq 14y$ $\Leftrightarrow$ $\frac{15}{7} \leq y$	$35 \geq 14y$ $\Leftrightarrow$ $\frac{5}{2} \geq y$

where y represents the interval of time (in min) between each 100 mL addition to Solution **A**.

y must be greater than or equal to $\frac{15}{7}$ min or 2 min 8 s, and less than or equal to $\frac{5}{2}$ min or 2 min 30 s.

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2. Solve the system of equations $D = 90\sqrt{2t}$, $D = \frac{9000t}{9t + 50}$ using a method of your choice (graphs, table of values, etc.)

Using a table of values:

$D = 90\sqrt{2t}$		$D = \frac{9000t}{9t+50}$	
t	D	t	D
0.616	≈ 99.90	0.616	≈ 99.81
0.617	≈ 99.98	0.617	≈ 99.96
0.618	≈ 100.06	0.618	≈ 100.10

- The first moment that the distances covered by the two groups of birds are identical is between 0.617 and 0.618 days after their departure.
- The second moment that the distances covered by the two groups of birds are identical is 50 days after departure.

$D = 90\sqrt{2t}$		$D = \frac{9000t}{9t+50}$	
t	D	t	D
49.75	897.75	49.75	899.55
50	900.00	50	900.0
50.25	902.25	50.25	900.45

Bank of problems *(cont'd)*

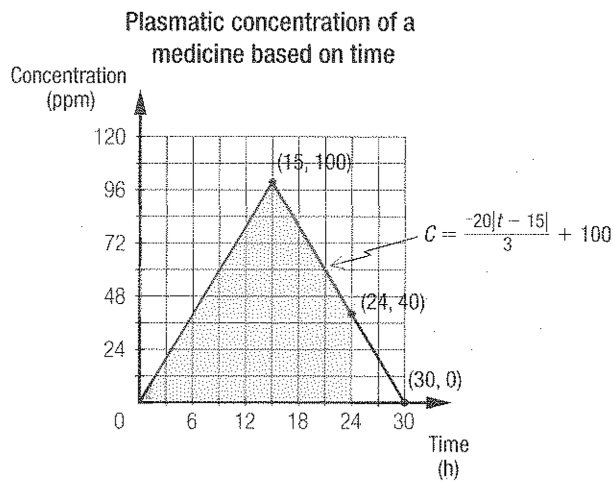
Page 87

3. • Calculate the area of the shaded region.

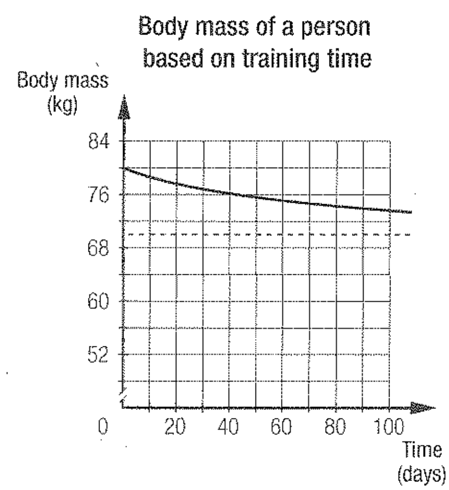
$A = \frac{30 \times 100}{2} - \frac{6 \times 40}{2}$

$A = 1380$ ppm × h.

• The total exposure to the medicine in the 24 h that follow its dosage is 1380 ppm × h.



4. Yes. The initial mass of this person is 80 kg. Yet, the body mass of this person evolves based on the rule $M = \frac{4800 + 70x}{x + 60}$. This rule corresponds to a decreasing rational function with an asymptote of $y = 70$, as shown in the graphical representation. A 15% loss of the person's initial mass would bring their body mass to 68 kg, which is impossible given the asymptote of this curve.



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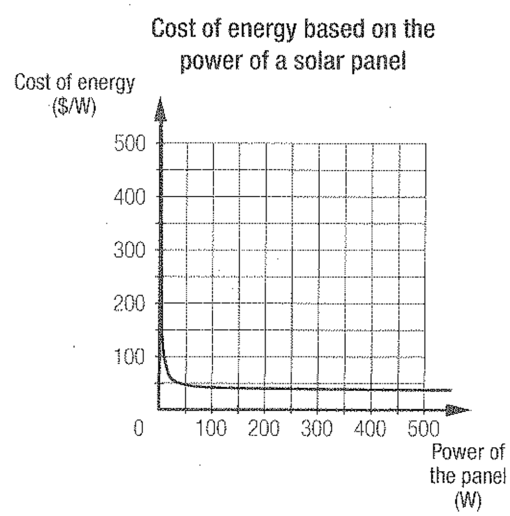
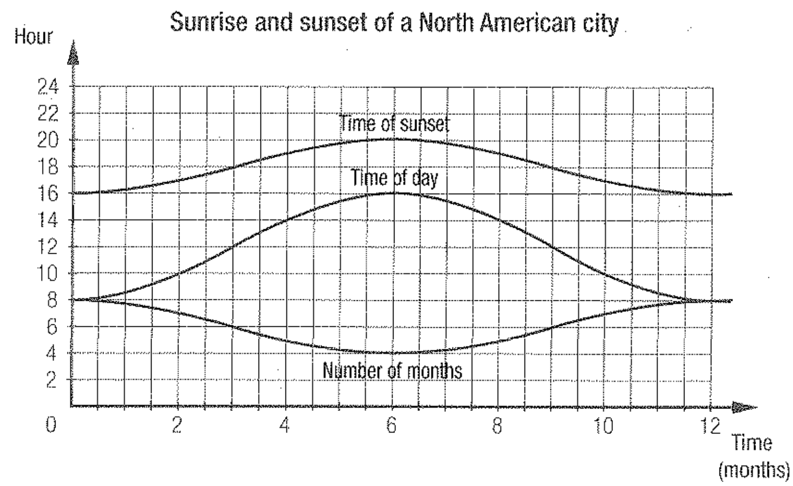
5. The relation $R = \frac{36x + 500}{x}$ provides the cost per watt ($/W), which is equivalent to the cost of one unit of electrical power. This relation corresponds to a decreasing function as shown in the adjacent graph.

The use of one solar panel of 1000 W costs $36,500, that is, $36.50/W, whereas the use of 100 solar panels of 10 W costs $86,000, that is $86/W.



6. The length of a day, based on the number of months elapsed since January 1, evolves according to a curve that corresponds to the difference between the time of sunset and time of sunrise.



Bank of problems (*cont'd*) **Page 88**

7. The rule that calculates the friction F of the air according to the speed S of the moving object can be obtained by determining the inverse of $S = \sqrt{\frac{-2F}{1.293}}$ by the following reasoning, in the case that $S \geq 0$:

$S = \sqrt{\frac{-2F}{1.293}}$ $F = -0.6465S^2$

8. • The area of the green region must be determined.

Area of triangle HCI

The coordinates of points H and I must be determined:

$1 = -2|x - 8| + 8$

$11.5 = x$ or $4.5 = x$

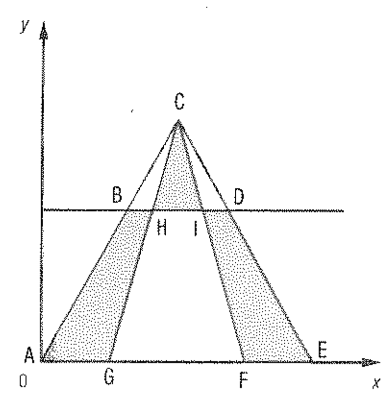
H(4.5, 1); I(11.5, 1)

Point C corresponds to the vertex of the curve associated with the function, in other words (8, 8).

$A_{\text{triangle}} = \frac{b \times h}{2}$

$A_{\text{triangle}} = 24.5$

The area of triangle HCI is 24.5 units².



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Area of quadrilateral ABHG

The area of trapezoid ABDE must be determined.
The coordinates of points B and D must first be determined:

$1 = -|x - 8| + 8$

$15 = x$ or $1 = x$

B(1, 1); D(15, 1).

The coordinates of points A and E are A(0, 0) and E(16, 0).

$A_{\text{trapezoid}} = \frac{(B + b) \times h}{2}$

$A_{\text{trapezoid}} = 15$

The area of trapezoid ABDE is 15 units^2.
The area of trapezoid GHIF must be determined.

The coordinates of points G and F must first be determined:

$0 = -2|x - 8| + 8$

$12 = x$ or $4 = x$

G(4, 0); F(12, 0).

The coordinates of points H and I are H(4.5, 1) and (11.5, 1).

$A_{\text{trapezoid}} = \frac{(B + b) \times h}{2}$

$A_{\text{trapezoid}} = 7.5$

The area of trapezoid GHIF is 7.5 units^2.

The sum of the areas of quadrilaterals ABHG and FIDE is $15 - 7.5 = 7.5$ units^2.

Quadrilaterals ABHG and FIDE are equivalent. The area of quadrilateral ABHG is equal to half the sum of the areas of quadrilaterals ABHG and FIDE, therefore 3.75 units^2.

The area of the green region is equal to the area of triangle HCI + the area of quadrilateral ABHG:
$24.5 + 3.75 = 28.25$ units^2.

- The area of the white region must be determined.

  The area of equilateral triangles BCH and ICD must be determined.

  The coordinates of points B, C and H are B(1, 1), C(8, 8) and H(4.5, 1).

  $A_{\text{triangle}} = \frac{b \times h}{2}$

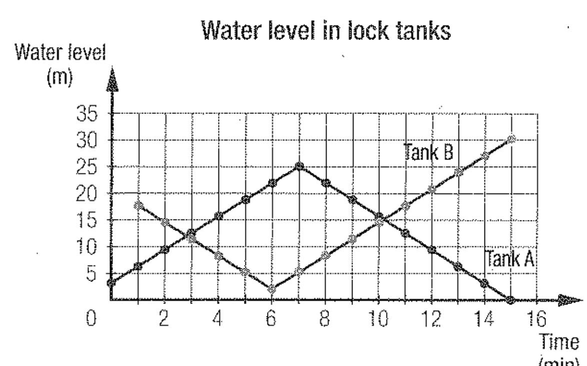
  $A_{\text{triangle}} = 12.25$

  The area of equilateral triangles BCH and ICD is 12.25 units^2.

  Quadrilateral FIDE is equivalent to quadrilateral ABHG, that is 3.75 units^2.

  The area of the white region is equal to the area of triangle BCH + the area of triangle ICD + the area of quadrilateral FIDE: $12.25 + 12.25 + 3.75 = 28.25$ units^2.

  The green region and white region have the same area, that is 28.25 units^2.



9. • Present the data from the table on a Cartesian plane.

- Determine the coordinates of the first intersection point of the curves.

  The equation of the segment of an increasing line of Equation **A** is $y = 3.125x + 3.125$;
  the equation of the segment of a decreasing line of Equation **B** is $y = -3.125x + 20.75$.
  Solve this system of equations:
  $3.125x + 3.125 = -3.125x + 20.75$

  At 2.82 min, the water level is the same in the two tanks, that is 11.94 m.

- Determine the coordinates of the second intersection point of the curves.

  The equation of the segment of an increasing line of Equation **B** is $y = 3.125x - 16.75$.
  The equation of the segment of a decreasing line of Equation **A** is $y = -3.125x + 46.875$.

Solve this system of equations:
$3.125x - 16.75 = -3.125x + 46.875$

At 10.18 min, the water level is the same in the two lock tanks, that is 15.06 m.

10. • Determine the speed (in m/s) of the space shuttle.

Mean altitude if the shuttle: 350 km
Mean radius of the Earth: 6371 km
Radius of the shuttle's orbit: 350 + 6371 = 6721 km.

The speed of a satellite: $S = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6\,721\,000}}$.
$S \approx 7697.21$ m/s

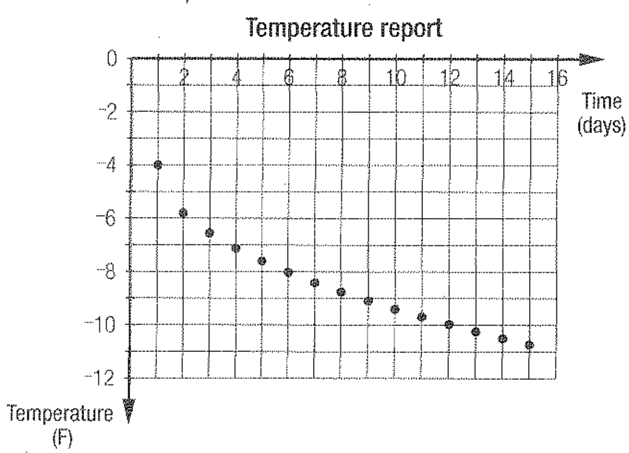
• Calculate the speed, in Mach of the shuttle.

$M = \frac{7697.21}{331.3} \approx 23.23$.

The space shuttle approximately 23.23 Mach.

11. • Determine the rule that calculates temperature (in °F) based on time in days.

By representing the scatter plot associated with the temperature report, one can see that the curve that best fits this situation is the square root function.



The rule of this function is $F = a\sqrt{x - 1} - 4$.

The curve passes through point (10, -9.4).

The rule that calculates the temperature, in °F, based on time (days) is $F = -1.8\sqrt{x - 1} - 4$.

• Determine the rule that calculates the temperature, in °C, based on time (days).

The rule that converts °F into °C is $C = \frac{5(F - 32)}{9}$.

The rule that calculates the temperature, in °C, based on time (days) is therefore $C(F(x))$. Thus:

$C(F(x)) = \frac{5((-1.8\sqrt{x-1} - 4) - 32)}{9}$

$C(F(x)) = -\sqrt{x - 1} - 20$

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