student book volume 1

Science

VISIONS

MATHEMATICS

Secondary Cycle Two, Year Three

ANSWER KEY

Vision 2

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PRELIMINARY VERSION

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The following is an example of an approach that results in the creation of the documents requested:

- Represent the constraints of Supplier **A** and the company's needs.

  The constraints of Supplier **A** can be translated by the following system of inequalities:

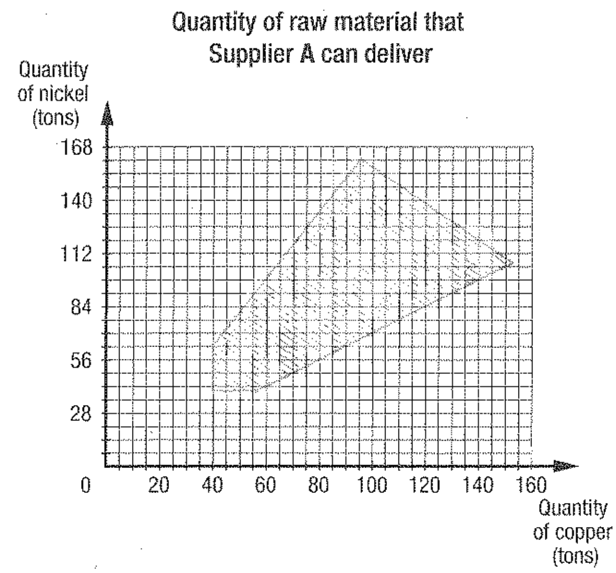
  x : quantity of copper (tons)
  y : quantity of nickel (tons)
  $x \geq 40$
  $y \geq 40$
  $x + y \leq 260$
  $y \leq 1.6x$
  $y \geq 0.7x$

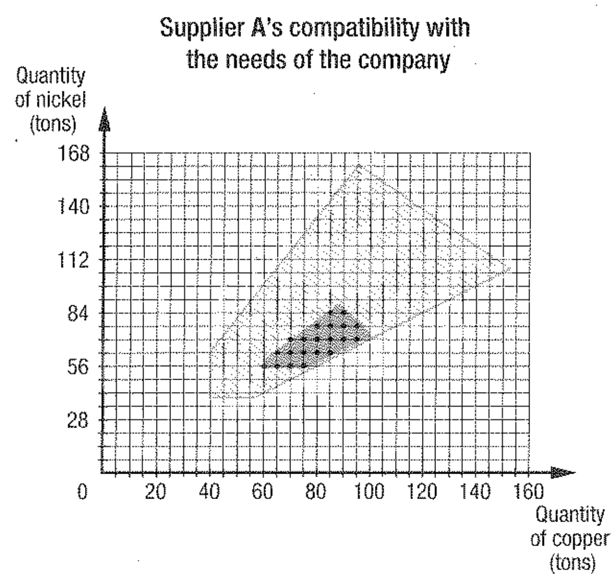
  Since each shipment of copper contains 5 tons and each shipment of nickel contains 7 tons, the polygon of constraints should be presented on a Cartesian plane scaled in tons and whose increment of change on the x-axis is 5 and whose increment of change on the y-axis is 7. Therefore, only the points located on the polygon of constraints and at the intersection of the lines on the Cartesian plane would be valid solutions for the supplier.

  The needs of the company can be translated by the following system of inequalities:

  $x \geq 60$
  $y \geq 55$
  $x + y \leq 200$
  $x \geq y$
  $x \leq 1.4y$
  $3500x + 10\,500y \leq 1\,327\,000$
  $\frac{x}{5} + \frac{y}{7} \leq 30$ (Since the total amount of raw material must be delivered in 5 days maximum, and Supplier **A** can deliver up to 6 shipments/day.)

  The polygon of constraints associated with the needs of the company can be presented by being superimposed onto the polygon of constraints that represents the constraints of the supplier. The black dots are associated with the ordered pairs that would satisfy both Supplier **A** and the company.





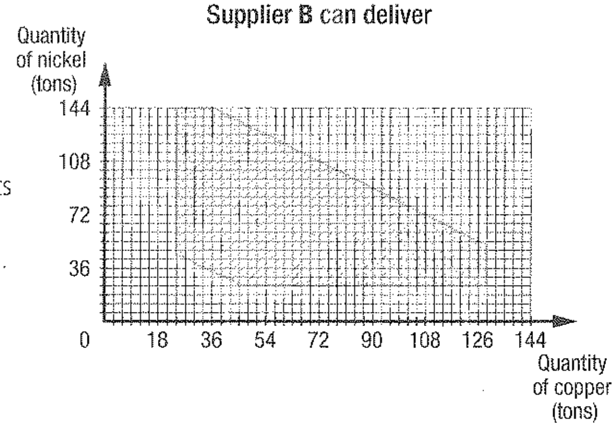
- Represent the constraints of Supplier **B** and the needs of the company.

  Since each shipment of copper contains 3 tons, and each shipment of nickel contains 6 tons, the polygon of constraints should be presented on a Cartesian plane scaled in tons and whose increment of change on the x-axis is 3 and whose increment of change on the y-axis is 6. Therefore, only the points that are located on the polygon of constraints and at the intersection of the lines on the Cartesian plane would be valid solutions for Supplier **B**.

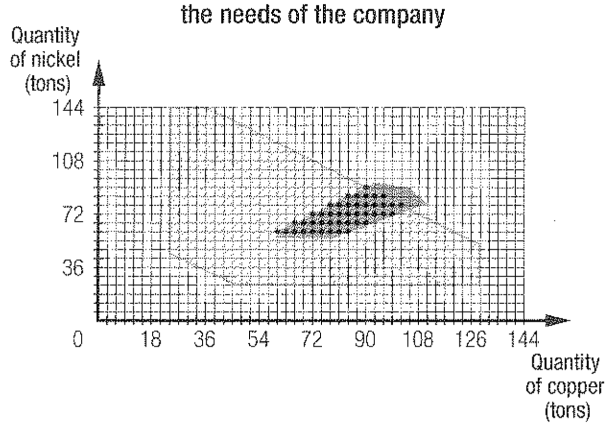
  The needs of the company can be translated by the following system of inequalities:
  $x \geq 60$
  $y \geq 55$
  $x + y \leq 200$
  $x \geq y$
  $x \leq 1.4y$
  $2800x + 11\ 500y \leq 1\ 327\ 000$
  $\frac{x}{3} + y \leq 50$ (since the total amount of raw material must be delivered in at most 5 days and Supplier **B** can deliver up to 10 shipments/day)

  The polygon of constraints associated with the needs of the company can be presented by being superimposed onto the polygon of constraints that represents the constraints of the supplier. The black dots are associated with the ordered pairs that would satisfy both Supplier **B** and the company.

Quantity of raw material that Supplier B can deliver



Supplier B's compatibility with the needs of the company



- Table of possibilities

Supplier A

Number of shipments			Quantity of raw material (tons)			Cost ($)	Moment when the total amount of raw material is delivered
Copper	Nickel	Total	Copper	Nickel	Total		
12	8	20	60	56	116	798,000	during 4th day
13	8	21	65	56	121	815,500	during 4th day
14	8	22	70	56	126	833,000	during 4th day
15	8	23	75	56	131	850,500	during 4th day
13	9	22	65	63	128	889,000	during 4th day
14	9	23	70	63	133	906,500	during 4th day
15	9	24	75	63	138	924,000	end of 4th day
16	9	25	80	63	143	941,500	during 5th day
17	9	26	85	63	148	959,000	during 5th day
14	10	24	70	70	140	980,000	end of 4th day
15	10	25	75	70	145	997,500	during 5th day
16	10	26	80	70	150	1,015,000	during 5th day
17	10	27	85	70	155	1,032,500	during 5th day
18	10	28	90	70	160	1,050,000	during 5th day
19	10	29	95	70	165	1,067,500	during 5th day
16	11	27	80	77	157	1,088,500	during 5th day
17	11	28	85	77	162	1,106,000	during 5th day
18	11	29	90	77	167	1,123,500	during 5th day
19	11	30	95	77	172	1,141,000	end of 5th day
17	12	29	85	84	169	1,179,500	during 5th day
18	12	30	90	84	174	1,197,000	end of 5th day

Supplier B

Number of shipments			Quantity of raw material (tons)			Cost ($)	Moment when the total amount of raw material is delivered
Copper	Nickel	Total	Copper	Nickel	Total		
20	10	30	60	60	120	858,000	end of 3rd day
21	10	31	63	60	123	866,400	during 4th day
22	10	32	66	60	126	874,800	during 4th day
23	10	33	69	60	129	883,200	during 4th day
24	10	34	72	60	132	891,600	during 4th day
25	10	35	75	60	135	900,000	during 4th day
26	10	36	78	60	138	908,400	end of 4th day
27	10	37	81	60	141	916,800	during 4th day
28	10	38	84	60	144	925,200	during 4th day
22	11	33	66	66	132	943,800	end of 4th day
23	11	34	69	66	135	952,200	during 4th day
24	11	35	72	66	138	960,600	during 4th day
25	11	36	75	66	141	969,000	during 4th day
26	11	37	78	66	144	977,400	during 4th day
27	11	38	81	66	147	985,800	during 4th day
28	11	39	84	66	150	994,200	during 4th day
29	11	40	87	66	153	1,002,600	end of 4th day
30	11	41	90	66	156	1,011,000	during 5th day
24	12	36	72	72	144	1,029,600	during 5th day
25	12	37	75	72	147	1,038,000	during 5th day
26	12	38	78	72	150	1,046,400	during 5th day
27	12	39	81	72	153	1,054,800	during 5th day
28	12	40	84	72	156	1,063,200	during 5th day
29	12	41	87	72	159	1,071,600	during 5th day
30	12	42	90	72	162	1,080,000	during 5th day
31	12	43	93	72	165	1,088,400	during 5th day
32	12	44	96	72	168	1,096,800	during 5th day
33	12	45	99	72	171	1,105,200	during 5th day
26	13	39	78	78	156	1,115,400	during 4th day
27	13	40	81	78	159	1,123,800	end of 4th day
28	13	41	84	78	162	1,132,200	during 5th day
29	13	42	87	78	165	1,140,600	during 5th day
30	13	43	90	78	168	1,149,000	during 5th day
31	13	44	93	78	171	1,157,400	during 5th day
32	13	45	96	78	174	1,165,800	during 5th day
33	13	46	99	78	177	1,174,200	during 5th day
34	13	47	102	78	180	1,182,600	during 5th day
28	14	42	84	84	168	1,201,200	during 5th day
29	14	43	87	84	171	1,209,600	during 5th day
30	14	44	90	84	174	1,218,000	during 5th day
31	14	45	93	84	177	1,226,400	during 5th day
32	14	46	96	84	180	1,234,800	end of 5th day
30	15	45	90	90	180	1,287,000	during 5th day

Maintenance engineers

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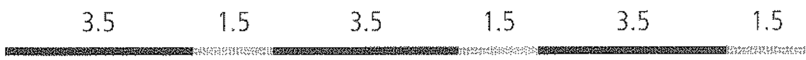
The following is an example of an approach that could result in solving the situational problem:

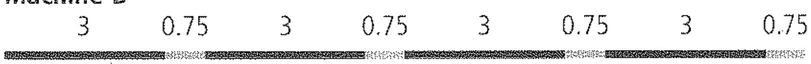
A) Analysis of a 15-day cycle

Machine A



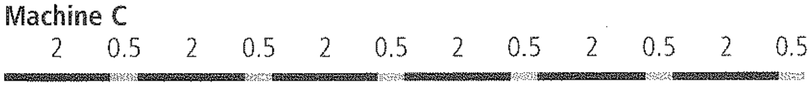
Machine **A** functions 10.5 days out of 15 and requires maintenance 3 times within each cycle of 15 days.

Machine B



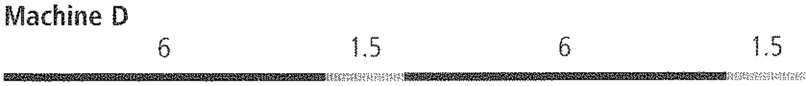
Machine **B** functions 12 days out of 15 and requires maintenance 4 times within each cycle of 15 days.

Machine C



Machine **C** functions 12 days out of 15 and requires maintenance 6 times within each cycle of 15 days.

Machine D



Machine **D** functions 12 days out of 15 and requires maintenance 2 times within each cycle of 15 days.

B) Combining Machines A and C

If x represents the number of Type-**A** machines and y represents the number of Type-**C** machines, the given constraints can be translated by the following system of inequalities:

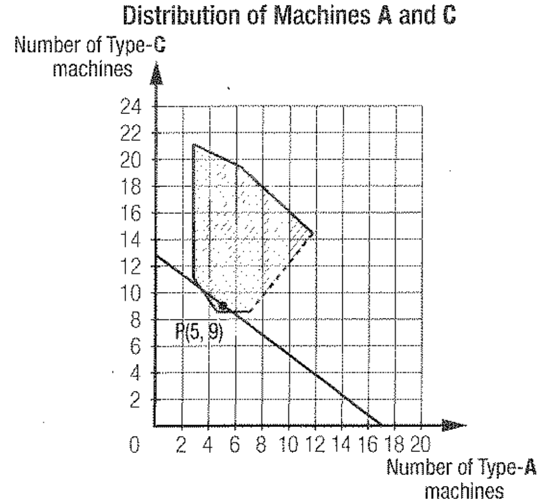
$10.5(405x) \geq 12\ 000$
$12(290y) \geq 30\ 000$
$10.5(405x) + 12(290)y \geq 50\ 000$
$12(290y) > 10.5(405x)$
$45x + 53y \leq 1300$

Since the costs related to the maintenance of the machines must be minimized, the optimizing function is given by the rule $C = 3(600x) + 6(400y)$.

The graph shown in the adjacent image illustrates the polygon of constraints as well as a scanning line associated with this situation.

In this context, only the points whose coordinates are whole numbers can be valid solutions, therefore the coordinates of point P(5, 9) generate the minimum of the optimizing function.

Distribution of Machines A and C



C) Combining Machines A and D

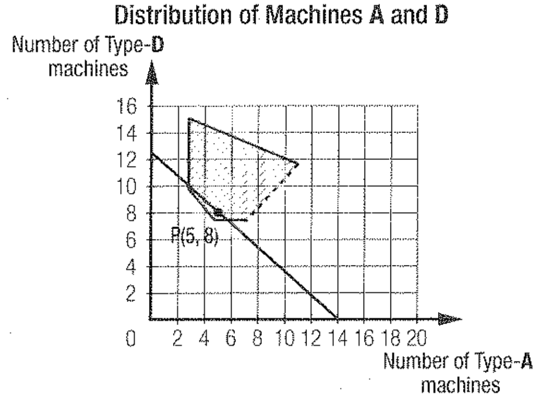
If x represents the number of Type-**A** machines and y represents the number of Type-**D** machines, the given constraints can be translated by the following system of inequalities:

$10.5(405x) \geq 12\ 000$
$12(325y) \geq 30\ 000$
$10.5(405x) + 12(325)y \geq 50\ 000$
$12(325y) > 10.5(405x)$
$45x + 57y \leq 1300$

Since the costs related to the maintenance of the machines must be minimized, the optimizing function is given by the rule $C = 4(500x) + 2(1000y)$.

The graph shown in the adjacent image illustrates the polygon of constraints as well as a scanning line associated with this situation.

Distribution of Machines A and D



In this context, only the points whose coordinates are whole numbers can be valid solutions; therefore the coordinates of point P(5, 8) generate the minimum of the optimizing function.

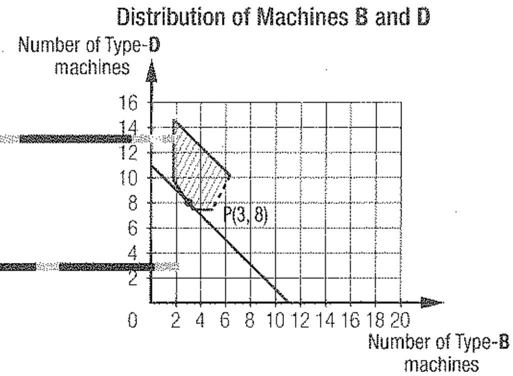
D) Combining Machines B and D

If x represents the number of Type-**B** machines and y represents the number of Type-**D** machines, the given constraints can be translated by the following system of inequalities:

$12(520x) \geq 12\ 000$
$12(325y) \geq 30\ 000$
$12(520x) + 12(325)y \geq 50\ 000$
$12(325y) > 12(520x)$
$38x + 57y \leq 1300$

Since the costs related to the maintenance of the machines must be minimized, the optimizing function is given by the rule $C = 4(500x) + 2(1000y)$.

The graph shown in the adjacent image illustrates the polygon of constraints as well as a scanning line associated with this situation.



In this context, only the points whose coordinates are whole numbers can be valid solutions; therefore the coordinates of point P(3, 8) generate a minimum of the optimizing function.

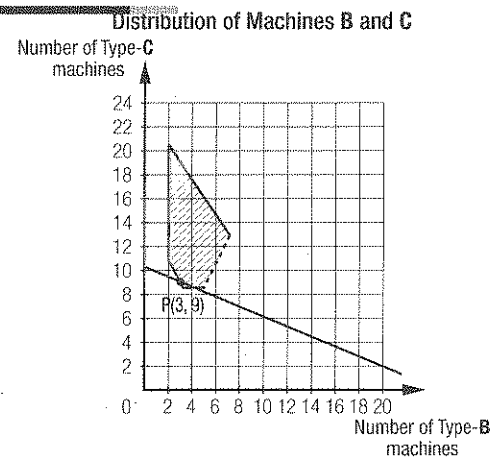
E) Combining Machines B and C

If x represents the number of Type-**B** machines and y represents the number of Type-**C** machines, the given constraints can be translated by the following system of inequalities:

$12(520x) \geq 12\ 000$
$12(290y) \geq 30\ 000$
$12(520x) + 12(290)y \geq 50\ 000$
$12(290y) > 12(520x)$
$38x + 53y \leq 1300$

Since the costs related to the maintenance of the machines must be minimized, the optimizing function is given by the rule $C = 4(500x) + 6(400y)$.

The graph shown in the adjacent image illustrates the polygon of constraints as well as a scanning line associated with this situation.



In this context, only the points whose coordinates are whole numbers can be valid solutions; therefore the coordinates of point P(3, 9) generate the minimum of the optimizing function.

F) Comparing the four optimal solutions

Distribution	Cost of purchase ($ in thousands)	Staff required	For a cycle of 15 days		
			Number of screw produced	Number of dials produced	Maintenance cost ($)
5 Type-**A** machines and 9 Type-**C** machines	702	60	21 262	31 320	30,600
5 Type-**A** machines and 8 Type-**D** machines	681	71	21 262	31 200	25,000
3 Type-**B** machines and 8 Type-**D** machines	570	77	18 720	31 200	22,000
3 Type-**B** machines and 9 Type-**C** machines	591	66	18 720	31 320	25,200

The company must obtain 3 Type-**B** machines and 8 Type-**D** machines. This set of machines:

- costs $570,000
- requires 77 employees
- produces 18 720 screws and 31 200 dials in 15 days
- has a maintenance cost of $22,000 for each 15-day cycle

REVISION 2

Prior learning 1

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a. 1) 12 m 2) 3 m

b. 1) $y = \frac{-3x}{14} + 12$ 2) $y = \frac{x}{6} + 3$

c. 1) $\frac{75}{7}$ m 2) 4 m

d. 1) At approximately 23.62 min. 2) Approximately 3.94 m.

Prior learning 2

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a.

Screen	A	B	C	D
Description	**The measure of the border must be less than 1 dm.**	The area of the red borders is at least 35 dm^2 greater than the area of the blue borders.	The perimeter of the screen, including the border, is at most 76 dm.	The sum of the areas of the red and blue borders is less than or equal to the area of the screen.
Inequality	$x < 1$	$2(24x) \geq 2(9x) + 35$	$\mathbf{2(2x + 8) + 2(2x + 20) \leq 76}$	$144 - 50x \geq 0$

b. A $x < 1$ B $x \geq \frac{7}{6}$ C $x \leq \frac{5}{2}$ D $x \leq \frac{72}{25}$

c. 1) No, because the border must have a width that is less than 1 dm.
2) Yes, because the border must have a width that is greater than or equal to $\frac{7}{6}$ dm.

d. 0 dm

e. $\frac{1826}{25}$ dm

Prior learning 3

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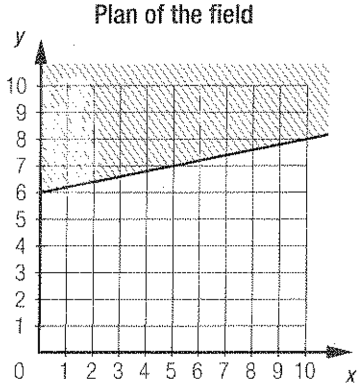
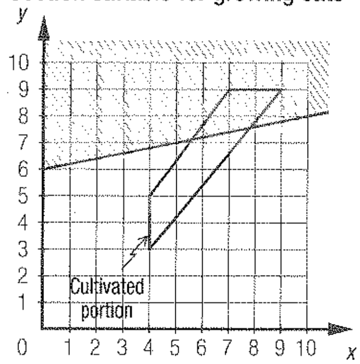
a. Plan of the field



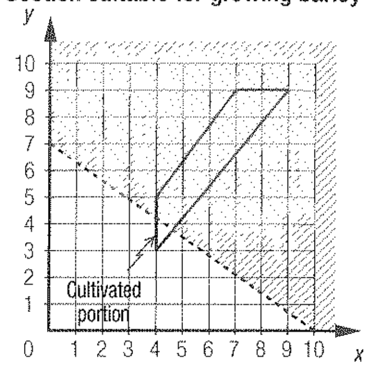
b. $y > -0.7x + 7$

c. The coordinates of the vertices of the quadrilateral that corresponds to the cultivated portion do not satisfy the inequality $y < -\frac{1}{2}x + 4.5$.

d. Section suitable for growing oats



Section suitable for growing barley



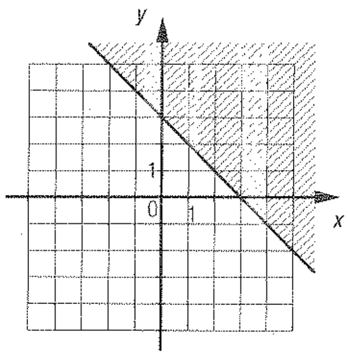
The section suitable for growing barley is more advantageous.

Knowledge in action

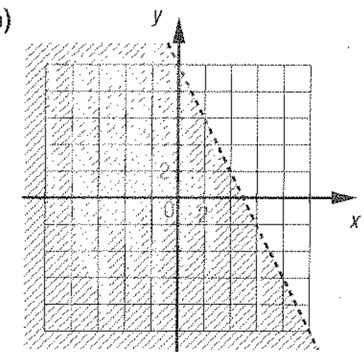
1. a) (-4.5, -11.5) b) $(4.\overline{8}, 27.\overline{5})$ c) (180, 30)
   d) There are no solutions to this system. e) $\left(\frac{510}{31}, -\frac{120}{31}\right)$ f) $\left(-\frac{445}{27}, -\frac{718}{27}\right)$

2. a) $x \geq -6$ b) $x < 2.5$ c) $x \geq -4$ d) $x > \frac{77}{3}$ e) $x \leq \frac{7}{3}$ f) $x \geq \frac{4}{3}$

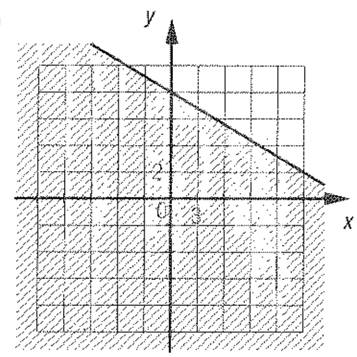
3. a)



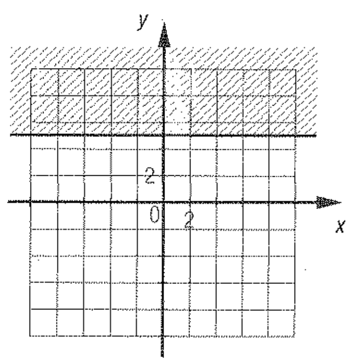
b)



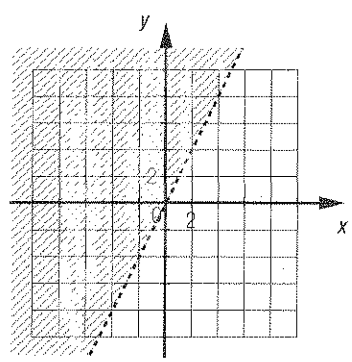
c)



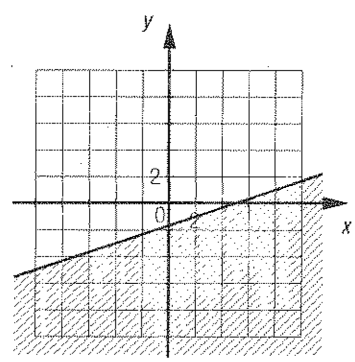
d)



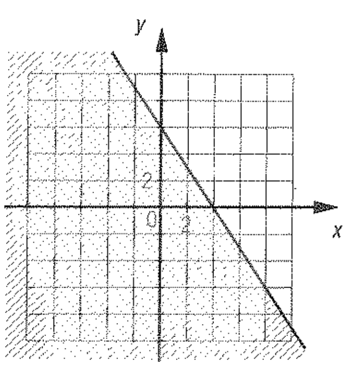
e)



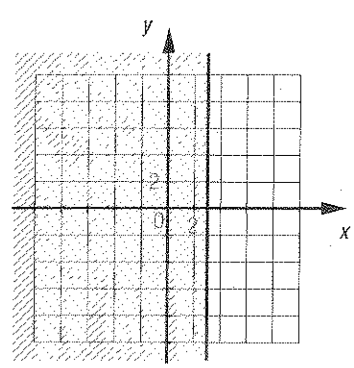
f)



g)



h)



4. Ⓐ ⑤, Ⓑ ①, Ⓒ ⑥, Ⓓ ③, Ⓔ ②, Ⓕ ④

Knowledge in action (*cont'd*)

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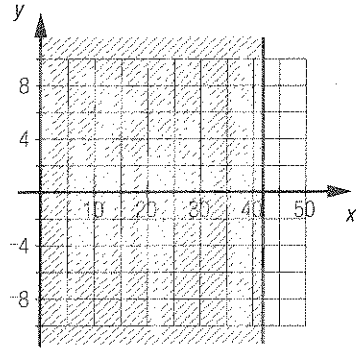
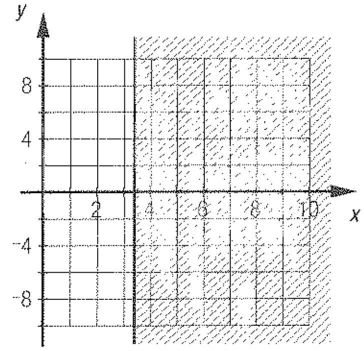
5. a) $y < -\frac{2}{3}x + 8$ b) $y \geq 0.5x - 20$ c) $y < 3x - 3$ d) $y \geq \frac{4}{3}x - 8$

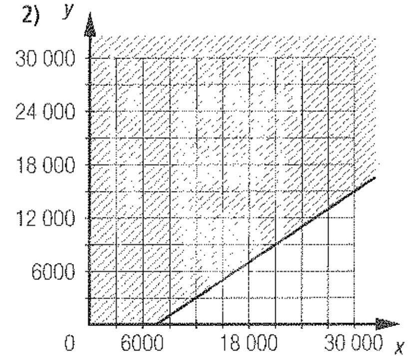
6. a) 1) $\frac{x}{2} + 6 \leq 27$

2) 

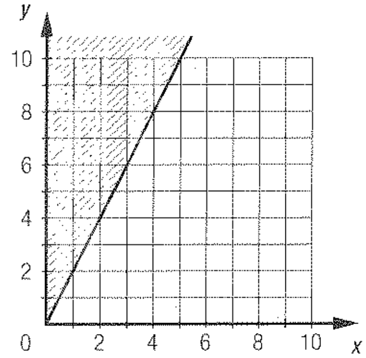
b) 1) $-2x \leq \frac{2x}{3} - 9$

2) 

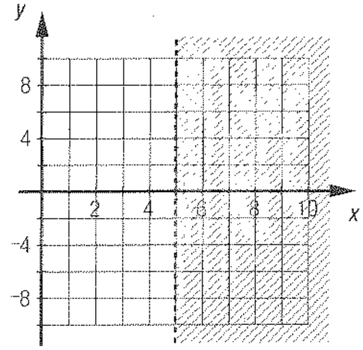
c) 1) $2x - 3y \leq 1500$

2) 

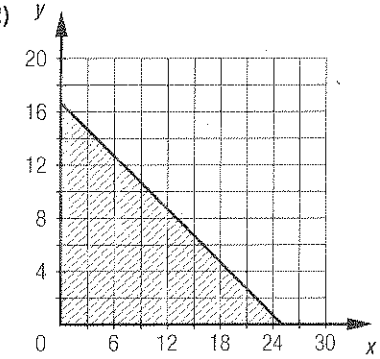
d) 1) $x \leq \frac{y}{2}$

2) 

e) 1) $2x - 10 > 0$

2) 

f) 1) $4x + 6y \leq 100$

2) 

Knowledge in action (*cont'd*)

Page 109

7. a) 1) x: time (in s) and y: height (in m) of an elevator.
2) $y = 23 - 0.75x$ and $y = 2 + 0.5x$.
3) The elevators meet at a height of 10.4 m after 16.8 s.

b) 1) x: time (in s) and y: temperature (in °C) of a liquid.
2) $y = 0.1x + 40$ and $y = 0.3x + 20$.
3) Both liquids are at the same temperature (50 °C) after 100 s.

c) 1) x: time (in s) and y: distance (in m) covered by the object.
2) $y = 8x + 100$ and $y = 10x$.
3) The second object would catch up to the first object after 50 s.

8. a) The values of angle B must be at least 37.5° and less than 60°. The values of angle C are associated with the values of angle B so that the sum of angles A, B and C is 180°.

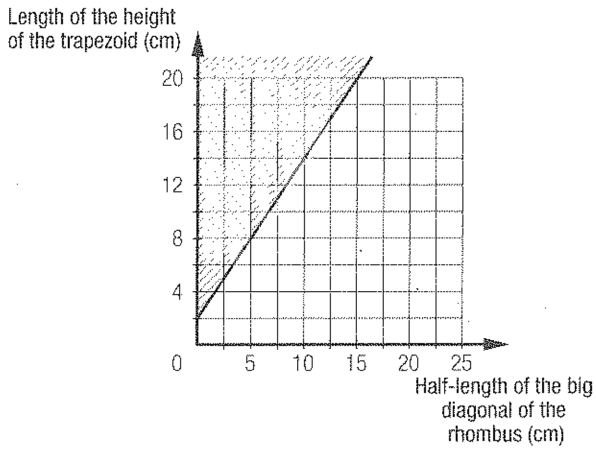
b) The possible values for angle B must be less than $\frac{300}{7}$° and greater than 0°. The values of angle C are associated with the values of angle B so that the sum of angles A, B and C is 180°.

9. a) $36p \geq 384$ and $36p \leq 476$. b) $\left[\frac{32}{2}, \frac{119}{9}\right]$ cm

Knowledge in action (*cont'd*)

Page 110

10. a) $6x \leq 5y - 10$

b) 

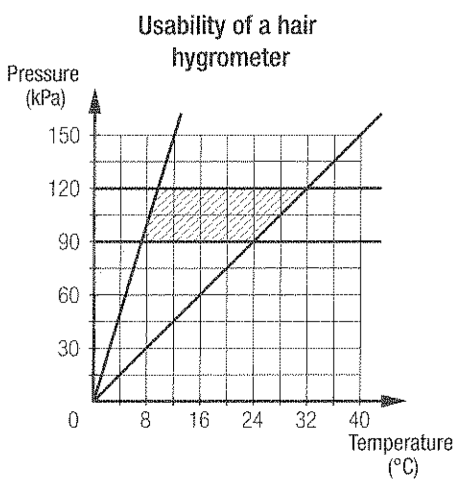
c) *Several answers possible. Example:* (3, 6), (6, 10) and (12, 17).

11. a) 1) $x^2 + 5^2 \leq 12^2$ 2) $\sqrt{119}$ dm

b) 1) $x^2 + y^2 \leq 144$ 2) On and under the curve. 3) On the curve.

SECTION 2.1 | Systems of inequalities and polygons of constraints

Problem Page 111



d) 1) x : montant des ventes (en $) d'un vendeur et y : salaire (en $) d'un vendeur.
2) $y = 0{,}2x +$

Activity 1 Page 112

a. x: number of saliva samples y : number of blood samples

b. Graph ①: $x < \frac{1}{3}y$, Graph ②: $x + y \leq 360$.

c.

	$x < \frac{1}{3}y$	$x + y \leq 360$
A(45, 180)	**Yes**	**Yes**
B(90, 235)	**No**	**Yes**
C(80, 290)	**Yes**	**No**
D(135, 235)	**No**	**No**

d. In order to represent the solution set that the two inequalities have in common.

e. 1) Ⓐ and Ⓓ 2) Ⓒ and Ⓓ 3) Ⓓ 4) Ⓑ

f. No. This point corresponds to the intersection point of the two lines. This point belongs to the solution set of the inequality $x + y \leq 360$, and not to the solution set of the inequality $x < \frac{1}{3}y$.

Activity 2

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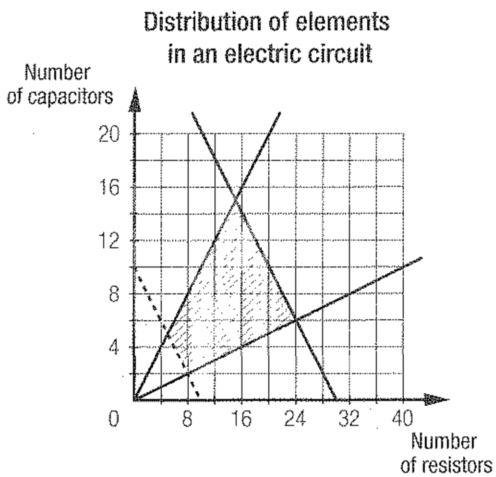
a. $x + y > 10$, $x + y \leq 30$, $x \geq y$ and $x \leq 4y$.

b. There cannot be a negative number of resistors and capacitors.

c.



d. Right trapezoid.

e. 1) Yes. 2) No.

f. $y \geq 0$, $x + y > 14$, $x + y \leq 28$ and $x \leq y$.

g.
- The number of capacitors must be greater than or equal to 0.
- The number of resistors combined with the number of capacitors must be greater than 14.
- The number of resistors combined with the number of capacitors cannot exceed 28.
- The number of resistors must be less than or equal to the number of capacitors.

h. A(14, 14), B(7, 7), C(0, 14), D(0, 28)

i. Points A and D are part of the solution set because they satisfy each of the system's inequalities. Points B and C are not part of the solution set because they do not satisfy the inequality $x + y > 14$.

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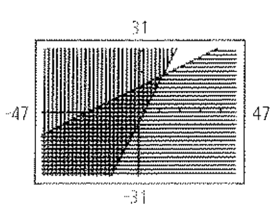
a. 1) The half-plane located above the boundary line must be shaded.
2) The half-plane located below the boundary line must be shaded.

b. 1) $y \geq 1.5x + 15$ 2) $y \leq -0.3x - 10$

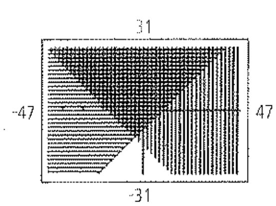
c. $y \geq x$ and $y \geq 30 - x$.

d. 1) Point (11, -12) does not belong to the system's solution set because it does not satisfy any of the inequalities: $y > x \Leftrightarrow -12 > 11$ (false) and $y > 30 - x \Leftrightarrow -12 > 19$ (false).
2) Le point (15, 26) belongs to the system's solution set because it satisfies both inequalities: $y > x \Leftrightarrow 26 > 15$ (true) and $y > 30 - x \Leftrightarrow 26 > 15$ (true).

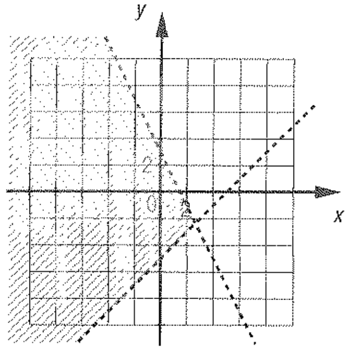
e. 1)

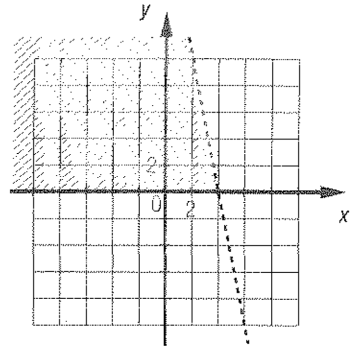


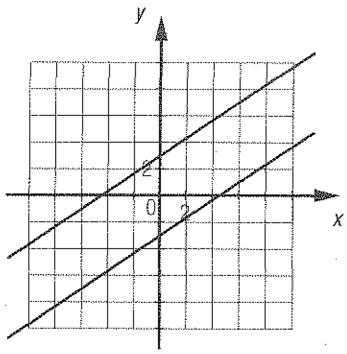
2)

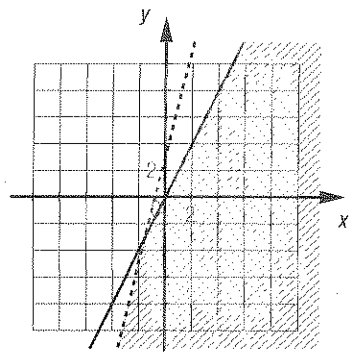


Practice 2.1

1. a)


b)


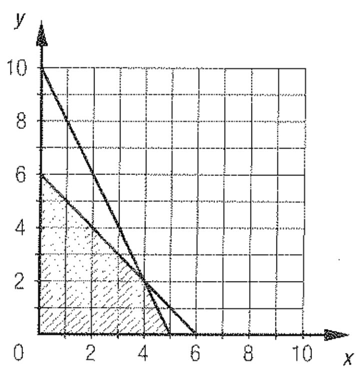
c)


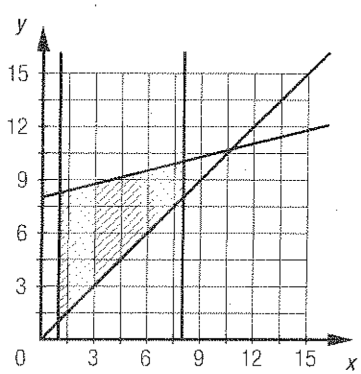
d)


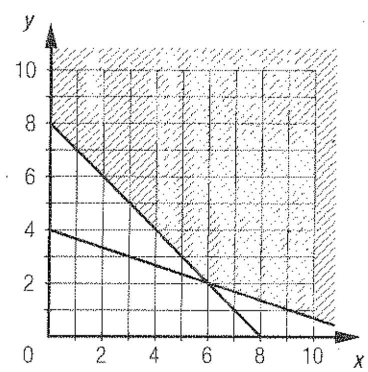
2. a) 1) $y < 2x - 2$ $y \leq -0.5x - 1$ 2) $y < 2x - 2$ $y \geq -0.5x - 1$
3) $y > 2x - 2$ $y \geq -0.5x - 1$ 4) $y > 2x - 2$ $y \leq -0.5x - 1$

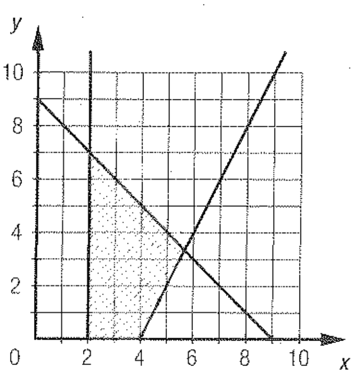
b) No, because the line with equation $y = 2x - 2$ is not part of the solution set, and therefore the intersection point does not belong to the solution set.

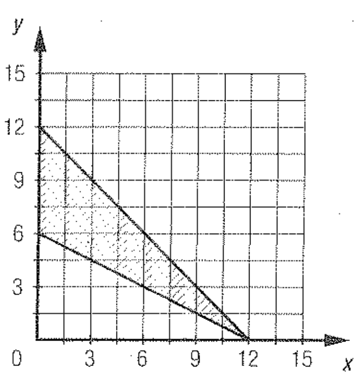
3. a) A(0, 0), C(-4, 4), E(-3, 2), F(-5, -6)
b) D(3, -2)
c) B(2, 3), C(-4, 4), E(-3, 2)
d) B(2, 3), D(3, -2)

4. a)


b)


c)


d)


e)


5. a) $x < -5$
$y \geq 6$
$3x + 2y < 18$
$y \geq \frac{-2x}{3} - \frac{20}{3}$

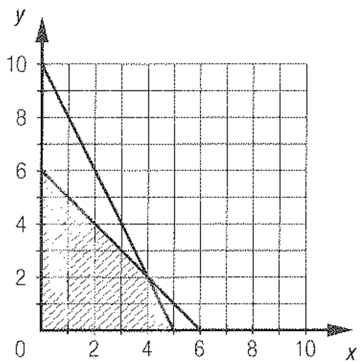
b) $y \geq 6$
$y < 5x - 20$
$y \geq 0.8x - 8$

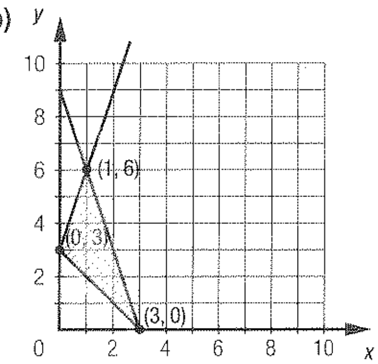
c) $y \leq 0.8x - 8$
$y < 5x - 20$
$y \geq \frac{-2x}{3} - \frac{20}{3}$
$3x + 2y < 18$

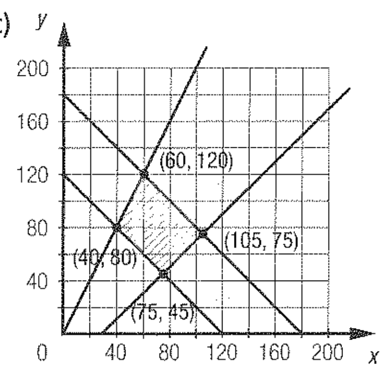
d) $y \leq 6$
$x > -5$
$y > 5x - 20$
$y \geq \frac{-2x}{3} - \frac{20}{3}$
$3x + 2y < 18$
$y \geq 0.8x - 8$

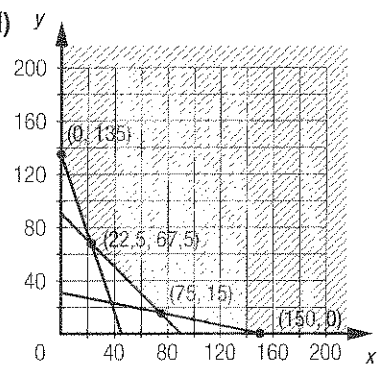
Practice 2.1 *(cont'd)*

6. a) A(0, 6), B(8, 10), C(20, 0), D(0, 4)
b) A(0, 12), B(3, 15), C(7,5 15), D$\left(\frac{8}{3}, \frac{16}{3}\right)$, E(0, 8)
c) A(0, 8), B$\left(\frac{8}{13}, \frac{72}{13}\right)$, C(8, 0)
d) A(0, 2), B(6, 1), C(9, 0), D(0, 0)

7. a)


b)


c)


d)


8. a) $x > 0$ and $y \geq 2x$
b) $y \leq 0$ and $x \leq \frac{y}{3}$
c) $y > x$ and $y \leq 4x$
d) $x + y > 0$ and $x + y \leq 12$
e) $y \geq x + 5$ and $y \leq x + 10$

Practice 2.1 *(cont'd)*

9. a) $y \leq -x^2$ and $y > -2(0,8)^x$.
b) $y \leq 2^x$ and $y > \frac{1}{x}$
c) $y \leq [x]$ and $y \geq x - 1$
d) $y \leq |x|$ and $y \geq |x|$

10. a) d_1: ⑤, the number of fitting rooms for women must be greater than the number of fitting rooms for men.
d_2: ④, there must be at least 3 fitting rooms for men.
d_3: ③, there must be at least 5 fitting rooms for women.
d_4: ①, the total number of fitting rooms must not exceed 15.
d_5: ②, the total number of fitting rooms must be greater than 10.

b) Constraint ③, there must be at least 5 fitting rooms for women. Without this constraint, the polygon would remain unchanged.

c) 1) Because the manager must not count the solutions associated with points (3, 7), (4, 6) and (5, 5), since these solutions do not satisfy Constraint ①, the total number of fitting rooms must be greater than 10.
2) 21 solutions.

Practice 2.1 (*cont'd*)

11. a)

Situation ①	Situation ②
$x \leq \frac{y}{2}$ $x + y \geq 15$ $x + y \leq 26$	$x \leq \frac{y}{2}$ $x + y \geq 15$ $x + y \leq 26$

Both situations are made up of the same inequalities.

b) 1) Yes, because the point satisfies each inequality.

2) No, because the number of fir trees and the number of maple trees must be whole numbers.

c) 1) $\mathbb{R}_+^*$ **2)** $\mathbb{N}$

12. a) (A), (D) **b)** (B) **c)** (C) **d)** (E)

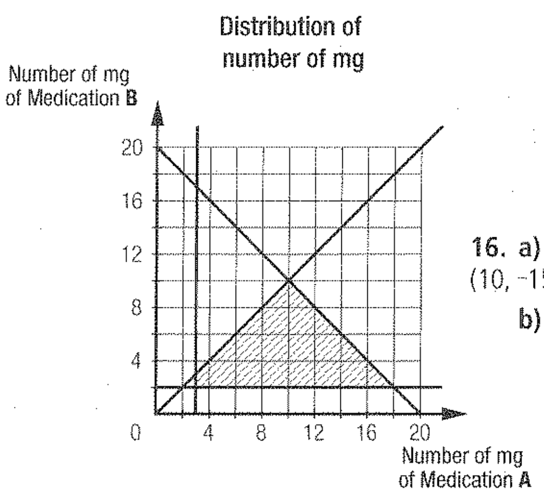
Practice 2.1 (*cont'd*)

13. a) 1) x: number of mg of Medication **A**

y: number of mg of Medication **B**

2) $x \geq 0$
$y \geq 0$
$x \geq 3$
$y \geq 2$
$x + y \leq 20$
$x \geq \frac{x + y}{2}$

3)



4) (3, 2), (3, 3), (10, 10) and (18, 2).

5) All the vertices are part of the solution set.

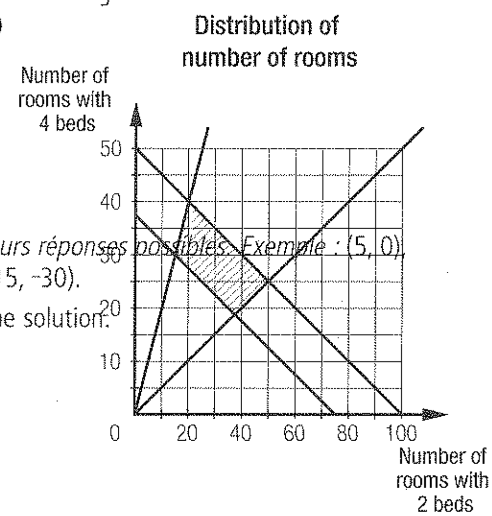
6) *Several answers possible. Example:* (10, 3), (12, 4) and (14, 5).

b) 1) x: number of rooms with 2 beds

y: number of rooms with 4 beds

2) $x \geq 0$
$y \geq 0$
$2x + 4y \geq 150$
$2x + 4y \leq 200$
$x \geq \frac{x + y}{3}$
$x \leq \frac{2(x + y)}{3}$

3)



4) (15, 30), (20, 40), (50, 25) et (37,5, 18,75).

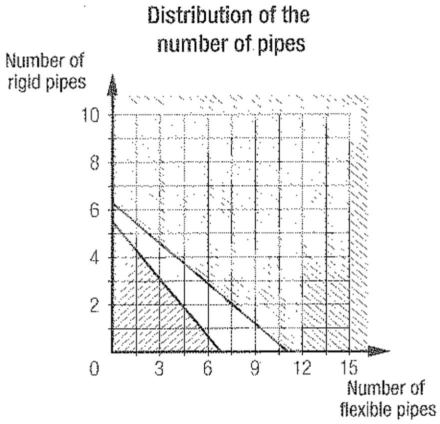
5) Only the vertex (37.5, 18.75) is not part of the solution set because the coordinates are not whole numbers.

6) *Several answers possible. Example:* (25, 30), (30, 30) and (40, 25).

16. a) *Plusieurs réponses possibles. Exemple :* (5, 0), (10, −15) et (15, −30).

b) Aucune solution.

14. a)



b) No. The two half-planes associated with each constraint do not intersect.

c) *Several answers possible. Example:*
The set of installed pipes must allow a minimum runoff of 6 L/min.

15. a) *Several answers possible. Example:* (5, 0), (4, 3) and (6, -3). **b)** No solution.

Practice 2.1 (*cont'd*) **Page 122**

16. a) 15 years old.

b) 1) Zone Ⓐ: intense training, Zone Ⓑ: improvement in cardiovascular capabilities, Zone Ⓒ: mass reduction, Zone Ⓓ: maintaining current physical condition

2) $x \geq 195$, $x \leq 215$, $y \geq 0.6x$ and $y \leq 0.65x$.

c) From 120 inclusive to 130 exclusive beats/min.

SECTION 2.2 Target objective and optimal solutions

Problem **Page 123**

Several answers possible. Example:
Each plate must be 20-cm thick and have a base with an area of 10 m^2.

Activity 1 **Page 124**

a. 1) $C = 1.5x + 0.5y$

2) No, because it determines the total cost of purchase based on the constraints.

b. $x \geq 0$
$y \geq 0$
$0.5x + 3y \geq 15$
$x + y \leq 15$
$y < \frac{2(x + y)}{3}$

c.

Point	Calculation of the cost of purchase ($ in millions)
A(2, 3)	$1.5 \times 2 + 0.5 \times 3 = 4.5$
B(3, 6)	**$1.5 \times 3 + 0.5 \times 6 = 7.5$**
C(4, 5)	**$1.5 \times 4 + 0.5 \times 5 = 8.5$**
D(5, 10)	**$1.5 \times 5 + 0.5 \times 10 = 12.5$**
E(6, 8)	**$1.5 \times 6 + 0.5 \times 8 = 13$**
F(7, 5)	**$1.5 \times 7 + 0.5 \times 5 = 13$**
G(10, 4)	**$1.5 \times 10 + 0.5 \times 4 = 17$**

d. **1)** A, B and D, because they are not part of the solution set.
2) G, because it is the most expensive among the possible solutions.
3) C, because it is the least expensive among the possible solutions.

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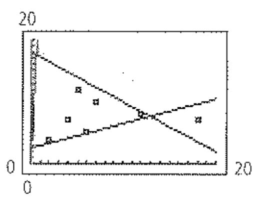
a. $y \geq 0.5x$, $y \geq 20 - 3x$ and $y \leq 18 - 0.5x$.

b. $5x + 3y$

c. **1)** (15, 8) **2)** (5, 6)

d. **1)**



2) **i)** The ordered pair (12, 8). **ii)** The ordered pair (2, 4).

15. *Plusieurs réponses possibles. Exemple :* (5, 0), (4, 3) et (6, ⁻3).

Practice 2.2

Page 127

1. a)

Ordered pair	$z = 4x - 2y$
(1, 0)	**4**
(1, 8)	**⁻12 (minimum)**
(3, 1)	**10**
(3, 3)	**6**
(4, 8)	**0**
(5, 2)	**16 (maximum)**
(8, 10)	**12**

b)

Ordered pair	$z = 7x + 9y$
(2, 2)	**32 (minimum)**
(6, 18)	**204**
(10, 4)	**106**
(12, 12)	**192**
(14, 16)	**242**
(18, 6)	**180**
(20, 12)	**248 (maximum)**

c)

Ordered pair	$z = ⁻1.2x + 0.4y + 2$
(0, 50)	**22 (maximum)**
(30, 20)	**⁻26**
(30, 70)	**⁻6**
(50, 40)	**⁻42**
(60, 90)	**⁻34**
(80, 10)	**⁻90 (minimum)**
(90, 40)	**⁻90 (minimum)**

Practice 2.2 (*cont'd*)

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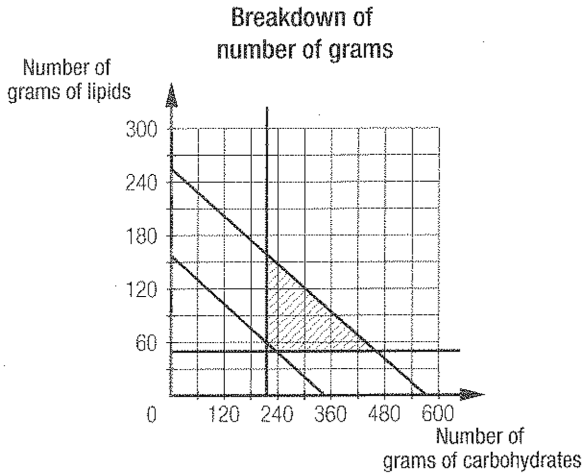
2. a) **1)** x: time (in min) devoted to sports news
y: time (in min) devoted to national news
$x \geq 0$, $y > 20$, $19x > y$, $4x < y$, $y \leq 35$ and $x + y \leq 75$.
2) The target objective is to produce a news program at the lowest cost.
3) $z = 25x + 15y$

b) **1)** x: number of Type-**A** airplanes produced
y: number of Type-**B** airplanes produced
$x \geq 0$, $y \geq 0$, $200x + 125y \leq 5000$, $x \geq 5 + 2y$ and $x + y \leq 30$.
2) The target objective is to minimize the airplanes' production time.
3) $z = 3x + 5y$

3. a)



b) $G = 0.04x + 0.01y$

c) **1)** Suggestion D minimizes the production of glycogen.

2) Suggestion B maximizes the production of glycogen.

Practice 2.2 *(cont'd)*

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4. a) $C = 150\ 000x + 225\ 000y$ **b)** Point B.

5. a) **1)** Point A. **2)** Point E.

b) **1)** *Several answers possible. Example:* $z = 2x + 3y$

2) *Several answers possible. Example:* $z = x - 4y$

3) $z = x + 2y$

Practice 2.2 *(cont'd)*

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6. a) *x* represents the number of full-time employees and *y* represents the number of part-time employees.

b) *Several answers possible. Example:*

- 5 full-time employees and 20 part-time employees.
- 8 full-time employees and 12 part-time employees.
- 11 full-time employees and 4 part-time employees.

c) Yes, if the company employs 1 full-time employee and 6 part-time employees.

7. a) **1)** $z = 12c + 18s$ **2)** $r = 20c + 25s$ **3)** $p = 8c + 7s$

b) **1)** Point D(100, 125), with production costs of $3,450.

2) Point A(75, 250), with revenues of $7,750.

3) Point C(150, 175), with profits of $2,425.

Practice 2.2 *(cont'd)*

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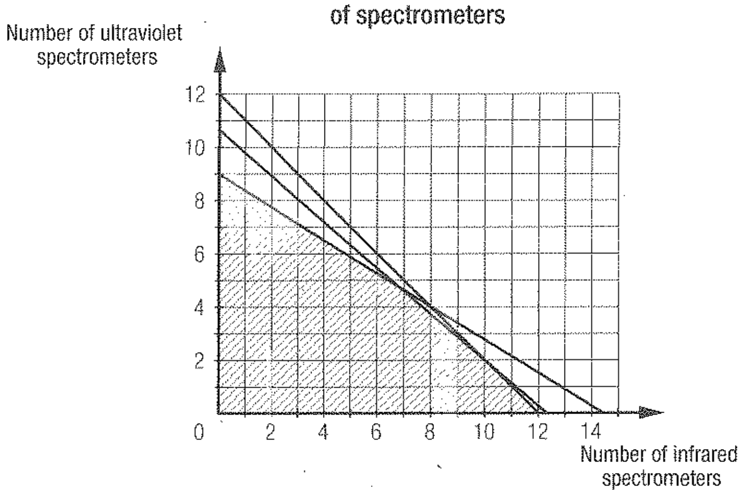
8. a) The objective is to minimize the quantity of plastic used to manufacture juice bottles.

b) $z = 150p + 250g$

c) Point E(40, 100) is the most optimal, with 31 000 cm^2 of plastic used.

9. a)

Distribution of number of spectrometers



b) No. Only ordered pairs made up of whole numbers must be considered.

c) 159 processed samples/h.

d) *Several answers possible. Example:* 10 infrared spectrometers and 1 ultraviolet spectrometer.

Practice 2.2 (*cont'd*)

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10. a)
- Side ① is matched with: Every thousand dollars spent on advertising increases the sales volume by at most 1200 units.
- Side ② is matched with: The investment in advertising is at most $60,000.
- Side ③ is matched with: Every thousand dollars spent on advertising increases the sales volume by at least 600 units.
- Side ④ is matched with: The investment in advertising is at least $30,000.

b) 1) Point D. 2) Point D.

c) *Several answers possible. Example:* The selling price could be $11.

Practice 2.2 (*cont'd*)

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11. a) Point A.

b) No. When comparing the value of the function evaluated at point C with the value of the function evaluated at point B, it is known that the term ax increases (because the x-coordinate of C is greater than the x-coordinate of B) and that the term by decreases (because the y-coordinate of C is less than the y-coordinate of B). Without knowing the extent of the increase and decrease, you cannot deduce which point would generate a maximum value.

12. a) *Several answers possible. Example:* The athlete would need to do 75 min of cardiovascular exercise and 15 min of muscular exercise.

b) $z = 10x + 6y$

c) Region F.

d) *Several answers possible. Example:* The athlete could do 60 min of cardiovascular exercise and 20 min of muscular exercise.

SECTION 2.3 Optimization using linear programming

Problem

Page 134

The tube should measure 3.75 cm in length and 0.375 cm in diameter.

Activity 1

Page 135

a. 1) This situation entails a kerosene consumption of 17.7 L/km.
   2) This situation entails a kerosene consumption of 18.5 L/km.
   3) This situation entails a kerosene consumption of 19.3 L/km.
   4) This situation entails a kerosene consumption of 20.1 L/km.

b. They are the coordinates corresponding to the altitude of the plane and the air resistance that would generate a fuel consumption of 19.3 L/km.

c. $\frac{2}{45}$

d. This number increases from 17.7 to 20.1.

e. 1) No, because line l_1 has no point in common with the polygon of constraints.
   2) No, because line l_4 has no point in common with the polygon of constraints.

f.

Vertex	$0.32x + 7.2y$	C
A(8.5, 2.185)	**0.32 × 8.5 + 7.2 × 2.185**	**18.452**
B(8.5, 2.375)	**0.32 × 8.5 + 7.2 × 2.375**	**19.82**
C(9, 2.25)	**0.32 × 9 + 7.2 × 2.25**	**19.08**
D(9, 2.09)	**0.32 × 9 + 7.2 × 2.09**	**17.928**

Activity 1 (*cont'd*)

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g. 1) B(8.5, 2.375) 2) D(9, 2.09)

h.

Kerosene consumption (L/km)	$C = 0.95x + 5y$	$y = \frac{C}{5} - 0.19x$
19	$19 = 0.95x + 5y$	$d_5: y = 3.8 - 0.19x$
19.3	$19.3 = 0.95x + 5y$	$d_6: y = 3.86 - 0.19x$
19.6	$19.6 = 0.95x + 5y$	$d_7: y = 3.92 - 0.19x$
20	$20 = 0.95x + 5y$	$d_8: y = 4 - 0.19x$

i. B(8.5, 2.375)

j. On segment AD.

k. 1) This point is on a vertex of the polygon of constraints.
   2) These points are located on a side of the polygon of constraints.

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a. The coordinates of the vertices are (2, 4), (6, 7) and (8, 2).

b. 1) $z = x + 2y$ 2) -0.5 3) (6, 7) 4) (2, 4)

c. *Several answers possible.* However, for any given value of A, the value of B must be equal to 0.4A. For example, one can enter A = 5 and B = 2.

d. 1) (6, 7) 2) (2, 4)

Practice 2.3

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1. a) 1) C(3, 3) 2) A(5, 9)
   b) 1) B(20, 40) 2) C(28, 12)
   c) 1) All the points located on segment AB. 2) C(18, 10)
   d) 1) D(40, 30) 2) All the points located on segment BC.

2. a) Vertex B. b) Vertex B.

Practice 2.3 (*cont'd*)

3. a) ① : C(6, 9), ② : B(2, 5) b) ① : 39, ② : -10

4. a) 1) Point E. 2) Point B. b) 1) Point C. 2) Point F.
   c) 1) Point D. 2) Point G. d) 1) Point G. 2) Point D.

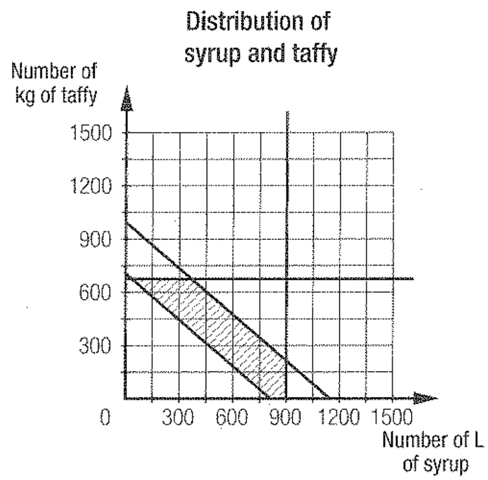
5. a) (17, 3) b) (80, 30) c) (0,9, 0,8) d) $\left(\frac{20}{73}, -\frac{317}{73}\right)$

Practice 2.3 (*cont'd*)

6. a) (2.5, 2.5) b) (10, 25)

7. a) (3, 6) b) (2, 5)

8. a) x: number of L of syrup y: number of kg of taffy
   b) $z = 3x + 8y$
   c) $x \geq 0$
   $y \geq 0$
   $35x + 40y \geq 28\ 000$
   $35x + 40y \leq 40\ 000$
   $x \leq 900$
   $y \leq 675$
   d)

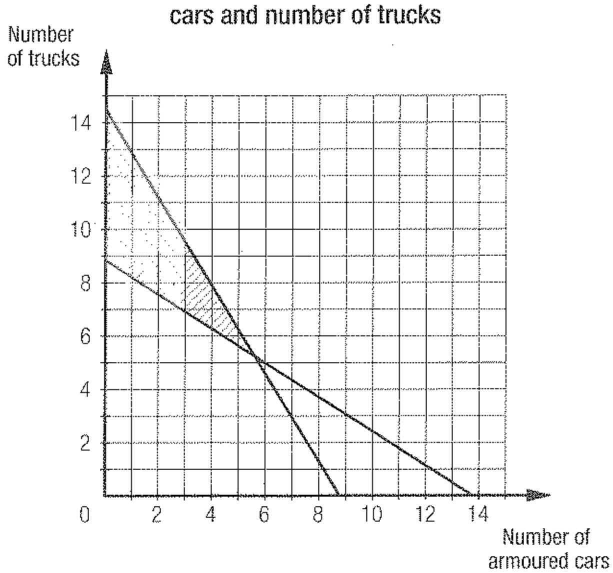


   e) The producer would have to produce 325 L of syrup and 675 kg of taffy.
   f) The producer can expect a profit of $6,375.

Practice 2.3 (*cont'd*)

9. a) 1) Yes. The slope of the scanning line associated with this situation is negative. The evaluation of this function would therefore determine the coordinates that would generate the minimum value.
   2) No. The slope of the scanning line associated with this situation is positive. The evaluation of this function would therefore not determine the coordinates that would generate a minimum value given that the polygon of constraints is unbounded.
   b) 1) No. The slope of the scanning line associated with this situation is negative. The evaluation of this function would therefore not determine the coordinates that would generate a maximum value given that the polygon of constraints is unbounded.
   2) No. The slope with the scanning line associated with this situation is positive. The evaluation of this function would therefore not determine the coordinates that would generate a minimum value given that the polygon of constraints is unbounded.

10. a) Distribution of number of armoured cars and number of trucks



b) $z = 20x + 12y$

c) Because only whole vehicles can be used for moving.

d) Two. Ordered pairs (1, 13) and (4, 8).

e) 176 soldiers.

f) 1 armoured car and 13 trucks.

11. a) *Several answers possible. Example:* $z = y - x$

b) $z = \frac{4x}{3} - y$

Practice 2.3 (*cont'd*)

Page 144

12. The company must manufacture 2160 screws and 3360 bolts to generate a maximum revenue of $936.

13. a) The research department must obtain 3 Model-**B** computers and no Model-**A** computers in order to maximize the calculation speed, which is 180 teraflops.

b) This department must obtain 8 Model-**B** computers and no Model-**A** computers in order to minimize its expenses, which are $192 million.

Practice 2.3 (*cont'd*)

Page 145

14. a) 1) 15 mg of Medication **A** and 21 mg of Medication **B** 2) $z = 0.952$

b) 1) 5 tablets of Medication **A** and 4 tablets of Medication **B** 2) $z = 0.881\ 25$

15. The temperature should be 303 K and the pressure 93.93 kPa for an approximate decrease of 5.95 min of drying time.

SPECIAL FEATURES 2

Chronicle of the past

Page 147

1. a) 2500 infantrymen and 1000 artillerymen.

b) In 9 days.

c) 15 days.

d) 2500 infantrymen and 2500 artillerymen.

2. a) $x \geq 0$, $y \geq 0$ and $z \geq 0$.

b) 1) A(0, 750, 375) 2) C(0, 0, 750) 3) D(375, 0, 375)

In the workplace

Page 149

1. $x \geq 36$, $x \leq 38$, $y \geq 0$ and $y \leq 0.2$

2. Treatments

	Medical follow-up	Daily dosage of Medication **A** (mg)	Daily dosage of Medication **B** (mg)
a)	Consultation in 13 days	No treatment	No treatment
b)	Consultation in 48 h	20.325	23.35
c)	Hospitalization recommended	60.284	99.5

Name: ______________________

Group: __________ Date: __________

4 **SERIES OF TRANSFORMATIONS** Julian applies a series of geometric transformations to the graph of the function $f(x) = x^2$ in the following order.

1) A translation of 2 units left and of 3 units downward.
2) A reflection over the y-axis.
3) A reflection over the x-axis.
4) A vertical stretch by a factor of 2.

Then he determines the rule of the function that corresponds to the resulting curve.

Julian then asks himself the following questions: "If I apply these geometric transformations in a different order, will I obtain the same rule? If not, what would not change in the rule?" Respond to Julian's questions while justifying your reasoning.

Name: ____________________

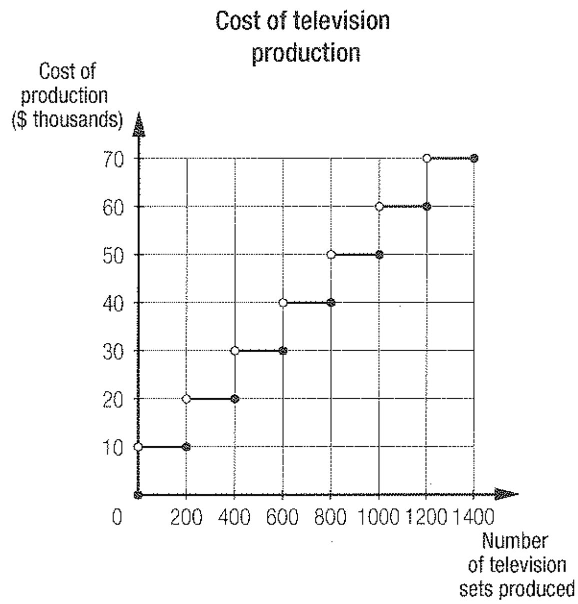
Group: ____________ Date: ____________

SNAPSHOT 2

(cont'd)

5 **COST AND REVENUE** The adjacent graph represents the cost of production for a company that only manufactures television sets. The company's revenue depends upon the number of television sets it produces. Taking the market into account, an economist modelled this relation using the following quadratic function.

$R(x) = {}^{-}0.08(x - 2500)^2 + 500\ 000$
where x is the number of television sets produced and $R(x)$ is the revenue (in \$).



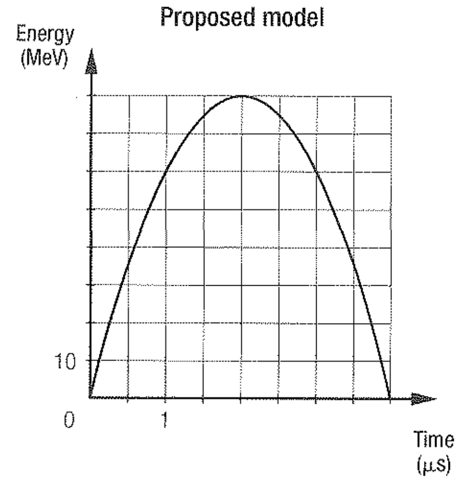
The company's profit is the difference between its revenue and its cost of production. Currently, the enterprise produces 1700 television sets. To maximize its profits, would it be advantageous to increase its production? If so, by how much?

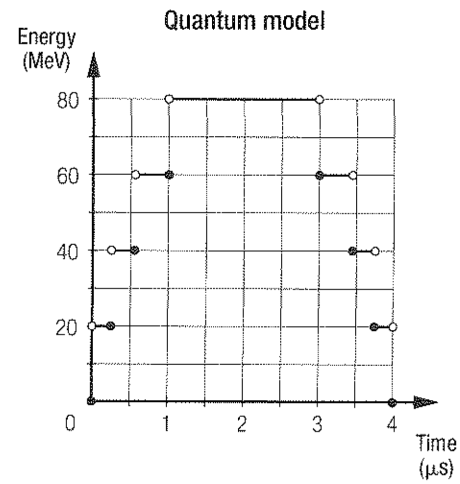
6 **PARTICLES AND ATOMS** The amount of energy an atom possesses can be defined as a function of time. The following is a proposed model:

The most recent model is the quantum model, according to which, particles contain only certain levels of energy. The second graph represents the new model.

What is the rule for this new model?

How did you proceed in determining this rule?



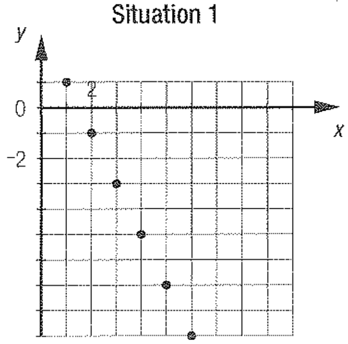


Support 2.1 — Page 1

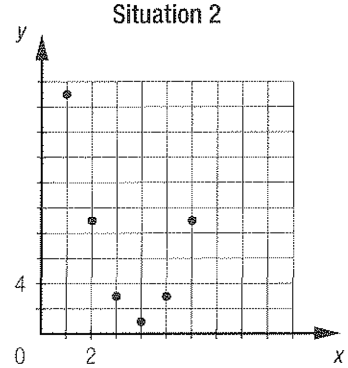
1. a) 1 b) 2 c) 1

2. a)

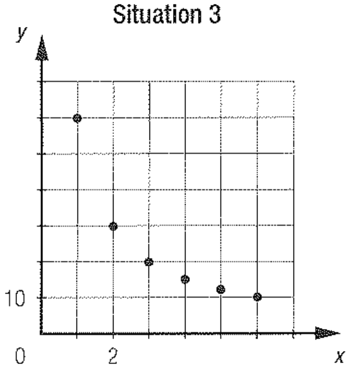
Situation 1



Situation 2



Situation 3



b) **Situation 1:** The linear model is the most suitable because the rate of change is constant. **Situation 2:** The quadratic model is the most suitable because there is a change in direction. **Situation 3:** The inverse variation model is the most suitable because there is neither a change in direction nor a constant rate of change.

Support 2.1 *(cont'd)* — Page 2

3. a) **Situation 3**

x	1	2	3	4	5	6
y	60	30	20	15	12	10
xy	**60**	**60**	**60**	**60**	**60**	**60**

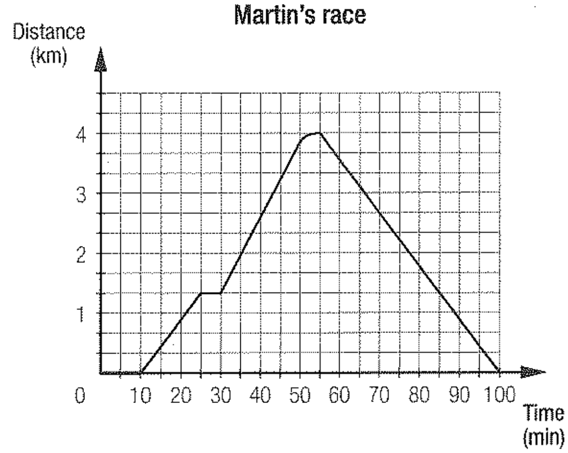
The product *xy* is constant for all ordered pairs.

b) In algebraic form, the product can be written $xy = 60$.
When *y* is isolated, $y = \frac{60}{x}$ is obtained.

c) To differentiate between an inverse variation from a partial variation, look for the following:
- The products of the ordered pairs are constant.
- The rate of change varies.

4. a)

Martin's race



b) The function is increasing over the interval [0, 55].

c) Over the interval [50, 55].

d) Part 1: Constant model.
Part 2: Linear model.
Part 3: Constant model.
Part 4: Linear model.
Part 5: Quadratic model.
Part 6: Linear model.

Consolidation 2.1 — Page 3

1. a) The probe will be able to collect the data at an altitude varying from 0 to 5 km, from 40 to 56 km or higher than 97 km.

b) The function is decreasing over the interval [0, 25] ∪ [50, 80]. It is increasing over the interval [10, 50] ∪ [80, 120].

2. a) The inverse variation model because the function decreases in a non-constant manner.

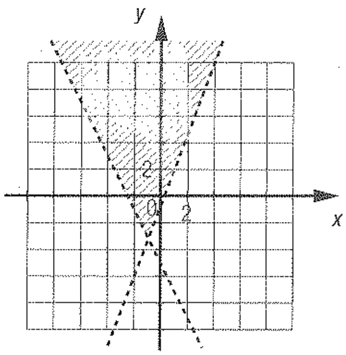
b) $I = \frac{12.5}{R}$

c) In theory, the intensity would never reach zero because the inverse model does not reach 0 except at infinity. Therefore, the resistance would also need to have an infinite value.

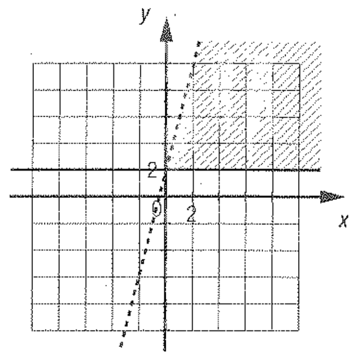
3. a) 21.4 mg **b)** 26.48 mg **c)** 110.5 mg

Overview

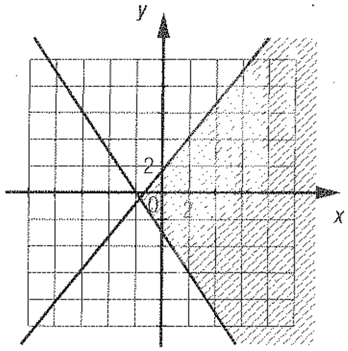
1. a)



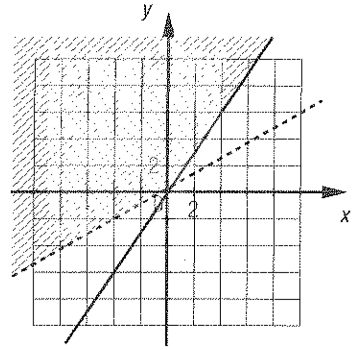
b)



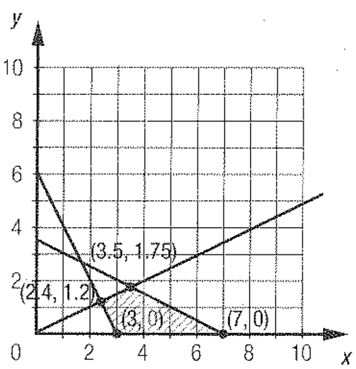
c)



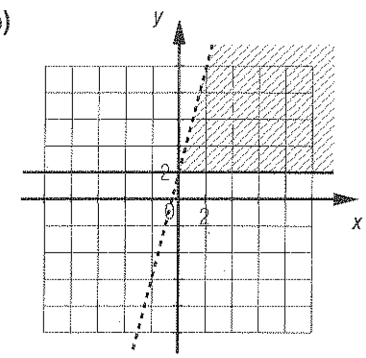
d)



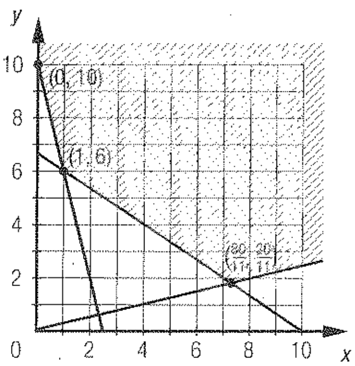
2. a)



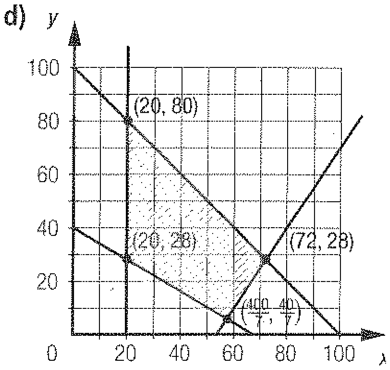
b)



c)



d)



3. a) 1) $x - y \leq -2$, $y \leq -2x + 20$, $4y \geq x - 4$ and $x + 2y > 10$. **2)** $D\left(\frac{28}{3}, \frac{4}{3}\right)$ **3)** C(6, 8) and $D\left(\frac{28}{3}, \frac{4}{3}\right)$.

b) 1) $3x - y < 0$, $y \leq 18$, $y < -x + 30$, $-2x + y \geq -24$ and $x + 6y \geq 38$. **2)** D(14, 4) **3)** D(14, 4)

4. a) A(-1, -6) and C(3, 4). **b)** A(-1, -6) and E(1, -16). **c)** A(-1, -6)

Overview (*cont'd*)

5. a) 1) *x*: number of wood chairs
y: number of wood stools
$x \geq 150$, $y \geq 100$, $x \geq 2y$ and $x + y \leq 1000$.
2) $z = 20x + 12y$

b) 1) *x*: number of part-time employees
y: number of full-time employees
$x \geq 0$, $y \geq 0$, $14x + 30y \geq 400$ and $x + y \leq 14$.
2) $z = 12x + 14y$

6. a) 1) All the points are found on segment BC. 2) 15
b) 1) D(8, 1) 2) 13
c) 1) B(6, 9) 2) 12.3
d) 1) C(8, 7) 2) 12.1

7.

System of inequalities representing the constraints	$y \leq -x + 15$ $y \leq 2x - 6$ $-x + 3y \geq -60$	$x \geq 0$ $x \geq 0$ $y \leq 15$ $x \leq 14$ $y \leq 2x + 4$	$y \geq -x - 2$ $y \leq x + 4$ $y \leq -3x + 8$
Rule of the optimizing Function	$z = 0.5x + 2y$	$z = y - 3x$	$z = -10x - 14y$
Target objective	Maximize	Minimize	Maximize
Solution	**(7, 8)**	**(14, 0)**	**(5, -7)**
Optimal value	**19,5**	**-42**	**48**

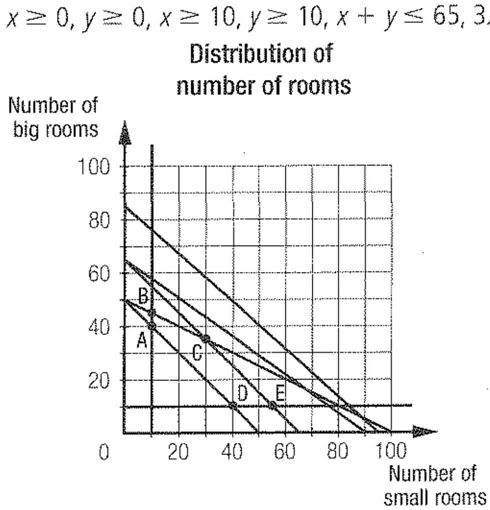
Overview (*cont'd*)

8. a) Graph ②.
b) This vertex is excluded from the system's solution set because it does not satisfy the constraint that the factory would like to obtain less than 12 machines in all.
c) 1) 6 Type-**A** machines and 5 Type-**B** machines. 2) 89 pieces/min.

9. 65 cubes (35 metal cubes and 30 wooden cubes).

Overview (*cont'd*)

10. a) *x*: number of small rooms *y*: number of big rooms
b) $10\,500x + 11\,550y = M$
c) $x \geq 0$, $y \geq 0$, $x \geq 10$, $y \geq 10$, $x + y \leq 65$, $3x + 6y \leq 300$ and $x + y \geq 50$.
d)



e) 30 small rooms and 35 big rooms.
f) A profit of $719,250.

11. a) The transportation company would need to obtain 22 Model-**A** trains and 10 of Model-**B** trains.
b) The transportation company would need to obtain 10 Model-**A** trains.

Overview *(cont'd)* **Page 154**

12. a) $358\ 000x + 378\ 000y = M$

b) The company must buy 3 Model-**A** airplanes and 10 Model-**B** airplanes.

c) 1) \$2.685 billion **2)** 5545 seats. **3)** 4 854 000 litres of kerosene.

13. a) z_1: (2.5, 2.25) z_2: (0, 4) **b)** z_1: (-0.75, ≈ -3.44) z_2: (0, -4)

Overview *(cont'd)* **Page 155**

14. a) 1) Point A$\left(\frac{\sqrt{\pi}}{\pi}, 1\right)$. **2)** Point C$\left(1, \frac{2}{\pi}\right)$.

b) 1) The cylinder must measure 1 m in height and $\frac{\sqrt{\pi}}{\pi}$ m.

2) The cylinder must measure $\frac{2}{\pi}$ m in height and 1 m in radius.

15. The dimensions of the screen, excluding the borders, must be 20 dm by 11.25 dm.

Overview *(cont'd)* **Page 156**

16. a) \$32,500 **b)** 2800 boxes.

c) The owner must buy 40 aluminum structures and 20 steel structures.

17. a) The material must be composed of at least 20% aluminum. It must be composed of at least 30% fibreglass but not more than 70%. There must be a maximum of 10% more aluminum than fibreglass in the material. The sum of the percentage of aluminum and the percentage of steel cannot exceed 100%.

b) 1) The material must be composed of 30% aluminum and 70% fibreglass for a density of 2.63 g/cm^3.

2) The material must be composed of 20% aluminum and 30% fibreglass for a rigidity of 34.6 GPa.

3) The maximum rigidity of the material is approximately 64.12 GPa for a density of 2.39 g/cm^3.

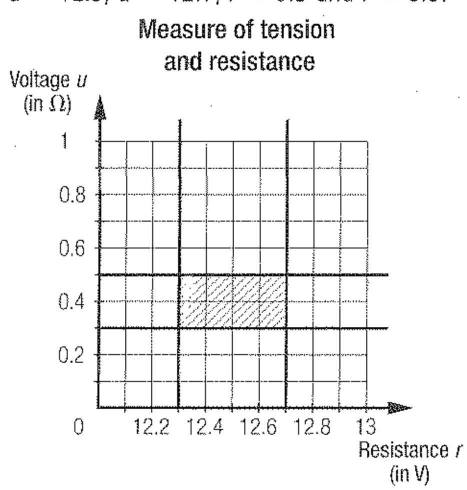
4) The minimum density of the material is approximately 1.81 g/cm3 for a rigidity of 48.3 GPa.

c) 1) 69.90 GPa **2)** 2.63 g/cm^3

Overview *(cont'd)* **Page 157**

18. a) $u \geq 12.3$, $u \leq 12.7$, $r \geq 0.3$ and $r \leq 0.5$.

b)



c) The uncertainty related to the circuit's residual power is 3.5 W.

19. a) The optimizing function is $M = 47x + 65y$. The coordinates (≈ 107.69, 0) maximize the profit. By rounding, one obtains a profit of \$5,029.

b) The optimizing function is $M = 13x + 22y$. The coordinates (≈ 65.33, 0) minimize the losses. The company would therefore have to use 65 of Size **A** sheets.

Overview (*cont'd*)

Page 158

20. The company's manoeuvrability in regards to the selling price is \$1.60. The minimum selling price can be \$1.90; whereas the maximum selling price can be \$3.50.

21. a) $B\left(500, \frac{16\,000}{9}\right)$, $C\left(\frac{3500}{3}, \frac{4000}{3}\right)$
 b) The coordinates of these points are not all whole numbers.
 c) The company would have to produce 1166 8-mL cartridges and 1333 14-mL cartridges.

Overview (*cont'd*)

Page 159

22. a) 90 children and 60 adults. b) 13 lifeguards.

23. a)

$E_r \geq 0$	The energy of the reactants is greater than or equal to 0.
$E_p \geq 0$	The energy of the products is greater than or equal to 0.
$E_p - E_r < 0$	The energy of the reactants subtracted from the energy of the products is less than 0.
$E_r - E_p < 300$	The energy of the products subtracted from the energy of the reactants is less than 300.
$\frac{E_r}{E_p} \leq 3$	The ratio of the energy of the reactants to the energy of the products is less than or equal to 3.

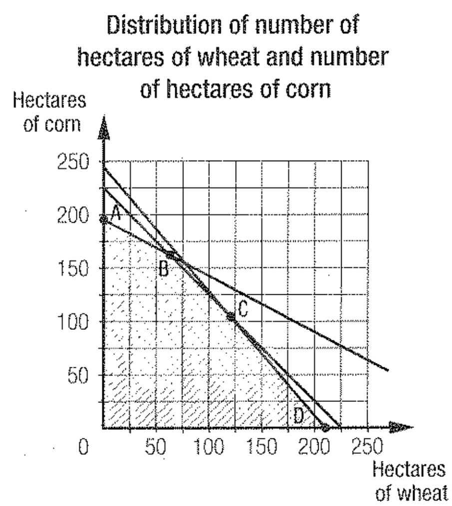
 b) E_r, because $E_p - E_r$ produces a number less than 0.
 c) No, because based on the constraints, the difference between the energy of the reactants and energy of the products would always be less than 300.

24. a) $(0, b)$, $\left(\frac{p - be}{d + ae}, \frac{ap - aeb}{d + ae} + b\right)$ and $\left(\frac{p - be}{d + ce}, \frac{cp - ceb}{d + ce} + b\right)$.
 b) The only vertex that is part of the solution set is $(0, b)$.

Bank of problems

Page 160

1. Let x be the number of hectares of wheat to sow and y, the number of hectares of corn to sow.
 - Write down the system of inequalities that translates the constraints of the problem.
 $x \geq 0$
 $y \geq 0$
 $119x + 102y \leq 25\,000$
 $31x + 62y \leq 12\,000$
 $x + y \leq 225$
 - Determine the optimizing function.
 $106.25x + 153.75y = M$
 - Represent the polygon of constraints.



 - Determine the coordinates of the vertices of the polygon of constraints.
 $A\left(0, \frac{6000}{31}\right)$
 $B\left(\frac{1950}{31}, \frac{5025}{31}\right)$
 $C\left(\frac{2050}{17}, \frac{1775}{17}\right)$
 $D\left(\frac{25\,000}{119}, 0\right)$
 - Evaluate the optimizing function at each vertex.

Vertex	Profit
$A\left(0, \frac{6000}{31}\right)$	Approximately \$29,758.06
$B\left(\frac{1950}{31}, \frac{5025}{31}\right)$	Approximately \$31,605.85
$C\left(\frac{2050}{17}, \frac{1775}{17}\right)$	Approximately \$28,865.80
$D\left(\frac{25\,000}{119}, 0\right)$	Approximately \$22,321.42

 - Formulate the answer.
 The farmer would need to sow approximately 62.9 hectares of wheat and 162.1 hectares of corn.

2. • Determine the equation of the line segment bounded by the points $\left(\frac{25}{8}, \frac{25}{4}\right)$ and $\left(\frac{125}{13}, \frac{25}{13}\right)$.
$y = \frac{-2}{3}x + \frac{25}{8}$

- Verify whether the coordinates (5, 5) and (8, 3) belong to the segment.
  Yes, because they satisfy the equation.
- Evaluate the profit generated by the production.

Point	Profit
(5, 5)	\$750
(8, 3)	\$750

- Evaluate the production time.

Point	Production time
(5, 5)	70 h
(8, 3)	67 h

- Determine the most advantageous breakdown.
  The carpenter would have to produce 8 stools and 3 chairs,
  because he or she would work 3 hours less for the same amount of profit.

Bank of problems (*cont'd*)

Page 161

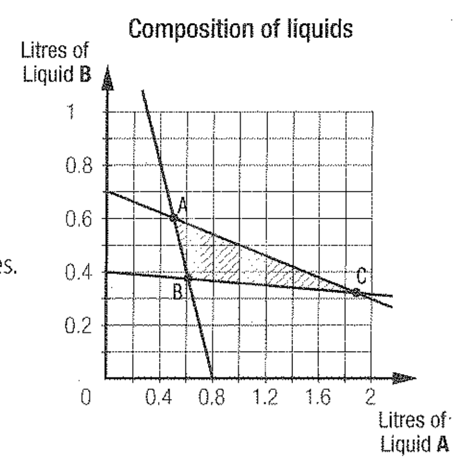
3. The coordinates of vertex B generate a smaller value of the function g than the coordinates of the vertex A if $cx_1 + dy_1 > cx_2 + dy_2$. This inequality can be manipulated in the following way:

$$cx_1 - dy_1 > cx_2 - dy_2$$
$$cx_1 - dy_1 - (cx_2 - dy_2) > 0$$
$$cx_1 - cx_2 - dy_1 - dy_2 > 0$$
$$cx_1 - cx_2 > dy_1 - dy_2$$
$$c(x_1 - x_2) > d(y_1 - y_2)$$
$$c < d\frac{y_1 - y_2}{x_1 - x_2}$$
$$\frac{c}{d} < \frac{y_1 - y_2}{x_1 - x_2}$$

Since the slope of the scanning line associated with the function $g(x, y)$ is $\frac{c}{d}$, it can be deduced that in order for the coordinates of vertex B to generate a smaller value of the function g than the coordinates of vertex A, the slope of segment AB must be greater than the slope of the scanning line.

4. Let x be the quantity of Liquid **A** and y the quantity of Liquid **B**.

- Establish the constraints under the form of a system of inequalities.
  $x \geq 0$
  $y \geq 0$
  $0.2x + y \leq 0.7$
  $40x + 20y \geq 32$
  $x + 25y \geq 10$
- Establish the function that calculates the minimum number of impurities.
  $23x + 12y = M$
- Draw the polygon of constraints associated with the system.



- Determine the coordinates of the vertices of the polygon.
  A(0.5, 0.6) $\quad$ $B\left(\frac{30}{49}, \frac{92}{245}\right)$ $\quad$ $C\left(\frac{15}{8}, \frac{13}{40}\right)$
- Among the vertices of the polygon, find the one that minimizes the impurities.

Vertex	Impurities (g/L)
A(0.5, 0.6)	18.7
$B\left(\frac{30}{49}, \frac{92}{245}\right)$	≈ 18.59
$C\left(\frac{15}{8}, \frac{13}{40}\right)$	≈ 47.2

- Formulate the answer.
  The ordered pair that generates the least elevated concentration of impurities is ordered pair$\left(\frac{30}{49}, \frac{92}{245}\right)$.
  One must therefore mix Liquids **A** and **B** in the following proportion: $\frac{30}{49} : \frac{92}{245}$.

Bank of problems (*cont'd*)

Page 162

5. For Monday:

The total mass of goods to transport must be at least 134 kg but less than 380 kg. The maximum mass of water to transport cannot exceed 240 kg. The minimum mass of food must be 44 kg, and the maximum mass must be 176 kg. Lastly, the mass of water must be at least equal to the mass of food.
The containers of water must have a mass of 30 kg, whereas the containers of food must have a mass of 22 kg.

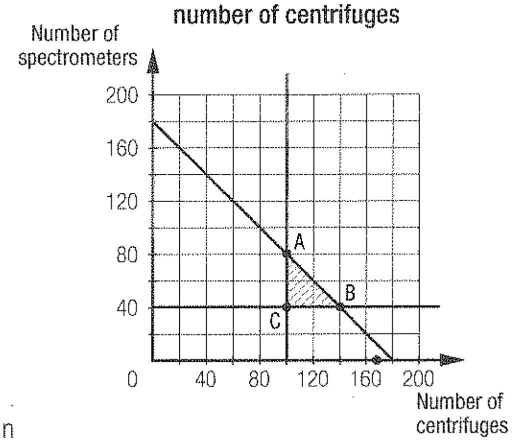
For Tuesday:

The total mass of goods to transport must be at most of 400 kg. The mass of water must be at least 40 kg, and the mass of food must be at least double the mass of water.
The containers of water must have a mass of 20 kg, whereas the containers of food must have a mass of 25 kg.

6. Let x be the number of centrifuges and y, the number of spectrometers.

- Establish the constraints under the form of a system of inequalities.
  $x \geq 100$
  $y \geq 40$
  $x + y \leq 180$
- Establish the function that calculates the revenue.
  $4000x + 5250y = M$
- Draw the polygon of constraints associated with the system.

Distribution of number of spectrometers and number of centrifuges



- Determine the coordinates of the vertices of the polygon.
  A(100, 80) B(140, 40) C(100, 40)
- Among the vertices of the polygon, find the one that maximizes the revenue.

Vertex	Revenue
A(100, 80)	\$820,000
B(140, 40)	\$770,000
C(100, 40)	\$610,000

- Establish the rule that calculates the profit.
  $(4000 - c)x + (5250 - s)y = M$
  Given that the revenue is maximal at A, the following must be done in order for the profit to be maximal in relation to point B:
  $100\,(4000 - c) + 80\,(5250 - s) > 140\,(4000 - c) + 40\,(5250 - s)$
- Solve this inequality to demonstrate that $c > s - 1250$.
  $80\,(5250 - s) - 40\,(5250 - s) > 140\,(4000 - c) - 100(4000 - c)$
  $40(5250 - s) > 40(4000 - c)$
  $(5250 - s) > (4000 - c)$
  $1250 - s > -c$
  $c > 1250 - s$
  Therefore, in order for the revenue and profit to be maximal A, you must have
  $c > 1250 - s$

7.
- The function that calculates the mean cost of production of an item must be minimized.
  The rule of this function is $z = \frac{ax + by}{x + y}$.
- For a constant value C of this cost, the set of ordered pairs (x, y) that generates this cost satisfies the equation $\frac{ax + by}{x + y} = C$. By manipulating this equation, the following is obtained:
  $ax + by = Cx + Cy$
  $ax - Cx = -by + Cy$
  $x(a - C) = y(-b + C)$
  $y = -\frac{a - C}{b - C}x$
- The slope of the segment joining the origin and a point whose coordinates satisfy this equation is $-\frac{a - C}{b - C}$.
  The lower the value of C, the closer the value of this expression is to $-\frac{a}{b}$. It can be concluded that the point on the polygon of constraints whose coordinates generate the smallest value of C would be the one to form a segment with the origin whose slope would be the closest to $-\frac{a}{b}$.

8. *Example of a possible approach:*
- If x represents the amount invested in Portfolio **A** ($ in thousands) and y represents the amount invested in Portfolio **B** ($ in thousands), the constraints can be translated by the following system of inequalities:
  $x \leq 120$
  $y \leq 100$
  $x + y \geq 120$
  $x + y \leq 180$
- The function that calculates the total investment risk (x, y) is
  $r = 0.3\frac{25}{100}x + 0.1\frac{15}{100}y$ or $r = 0.075x + 0.015y$.
- The function that calculates the total profit of an investment (x, y) is
  $p = 0.4\frac{10}{100}x + 0.2\frac{15}{100}x + 0.1\frac{20}{100}x + 0.5\frac{10}{100}y + 0.1\frac{15}{100}y + 0.3\frac{20}{100}y$ or $p = 0.09x + 0.125y$.

The following graph shows the polygon of constraints as well as two scanning lines associated with the risk and profit.
The graph demonstrates the following:
- The coordinates (40, 100) minimize the risk.
- The coordinates (80, 100) maximize the profit.

The amount to invest in Portfolio **A** is therefore the average of $40,000 and $80,000, meaning $60,000, and the amount to invest in Portfolio **B** is $100,000.