LES 5 | *Bio-indicators of pollution*

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Population environment A

On average, from year to year, the Population of environment **A** corresponds to 95% of the population of the previous year. Thus, population P_A (in number of individuals) evolves based on the rule $P_A = 5480(0.95)^t$ where t represents time (in years).

Population environment B

On average, from year to year, the Population of environment **B** corresponds to 98% of the population of the previous year. Thus, population P_B (in number of individuals) evolves based on the rule $P_B = 7475(0.98)^t$ where t represents time (in years). This rule is only valid for $t \geq 4$.

Population environment C

On average, from year to year, the Population of environment **C** corresponds to 90% of the population of the previous year. Thus, population P_C (in number of individuals) evolves based on the rule $P_C = 5556(0.9)^t$ where t represents time (in years). This rule is only valid for $t \geq 2$.

Population environment D

On average, from year to year, the Population of environment **D** corresponds to 98% of the population of the previous year. Thus, population P_D (in number of individuals) evolves based on the rule $P_D = 2580(0.98)^t$ where t represents time (in years).

Population environment E

On average, from year to year, the Population of environment **E** corresponds to 85% of the population of the previous year. Thus, population P_E (in number of individuals) evolves based on the rule $P_E = 3611(0.85)^t$ where t represents time (in years).

Population of environments as a whole

To determine the population of all the environments, you must first calculate the sum of the populations of each environment.

Amphibian populations in different environments

Time (years)	Population environment **A**	Population environment **B**	Population environment **C**	Population environment **D**	Population environment **E**	**Population environments as a whole**
0	5480	Data unavailable	Data unavailable	2580	3611	**Impossible to determine**
1	5206			2541	3069	
2	4946		4500	2503	2609	
3	4698		4050	2466	2218	
4	4463	6895	3645	2429	1885	**19 317**
5	4240	6757	3281	2392	1602	**18 272**
6	4028	6622	2952	2356	1362	**17 320**
7	3827	6490	2657	2321	1158	**16 453**
8	3636	6360	2391	2286	984	**15 657**
9	3454	6233	2152	2252	836	**14 927**
10	3281	6108	1937	2218	711	**14 255**
11	3117	5986	1743	2185	604	**13 635**
12	2961	5866	1569	2152	514	**13 062**
13	2813	5749	1412	2120	437	**12 531**
14	2672	5634	1271	2088	371	**12 036**

On average, from year to year, the population of all the environments corresponds to 95% of the population of the previous year. Thus, population P_T (in number of individuals) evolves based on the rule $P_T = 23\ 716(0.95)^t$ where t represents time (in years).

LES 6 Locust invasions

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Time (months)	Calculation of the population of locusts	Population of locusts (in number of insects)
0	4250	4250
1	4250 ÷ 2	2125
2	2125 × 32 ÷ 2	34 000
3	34 000 ÷ 2	17 000
4	17 000 × 32 ÷ 2	272 000
5	272 000 ÷ 2	136 000
6	136 000 × 32 ÷ 2	2 176 000
7	2 176 000 ÷ 2	1 088 000
8	1 088 000 × 32 ÷ 2	17 408 000
9	17 408 000 ÷ 2	8 704 000
10	8 704 000 × 32 ÷ 2	139 264 000
11	139 264 000 ÷ 2	69 632 000
12	69 632 000 × 32 ÷ 2	1 114 112 000

Even with the monthly spraying of pesticides, it only takes 6 months for the population to reach its critical point and remain at the same critical level.

REVISION 3

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Prior learning 1

a. 5088 ppm

b. **1)** 159 ppm **2)** ≈ 4.97 ppm **3)** ≈ 3.03×10^{-4} ppm

c. 6 h

d. **1)** 7 h after being administered. **2)** 39.75 ppm

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Prior learning 2

a. The number of cells double every day.

b. 7800 cells.

c. **1)** 62 400 cells. **2)** 7 987 200 cells. **3)** 800 cells.

d. 8 days after the bioreactor turns on.

e. **1) Evolution of the number of cells in a bioreactor from the time of activation**

Time (days)	0	1	2	3	4
Number of cells per millilitre of culture	**7800**	**23 400**	**70 200**	**210 600**	**631 800**

2) 6 days after the bioreactor turns on.

Knowledge in action

1. a) -2^6 **b)** $2^3 \times 3^3$ **c)** -2^{11} **d)** $2^{10} \times 3^2 \times 5$ **e)** $-2^4 \times 3^2 \times 5$ **f)** $-2^4 \times 3^4$

2. a) 27 **b)** 32 **c)** 225 **d)** 1.44
e) 0.027 **f)** 0 **g)** 1 **h)** 1

3. a) 2^9 **b)** 9^8 or 3^{16} **c)** 3^4 **d)** 4^2 or 2^4
e) 4^{-24}, 2^{-48}, $\left(\frac{1}{4}\right)^{24}$ or $\left(\frac{1}{2}\right)^{48}$ **f)** 3^{-21} or $\left(\frac{1}{3}\right)^{21}$ **g)** 6^0 or 1 **h)** 12^{-26} or $\left(\frac{1}{12}\right)^{26}$

4. a) a^7 **b)** a^7 **c)** 2^{3a} **d)** a^{-2} or $\left(\frac{1}{a}\right)^2$
e) 3^{a+4} **f)** a^{2b+1} **g)** a^3 **h)** a^0 or 1 (if $a \neq 0$)

5. (1) and (B), (2) and (D), (3) and (A), (4) and (E), (5) and (C)

6. a) $x = 3$ **b)** $x = 6$ **c)** $x = 32$ **d)** $x = 4$ **e)** $x = 4$ **f)** $x = 2401$

Knowledge in action (*cont'd*)

7. a) $\sqrt{3}$ **b)** $\sqrt[3]{25}$ or $\sqrt[3]{5^2}$ **c)** $\sqrt[5]{16}$ or $\sqrt[5]{2^4}$
d) $\sqrt{7^5}$ or $\sqrt{16\,807}$ **e)** $\sqrt{3^3}$ or $\sqrt{27}$ **f)** $\sqrt[4]{36}$ or $\sqrt{6}$

8. a) $3^{\frac{1}{2}}$ **b)** $9^{\frac{1}{3}}$ or $3^{\frac{2}{3}}$ **c)** $5^{\frac{2}{5}}$ or $25^{\frac{1}{5}}$ **d)** $\left(\frac{2}{3}\right)^{\frac{3}{2}}$ or $\left(\frac{8}{27}\right)^{\frac{1}{2}}$ **e)** $5^{-\frac{1}{6}}$ **f)** $8^{\frac{1}{4}}$ or $2^{\frac{3}{4}}$

9. a) a **b)** b^6 **c)** $2^3c^{\frac{2}{3}}$ **d)** 1 **e)** $\frac{\sqrt{3}}{6e}$

10. a) $x = 5$ **b)** $x = -4$ **c)** $x = 6$ **d)** $x = \frac{1}{4}$ **e)** $x = -9$ **f)** $x = -\frac{1}{4}$

11. Statement (A) are (C) are true.
Statement (B) is true only if $a \neq 0$.

12. a) 1) 16 bacteria. **2)** 1024 bacteria.
b) 1) 3 h later. **2)** 5 h 30 min later.

Knowledge in action (*cont'd*)

13. a) $\left(\frac{1}{3}\right)^8$ **b)** $\left(\frac{1}{2}\right)^3$ **c)** 3^3 **d)** 5^3 **e)** $-\left(\frac{1}{2}\right)^{16}$ **f)** 5^3

14. a) $a = 4$ **b)** $a = 2$ **c)** $a = 3$ or $a = -3$
d) $a = 8$ **e)** $a = 2$ **f)** $a = 25$ or $a = -25$

15. No. The area of the left pyramid is $x^2 + 4 \times \frac{x \times 2x}{2} = 5x^2$ whereas, the area of the right pyramid is $(2x)^2 + 4 \times \frac{x \times 2x}{2} = 8x^2$.

16. a)

Percentage of light according to the depth of a lake

Depth (cm)	Percentage of light
0	100
50	98.5
100	97.0225
150	**≈ 95.57**
200	**≈ 94.13**

b) The percentage of light is approximately 22%.

c) 1) The visibility is not zero.
2) The visibility is almost zero (the percentage of light is approximately 5.66%).
3) The visibility is zero (the percentage of light is approximately 4.87%).

Knowledge in action *(cont'd)*

17. a) No real number multiplied by itself can produce a negative number.

b) A negative number multiplied by itself 3 times can produce a negative number.

18. The time required for:

- Krypton-85 is 32.1 years
- Plutonium-239 is 72 000 years
- Iodine-129 is 5.1×10^7 years
- Uranium-235 is 2.13×10^9 years
- Uranium-238 is 1.35×10^{10} years

19. a) **Number of computers infected with a computer virus according to time**

Time (h)	Number of infected computers
0	1
1	2
2	4
3	8
4	**16**
5	**32**
6	**64**

b) 1) 1024 computers.

2) 16 777 216 computers.

3) 4.72×10^{21} computers.

4) 2^n computers.

Knowledge in action *(cont'd)*

20. a) **Value of a car according to time elapsed since purchase**

Time (years)	Value ($)
0	35,000
1	**29,750**
2	**25,287.50**
3	**21,494.38**
4	**18,270.22**
5	**15,529.69**

b) 1) $3,057.40

2) $1,356.58

21. a) 256 plantes.

b) 1024 cm^2

c) $\approx$ 99.18 m^2

22. a) The ball is at a height of 8 m after the 1st bounce.

b) The ball is at a height of 4.096 m after the 4th bounce.

c) The ball is at a height of $10\left(\frac{4}{5}\right)^n$ after the *n*th bounce.

SECTION 3.1 The exponential function

Problem

Enough energy can be produced to bring 3 L of water to a boil in approximately 3.57×10^{-6} s.

Activity 1

a. 12 mol.

b. The quantity of ^{60}Co decreases in half every 64 months.

c. **Quantity of ^{60}Co according to time**

Time (months)	Calculations	Quantity of ^{60}Co (mol)
0	$12 \times 0{,}5^0$	12
64	$12 \times 0.5 = 12 \times 0.5^1 = 12 \times 0.5^{\frac{64}{64}}$	6
128	$12 \times 0.5 \times 0.5 = 12 \times 0.5^2 = 12 \times 0.5^{\frac{128}{64}}$	3
192	$12 \times 0.5 \times 0.5 \times 0.5 = 12 \times 0.5^3 = 12 \times 0.5^{\frac{192}{64}}$	**1.5**
256	$\mathbf{12 \times 0.5^{\frac{256}{64}}}$	**0.75**
320	$\mathbf{12 \times 0.5^{\frac{320}{64}}}$	**0.375**
384	$\mathbf{12 \times 0.5^{\frac{384}{64}}}$	**0.1875**
...	...	...
n	$\mathbf{12 \times 0.5^{\frac{n}{64}}}$	

d. By $0.5^{\frac{2048}{64}}$.

e. The curve progressively approaches the x-axis but never touches it.

Activity 2

a. **Temperature changes for the two types of lava**

Time (h)	0	1	2	3	4
Terrestrial lava temperature (°C)	**1200**	**972**	**787.32**	**≈ 637.73**	**≈ 516.56**
Undersea lava temperature (°C)	**1200**	**491.52**	**≈ 201.33**	**≈ 82.46**	**≈ 33.78**

b. **1)** 19% **2)** 59.04%

c. $T = 1200(0.8)^{4x}$

d. Yes. Using the law of exponents, you can transform the rule $T = 1200(0.9)^{2x}$ to $T = 1200(0.9^2)^x$ and then finally $T = 1200(0.81)^x$.

e. $1200(0.8)^{4x} = 1200(0.8^4)^x = 1200(0.4096)^x$.

Activity 3

a. **Value of an investment based on interest calculation period**

Number of periods for each year	Interest calculated at each period (%)	Calculation	Value of the investment at year end ($)
1 (yearly)	100	1×2	2
2 (biannually)	50	1×1.5^2	2.25
4 (quarterly)	25	1×1.25^4	**2.44**
12 (monthly)	**$8.\overline{3}$**	$\mathbf{1 \times (1.08\overline{3})^{12}}$	**2.61**
52 (weekly)	**≈ 1.92**	$\mathbf{1 \times 1.092^{52}}$	**2.69**
365 (daily)	**≈ 0.27**	$\mathbf{1 \times 1.0027^{365}}$	**2.71**
8760 (hourly)	**≈ 0.01**	$\mathbf{1 \times 1.0001^{8760}}$	**2.72**
n	$\frac{100}{n}$	$1 \times \left(1 + \frac{1}{n}\right)^n$	

b. 1) 2 2) 2.25 3) ≈ 2.44 4) ≈ 2.61
5) ≈ 2.69 6) ≈ 2.71 7) ≈ 2.72

c. The results are the same.

d. Toward a value of approximately 2.7183.

e. It is the same value as in **d.**

f. $2.72

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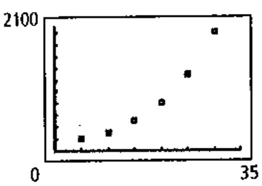
Technomath

a. The value of **a** represents the initial value of the curve that is used to model this situation. The value of **b** represents the base.

b. 1)

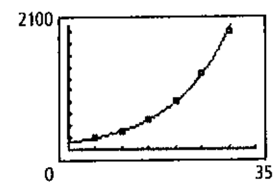


2)

```
ExpReg
 y=a*b^x
 a=143.7306783
 b=.5771366719
```

$y = 122.39(1.1)^x$

3)



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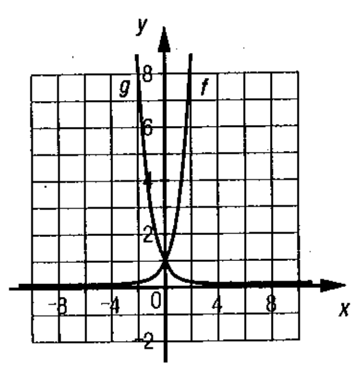
Practice 3.1

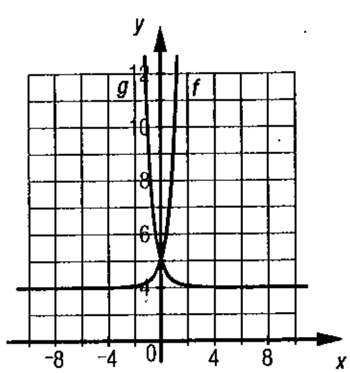
1.

	Rule of the function	Domain	Range	Initial value	Variation	Equation of the asymptote
a)	$y_1 = 3\left(\frac{1}{5}\right)^x$		]0, +∞[	3	Decreasing	$y = 0$
b)	$y_2 = 2.5^x$		]0, +∞[	1	Increasing	$y = 0$
c)	$y_3 = 3(5)^{x-3} + 1$		]1, +∞[	1.024	Increasing	$y = 1$
d)	$y_4 = 4(0.3)^{-(x-4)} + 2$		]2, +∞[	2.0324	Increasing	$y = 2$
e)	$y_5 = 2.5(1.01)^{12x}$		]0, +∞[	2.5	Increasing	$y = 0$
f)	$y_5 = 3000(0.95)^{\frac{x}{6}}$		]0, +∞[	3000	Decreasing	$y = 0$

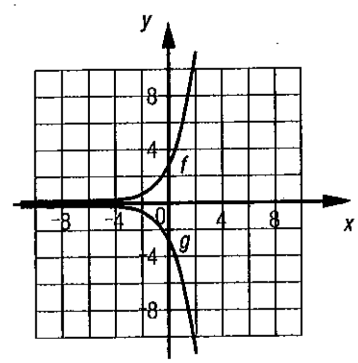
2. a) 1)



2)



3)



b) 1) A reflection about the y-axis.
2) A reflection about the y-axis.
3) A reflection about the x-axis.

3. a) Decreasing **b)** Increasing **c)** Decreasing
d) Increasing **e)** Increasing **f)** Increasing

4. a) 1) Domain: ℝ; range:]-250, +∞[. 2) ≈ -249.99
3) This function is decreasing. 4) $y = -250$

b) 1) Domain: ℝ; range:]-∞, 1.28[. 2) 0
3) This function is increasing. 4) $y = 1.28$

c) 1) Domain: ℝ; range:]-207.36, +∞[. 2) -87.36
3) This function is increasing. 4) $y = -207.36$

d) 1) Domain: ℝ; range:]-∞, 337.5[. 2) ≈ 324.33
3) This function is decreasing. 4) $y = 337.5$

e) 1) Domain: ℝ; range:]-10 711.05, +∞[. 2) -211.05
3) This function is increasing. 4) $y = -10\,711.05$

f) 1) Domain: ℝ; range:]-32, +∞[. 2) 4064
3) This function is decreasing. 4) $y = -32$

Practice 3.1 (*cont'd*)

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5. a) $f(x) = 2(3)^x + 2$ **b)** $f(x) = 25(5)^x - 2$ **c)** $f(x) = -25(0.2)^x + 1$
d) $f(x) = 32(0.5)^x - 8$ **e)** $f(x) = 1.5(2)^x - 1$ **f)** $f(x) = -32(0.5)^x - 8$

6. a) 1) $f(x) = 4(6)^x$ 2) $f(x) = 3(1.5)^x$ 3) $f(x) = -(3)^x$ 4) $f(x) = 2(0.5)^x$
b) 1) $f(x) = 3(2)^x + 7$ 2) $f(x) = 10(5)^x - 15$ 3) $f(x) = 0.5(10)^x + 300\,000$ 4) $f(x) = 3(4)^x - 5$

Practice 3.1 (*cont'd*)

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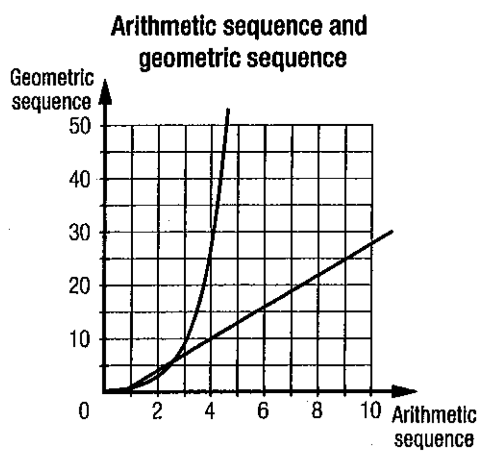
7. a) 1) $f \times g = -0.25(2)^{4x+5}$ 2) $\frac{f}{g} = -4(4)^{2x-5}$

b) 1) Domain: ℝ; range:]-∞, 0[. 2) Domain: ℝ; range:]-∞, 0[.

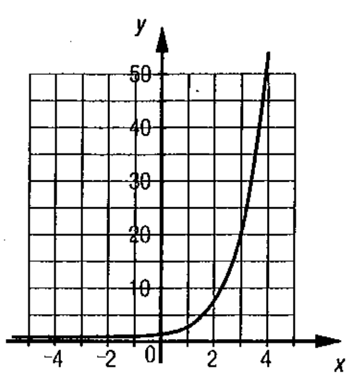
8. a)



b) A first-degree polynomial function is associated with the arithmetic sequence; whereas, an exponential function is associated with the geometric sequence.

c) Arithmetic sequence: $y = 3x - 2$; geometric sequence: $y = 3^{x-1}$ or $y = \frac{1}{3}(3)^x$.

9. a)



b) $y = 0$

c) **1)** Domain: ℝ; range:]0, +∞[. **2)** This function is increasing.
3) The initial value is 1.

10. ≈ $7691.15

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Practice 3.1 *(cont'd)*

11. **a)** $26,500 **b)** 106% **c)** $26,522.50 **d)** 106.09%

e)

Plan A				Plan B			
Time (months)	Time (years)	Calculation	Value of investment ($)	Time (months)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.06)^0$	25,000	0	0	$25\,000(1.03)^0$	25,000
				6	0.5	$25\,000(1.03)^1$	**25,750**
12	1	$\mathbf{25\,000(1.06)^1}$	**26,500**	12	1	$25\,000(1.03)^2$	**26,522.50**
				18	1.5	$\mathbf{25\,000(1.03)^3}$	**27,318.18**
24	2	$\mathbf{25\,000(1.06)^2}$	**28,090**	24	2	$\mathbf{25\,000(1.03)^4}$	**28,137.72**
				30	2.5	$\mathbf{25\,000(1.03)^5}$	**28,981.85**
36	3	$\mathbf{25\,000(1.06)^3}$	**29,775.40**	36	3	$\mathbf{25\,000(1.03)^6}$	**29,851,31**
				42	3.5	$\mathbf{25\,000(1.03)^7}$	**30,746.85**
48	4	$\mathbf{25\,000(1.06)^4}$	**31,561.92**	48	4	$\mathbf{25\,000(1.03)^8}$	**31,669.25**
...	...	...	...	...	...	...	...
	x	$\mathbf{25\,000(1.06)^x}$			x	$\mathbf{25\,000(1.03)^{2x}}$	

f) Investment **B** is the most profitable because in this plan, the second portion of the 3% interest is calculated on an amount to which 3% has already been added.

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Practice 3.1 *(cont'd)*

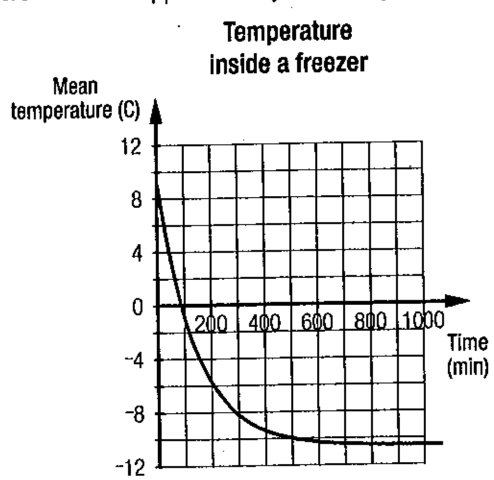
12. **a)** **1)** 13.5 V **2)** ≈ 3.50 V

b) This function is decreasing.

c) Domain: [0, 216]; range: [4.87×10^{-8}, 13.5].

13. There would be approximately 1197 frogs remaining.

14. **a)**



b) **1)** $y = -10.5$

2) Even if theoretically this temperature would never be reached, it is the "minimum" temperature of the freezer.

c)]-10.5, 9]

d) 9 °C

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Practice 3.1 (*cont'd*)

15. a) ≈ 29.53% **b)** ≈ 50.34% **c)** ≈ 82.62%

16. $V = V_0(1.005)^{3x}$

17. a) $p = 10^d$ **b)** The opaqueness is approximately 316.23 units. **c)** No.

18. a) $I = (1.02)^x$ where I represents the interval (in h) and x represents the number of days elapsed.

b) 1) ≈ 1.15 h or ≈ 1 h 9 min. **2)** ≈ 1.81 h or ≈ 1 h 49 min. **3)** ≈ 19.50 h or ≈ 19 h 30 min.

SECTION 3.2 The logarithmic function

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Problem

An earthquake measuring 10 on the Richter scale releases approximately 984 560 times more energy than an earthquake measuring 6 on the Richter scale.

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Activity 1

a.

Evolution of percentage of ^{14}C in bones

Time (years)	Percentage de ^{14}C
0	10^{-10}
5 730	5×10^{-11}
11 460	2.5×10^{-11}
17 190	1.25×10^{-11}
22 920	$\mathbf{6.25 \times 10^{-12}}$
28 650	$\mathbf{3.125 \times 10^{-12}}$
34 380	$\mathbf{1.5625 \times 10^{-12}}$

b. This percentage decreases by half every 5730 years.

c. An exponential function.

d. 1) $\approx 3.91 \times 10^{-13}$ % **2)** $\approx 9.77 \times 10^{-14}$ %

e. 1) 40 110 years **2)** 45 840 years

f. Both of these graphs represent functions that are inverses of each other since they are symmetrical in relation to the equation of the line $y = x$.

g. 1) f^{-1} **2)** f

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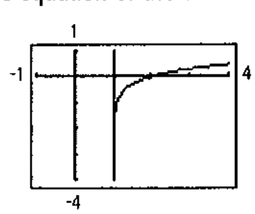
Technomath

a. Y_1: b = 1 and h = 2.4 ; Y_2: b = 2 and h = -1.8; Y_3: b = -2 and h = 4.2.

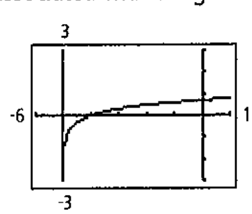
b. 1) $y = -4.2$ **2)** $y = -1.8$ **3)** $y = 2.4$

c. The equation of the vertical asymptote associated with a logarithmic function in the form $y = a \log b(x - h)$ is $x = h$.

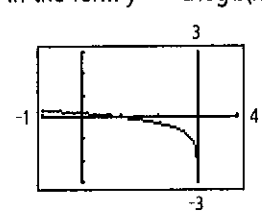
d. 1)



2)



3)

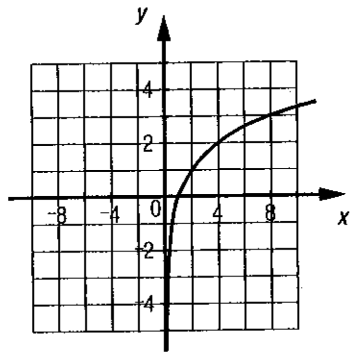


Practice 3.2

1. a) $4 = \log_3 81$ b) $6 = \log_2 64$ c) $\frac{2}{3} = \log_5 \sqrt{125}$ d) $\frac{1}{2} = \log_{144} 12$
   e) $-2 = \log 0.01$ f) $3 = \log_{\frac{1}{3}} \frac{1}{27}$ g) $0 = \log_3 1$ h) $-4 = \log_{\frac{1}{4}} 256$
2. a) $2^5 = 32$ b) $10^3 = 1000$ c) $4^{-1} = \frac{1}{4}$ d) $10^{-4} = 0.0001$
   e) $10^1 = 10$ f) $5^0 = 1$ g) $2^{-4} = \frac{1}{16}$ h) $3^4 = 3^4$
3. a) 4 b) 3 c) 3 d) 3
   e) -2 f) -4 g) -3 h) 1
4. a) 4 b) 100 c) -2 d) $\frac{3}{2}$
   e) 10 f) $\frac{1}{81}$ g) 3 h) $\sqrt{12}$
5. a) 1)

x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

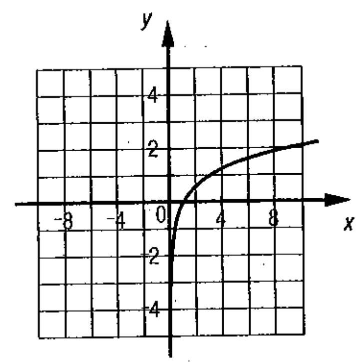
2)



b) 1)

x	y
$\frac{1}{27}$	-3
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2
27	3

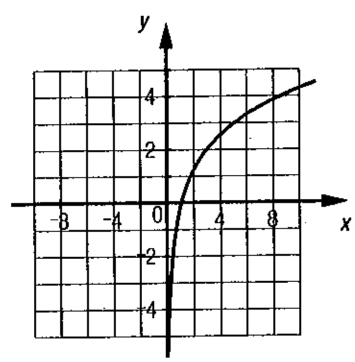
2)



c) 1)

x	y
$\frac{1}{125}$	-3
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2
125	3

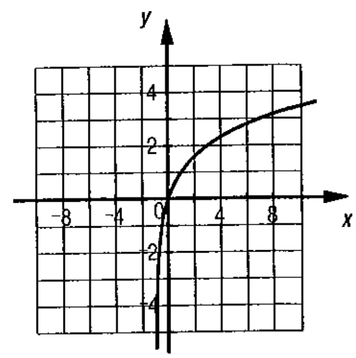
2)



d) 1)

x	y
$-\frac{7}{8}$	-3
$-\frac{3}{4}$	-2
$-\frac{1}{2}$	-1
0	0
1	1
3	2
7	3

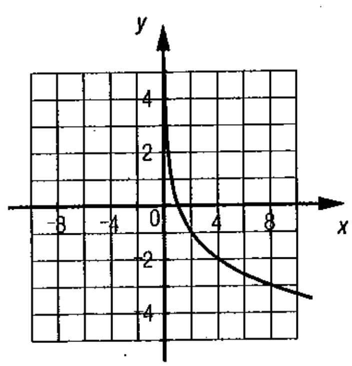
2)



e) 1)

x	y
$\frac{1}{8}$	3
$\frac{1}{4}$	2
$\frac{1}{2}$	1
1	0
2	-1
4	-2
8	-3

2)



Practice 3.2 *(cont'd)*

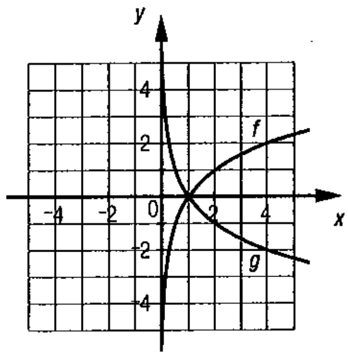
6. a) ≈ 1.49 b) ≈ 1.79 c) ≈ 0.40 d) ≈ 1.91
   e) ≈ -0.69 f) ≈ 0.43 g) ≈ 2.30 h) ≈ -0.74
7. a) $f^{-1}(x) = \log_3 x$ b) $g^{-1}(x) = \log_{0.8}(x - 7)$ c) $h^{-1}(x) = \frac{1}{3}\ln x$
   d) $i^{-1}(x) = \log \frac{200}{9}(x + 5)$ e) $j^{-1}(x) = \log_{\frac{1}{3}} \frac{20}{3} x$ f) $k^{-1}(x) = 2\ln\frac{x}{5}$

8. a) $f^{-1}(x) = 5^x$ b) $g^{-1}(x) = 10^{\frac{2x}{9}} + 3$ c) $h^{-1}(x) = e^{\frac{20x}{47}}$
d) $i^{-1}(x) = \frac{1}{2}(2)^{\frac{2}{15}(x-5)}$ e) $j^{-1}(x) = 10^{\frac{1}{2}(x-1)} + 4$ f) $k^{-1}(x) = 2e^{2x}$

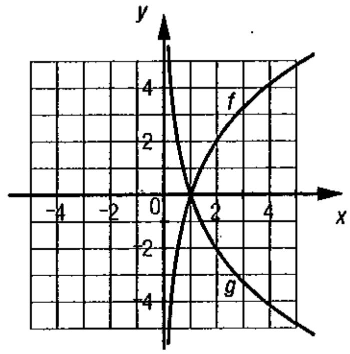
9.

	Rule of the function	Base	Equation of the asymptote	Domain	Range
a)	$f(x) = 2\log_2 x$	2	$x = 0$	]0, +∞[	ℝ
b)	$g(x) = \log x$	10	$x = 0$	]0, +∞[	ℝ
c)	$h(x) = 3\log_{1.5}(x-4) + 2$	1.5	$x = 4$	]4, +∞[	ℝ
d)	$i(x) = \log_{0.5}x - 1$	0.5	$x = 0$	]0, +∞[	ℝ
e)	$j(x) = \ln x$	e	$x = 0$	]0, +∞[	ℝ
f)	$k(x) = -\log_3(x+1) - 5$	3	$x = -1$	]-1, +∞[	ℝ

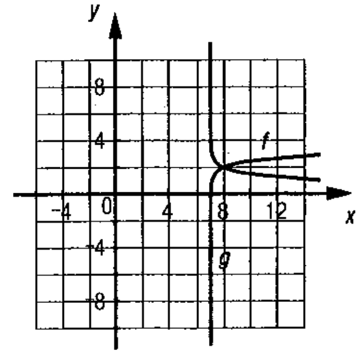
10. a) 1)



2)



3)



b) 1) A reflection about the *x*-axis. 2) A reflection about the *x*-axis.
3) A reflection about the line with equation $y = 2$.

11. a) Increasing b) Decreasing c) Increasing
d) Increasing e) Decreasing f) Increasing

Practice 3.2 (*cont'd*)

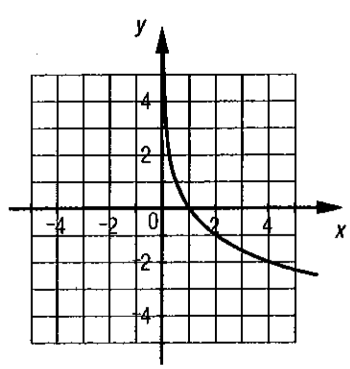
12. a) $f(x) = \log_2 \frac{1}{2}(x-3)$ b) $f(x) = \log_{\frac{1}{2}}(x+2)$ c) $f(x) = \log_{\frac{3}{2}} -(x-5)$
d) $f(x) = \log_7 -\frac{1}{3}(x+7)$ e) $f(x) = \log_{24} 2(x-8)$ f) $f(x) = \log_{\frac{1}{2}} \frac{1}{7}(x+10)$

13.

Function	f	f^{-1}
Rule	$f(x) = -1.5(2)^x + 4$	$f^{-1}(x) = \log_2 \frac{2}{3}(x+4)$
Domain	ℝ	]-∞, 4[
Range	]-∞, 4[	ℝ
Initial value	**2.5**	**No initial value**

Practice 3.2 (*cont'd*)

14. a) The two curves are superimposed.



b) $y = -\log_2 x \Leftrightarrow -y = \log_2 x$
$\Leftrightarrow 2^{-y} = x$
$\Leftrightarrow \left(\frac{1}{2}\right)^y = x$
$\Leftrightarrow y = \log_{\frac{1}{2}} x$

15. a) 1) No initial value. 2) $x = 0$
b) 1) The initial value is 3. 2) $x = 4$
c) 1) No initial value. 2) $x = 0$
d) 1) No initial value. 2) $x = 5$
e) 1) No initial value. 2) $x = 0$
f) 1) The initial value is 1. 2) $x = e$

16. **Intensity of sound according to acoustic pressure**

Nature of the sound	Pression (Pa)	Intensity (dB)	Perception
Thunder	11.25	**115**	**Dangerous**
Fire truck siren	35.56	**≈ 125**	**Unbearable**
Normal conversation	0.02	**60**	**Normal**
Crowded highway	1.12	**≈ 95**	**Painful**
Night club	5.02	**≈ 108**	**Dangerous**
Rock concert	63.25	**130**	**Unbearable**

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Practice 3.2 *(cont'd)*

17. a) 1) 5000 V 2) 4.74×10^{-15} V
b) 1) $x = -\frac{10}{83}\ln\left(\frac{T}{5000}\right)$ 2) 8.35×10^{-2} ms

18. a) $t = 2\log_{\frac{1}{10}}\left(\frac{Q}{100}\right)$ b) $t = -2\log\left(\frac{Q}{100}\right)$
c) 1) ≈ 0.25 h or ≈ 15 min. 2) ≈ 0.60 h or 36 min.
3) 2 h 4) ≈ 2.60 h or 2 h 36 min.

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Practice 3.2 *(cont'd)*

19. a) ≈ 27.38 MJ
b) 1) $x = 4095\ln 0.1\,(E+10)$ 2) ≈ 7337.26 turns/min

20. a) $[H^+] = 10^{-(pH)}$
b) **Characteristics of certain fluids**

Fluid	$[H^+]$ (mol/L)	pH
Milk	**≈ 1.74×10^{-7}**	6.76
Orange juice	1.95×10^{-4}	**≈ 3.71**
Bleach	1.78×10^{-13}	**≈ 12.75**
Coffee	**≈ 1.29×10^{-5}**	4.89
Human blood	4.57×10^{-8}	**≈ 7.34**
Gastric acid	6.17×10^{-2}	**≈ 1.21**
Distilled water	**1×10^{-7}**	7
Tea	**≈ 3.16×10^{-6}**	5.5

Exponential and logarithmic situations

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Problem

A scalpel must spend at least 8 min (approximately 7.60 min) in the autoclave.

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Activity 1

a. You can proceed as follows:
- from ① to ②, since $m = c^n$
- from ② to ③, using the law of exponents $(c^n)^x = c^{xn}$
- from ③ to ④, using the equivalence $m^x = c^{xn} \Leftrightarrow xn = \log_c m^x$
- from ④ to ⑤, since $n = \log_c m$

b. 1) 36 2) 7.5 3) -6 4) -6.8

c. Since $\log 9^{5000}$ is equivalent to $5000 \log 9$, you must calculate log 9 and multiply this number by 5000. Thus, $\log 9^{5000} = 5000 \log 9 \approx 5000 \times 0.95 \approx 4771.21$.

d. You can proceed as follows:
- from ① to ②, since $m = c^n$
- from ② to ③, since $\log_d c^n = n \log_d c$ (equivalence was seen previously)
- from ③ to ④, by dividing both sides of the equation by $\log_d c$
- from ④ to ⑤, since $n = \log_c m$

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Activity 1 *(cont'd)*

e. By using the equivalence $\log_c m = \frac{\log_d m}{\log_d c}$ where $d = 10$ or $d = e$, it is possible to calculate the logarithm of a number over any base.

f. 1) $\log_6 77$ 2) $\log_5 0.7$ 3) $\log_{\frac{1}{3}} 8$

g. 1) $y = \log_5 x$ 2) $y = \log_{5.2}(x + 4)$ 3) $y = \log_{0.2}\left(\frac{x}{5}\right)$

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Activity 2

a. 20 °C

b. 1) $0.94 = 0.94^x$ 2) Because $0.94^1 = 0.94^x$. Therefore, $x = 1$.

c. 1) You can proceed as follows:
- from ① to ②, by adding 200 to both sides of the equation
- from ② to ③, by dividing both sides of the equation by 220
- from ③ to ④, since $\frac{1}{55} = 0.94^x \Leftrightarrow x = \log_{0.94}\frac{1}{55}$
- from ④ to ⑤, since $\log_{0.94}\frac{1}{55} \approx 64.76$

2) It is the time required (in min) for the specimen to reach a temperature of -196°C.

d. $-196 \geq 220(0.94)^x - 200$

e. $[64.76, +\infty[$

Activity 3

a. 1) The zero.

2) You can proceed as follows:

- from ① to ②, by adding 18 to both sides of the equation
- from ② to ③, by dividing both sides of the equation by 9
- from ③ to ④, since $\log(x + 5) = 2 \Leftrightarrow x + 5 = 10^2$
- from ④ to ⑤, because $10^2 = 100$
- from ⑤ to ⑥, by subtracting 5 from both sides of the inequality

b. $]-5, +\infty[$

c. The interval where f is negative.

d. $]-5, 95[$

Practice 3.3

1. a) $c\log_a(4b)$ **b)** $2\log x$ **c)** $3\ln(2 + x)$ **d)** $\frac{1}{2}\ln 3x$
e) $-\log_c 3x$ **f)** $3\ln\left(\frac{y}{x}\right)$ **g)** $d\log_a y$ **h)** $-2\ln x$

2. a) $\log_3 6^4$ **b)** $\log_7 5^2$ **c)** $\ln(3t)^2$
d) $\log 5^3$ **e)** $\log_m x^2$ **f)** $\log 2^4$
g) $\log_3 9^0$ **h)** $\log_5 10^0$ **i)** $\log_6 y^4$

3. a) ≈ 5.91 **b)** ≈ 2.77 **c)** ≈ 2.32 **d)** ≈ 3.21
e) 0.5 **f)** -1 **g)** -6 **h)** ≈ -8.38

4. a) ≈ 2.3980 **b)** ≈ 3.513 **c)** ≈ 2.2695 **d)** ≈ -0.7565
e) ≈ 44.5977 **f)** ≈ -2.3980 **g)** ≈ 1.0619 **h)** ≈ -0.8783
i) ≈ 0.3997 **j)** -4.2474 **k)** ≈ 0.8905 **l)** ≈ 16.8483

Practice 3.3 *(cont'd)*

5. a) $x \approx -6.81$ **b)** $x = 10^{46}$ **c)** $x \approx 3.53$ **d)** $x \approx -5$
e) $x = -24$ **f)** $x \approx -0.09$ **g)** $x = -81$ **h)** $x \approx -0.34$

6. a) $x > 2.15$ **b)** $x \geq e^{18}$ **c)** $x > 33.\overline{\ }$ **d)** $x \geq 6$
e) $-99\,998 > x > 2$ **f)** $x > -1.29$ **g)** $x \geq -2.40$ **h)** $x > \frac{9}{8}$

7. a) 1) ≈ 6.14 **2)** $f(x) \geq 0$, if $x \in [\approx 6.14, +\infty[$ and $f(x) \leq 0$, if $x \in]-\infty, \approx 6.14]$.
b) 1) -1 **2)** $g(x) \geq 0$, if $x \in [-1, +\infty[$ and $g(x) \leq 0$, if $x \in]-2, -1]$.
c) 1) ≈ 3.74 **2)** $h(x) \geq 0$, if $x \in]-\infty, \approx 3.74]$ and $h(x) \leq 0$, if $x \in [\approx 3.74, +\infty[$.
d) 1) ≈ 7.04 **2)** $i(x) \geq 0$, if $x \in [\approx 7.04, +\infty[$ and $i(x) \leq 0$, if $x \in]7, \approx 7.04]$.
e) 1) ≈ 1.16 **2)** $j(x) \geq 0$, if $x \in]-\infty, \approx 1.16]$ and $j(x) \leq 0$, if $x \in [\approx 1.16, +\infty[$.
f) 1) $\frac{1}{e^2}$ **2)** $k(x) \geq 0$, if $x \in \left[\frac{1}{e^2}, +\infty\right[$ and $k(x) \leq 0$, if $x \in \left]0, \frac{1}{e^2}\right]$.

8. a) $x = \sqrt{3}$ **b)** $x = \sqrt[5]{625}$ **c)** $x = \frac{1}{6}$ **d)** $x = \sqrt{6} - 4$

9. a) $x = 8$ **b)** $x = 2$ **c)** $x = \sqrt{10} - 2$
d) $x = -4$ **e)** $x = 1002$ **f)** $x = 6$
g) $x = e$ or $x = -e$. **h)** $x = \frac{1}{3}$ **i)** $x = 5$
j) $x = 2$ or $x = 5$.

10. a) At $t = 0$ years. **b)** At approximately 9.66 years.
c) At approximately 12.86 years.

Practice 3.3 *(cont'd)*

11.

	A (dB)	*T* (V)	T_0 (V)
a)	30	**≈ 395.28**	12.5
b)	**≈ 4.08**	16	10
c)	60	18	**0.018**
d)	15	**≈ 84.35**	15
e)	**≈ 6.02**	36	18
f)	45	9	**≈ 0.05**

12. After approximately 15 years, more specifically after 14.72 years.

13. a) 1) ≈ -1.51 **2)** -7.5 **3)** -12.5 **4)** ≈ 1.51
b) 1) ≈ 19.06 times. **2)** ≈ 8.3×10^{-4} times. **3)** ≈ 10 964.78 times.

Practice 3.3 *(cont'd)*

14. The amount of time required for total decomposition:
- of a plastic bag is approximately 461.75 years
- of a tissue is approximately 0.25 years (3 months)
- of a carton of milk is approximately 49.88 years
- of a piece of chewing gum is approximately 5 years
- of an alkaline battery is approximately 6931.13 years

15. The warning should be issued approximately 6.44 weeks after May 1.

16. a) 1) 64 **2)** ≈ 51.98 **3)** ≈ 37.39 **4)** ≈ 32.08
b) 1) 1 048 576 **2)** ≈ 104 031.92 **3)** ≈ 2671.54 **4)** ≈ 486.71
c) 1) $A = 2^{-0.3\log_2 B + 6}$ **2)** $B = 2^{\frac{\log_2 A - 6}{-0.3}}$

17. The network reaches its maximum capacity approximately 25.09 years after it is put in place.

Practice 3.3 *(cont'd)*

18. a) 1) 0.6990, 1.6990, 2.6990, 3.6990 **2)** 0.9031, 1.9031, 2.9031, 3.9031
b) A multiplication of the argument by 10 is associated with an increase of 1 of the logarithm.

19. a) 1) 60 min **2)** 42 min **3)** ≈ 2.42 min
b) 1) At least 2 pieces. **2)** At least 3 pieces. **3)** At least 5 pieces.

20. The temperature is 0°C, approximately 3.03 h after it is switched on.

21. a) 1) ≈ 8.11 % **2)** ≈ 5.78 % **3)** ≈ 4.58 %
b) 1) ≈ 4.96 years. **2)** ≈ 15.4 years. **3)** ≈ 24.41 years.
c) 1) $r = \frac{\ln 2}{t}$ **2)** $t = \frac{\ln 2}{r}$

Practice 3.3 *(cont'd)*

22. a) The initial temperature of the first alloy is 20°C;the temperature of the second alloy is 40°C.
b) After 5 h, both alloys have reached the same temperature.
c) After 10 h.

23. a) 1) ≈ 18.99 cm **2)** ≈ 45.75 cm
b) 1) ≈ 91.50 cm **2)** ≈ 12.30 cm

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Chronicle of the past

1. a) \$717.32 b) \$129,116.70
2. a) 2 584 929 b) 45 c) 262 144
   d) 16 807 e) 21 f) 2 585 869

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In the workplace

1. a) Light deafness (threshold approximately 34.81 dB).
   b) Moderate deafness (threshold approximately 49.97 dB).
   c) Light deafness (threshold 20 dB).
2. $F = 62.5(2)^x$ where F represents the frequency (in Hz) and x represents the stage number of the hearing test.

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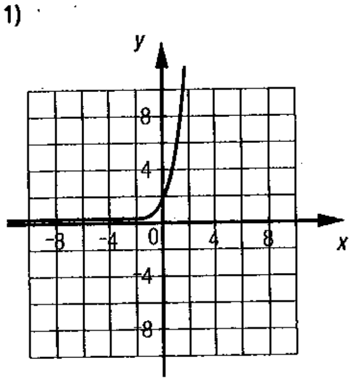
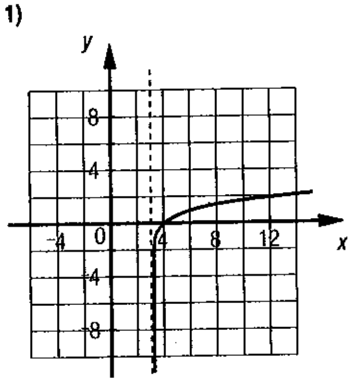
Overview

1. a) 1)



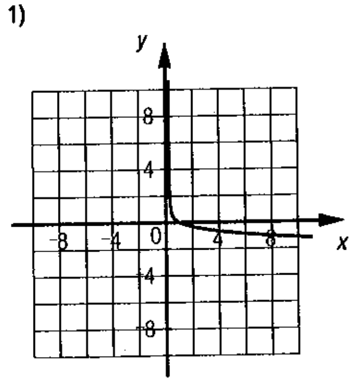
2) Domain: $\mathbb{R}$; range: $]0, +\infty[$.
3) 1.8
4) No zero.
5) $f(x) \geq 0$ if $x \in \mathbb{R}$.

b) 1)



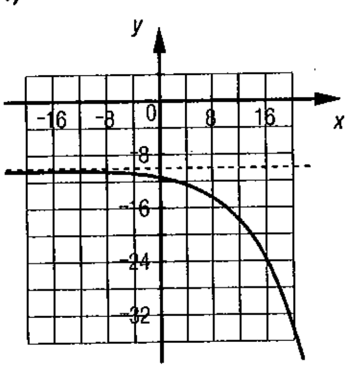
2) Domain: $]3, +\infty[$; range: $\mathbb{R}$.
3) No initial value.
4) 4
5) $f(x) \geq 0$ if $x \in [4, +\infty[$ and $f(x) \leq 0$ if $x \in]3, 4]$.

c) 1)



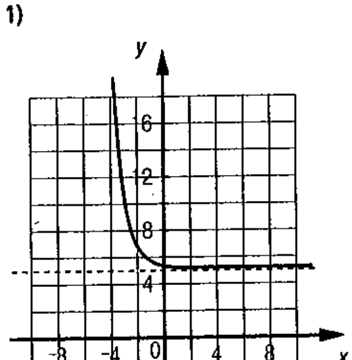
2) Domain: $]0, +\infty[$; range: $\mathbb{R}$.
3) No initial value.
4) 1
5) $f(x) \geq 0$ if $x \in]0, 1]$ and $f(x) \leq 0$ if $x \in [1, +\infty[$.

d) 1)



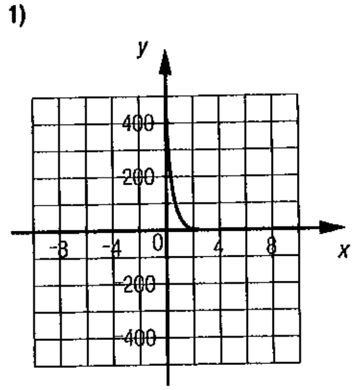
2) Domain: $\mathbb{R}$; range: $]-\infty, -10[$.
3) ≈ -11.49
4) No zero.
5) $f(x) \leq 0$ if $x \in \mathbb{R}$.

e) 1)



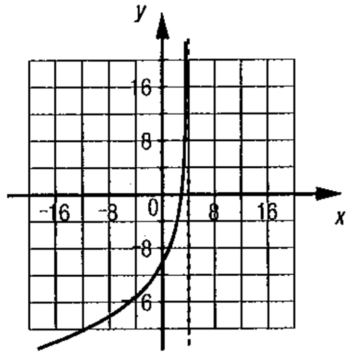
2) Domain: $\mathbb{R}$; range: $]5, +\infty[$.
3) 5.15
4) No zero.
5) $f(x) \geq 0$ if $x \in \mathbb{R}$.

f) 1)



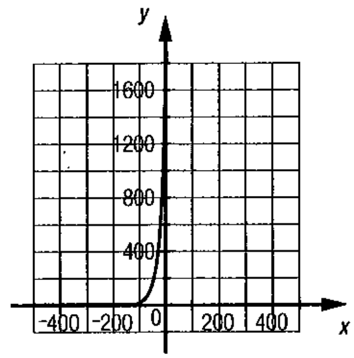
2) Domain: $\mathbb{R}$; range: $]0, +\infty[$.
3) 450
4) No zero.
5) $f(x) \geq 0$ si $x \in \mathbb{R}$.

g) 1)



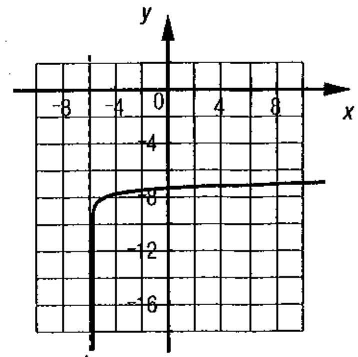
2) Domain:]-∞, 4[; range: ℝ.
3) -10
4) 3
5) $f(x) \leq 0$ if $x \in$]-∞, 3] and $f(x) \geq 0$ if $x \in$ [3, 4[

h) 1)



2) Domain: ℝ; range:]0, +∞[.
3) 1500
4) No zero.
5) $f(x) \geq 0$ if $x \in \mathbb{R}$.

i) 1)



2) Domain:]-6, +∞[; range: ℝ.
3) ≈ -7.33
4) ≈ 2 087 372 975.67
5) $f(x) \leq 0$ if $x \in$ [-6, ≈ 2 087 372 975,67] and $f(x) \geq 0$ if $x \in$ [≈ 2 087 372 975.67, +∞[.

2. a) $f(x) = 2(3)^x - 5$ **b)** $f(x) = \log_2 x$ **c)** $f(x) = \log_{\frac{9}{4}}(x + 2)$

d) $f(x) = 4\left(\frac{1}{2}\right)^x - 4$ **e)** $f(x) = 1500\left(\frac{82}{75}\right)^{\frac{x}{2}}$ **f)** $f(x) = \log_{\frac{1}{2}}(x + 4)$

g) $f(x) = -2(4)^x$ **h)** $f(x) = \log_2 \frac{1}{8}(x - 1)$ **i)** $f(x) = -2^x + 5$

Overview (*cont'd*)

3. a) $x \approx 13.64$ **b)** $x = 15$ **c)** $x \approx 1.62$ **d)** $x = 125\ 000$
e) $x \approx 0.20$ **f)** $x \approx 0.74$ **g)** $x = -79$ **h)** $x \approx 0.85$

4. a) $x > 99\ 998$ **b)** $x > 7$ **c)** $x < 5.27$ **d)** $x \leq -252$
e) $x < 81\ 337.40$ **f)** $x \geq 3.42$ **g)** $x \leq 2$ **h)** $x \geq 10^{-7}$

5. a) $f^{-1}(x) = \log_{0.7}\frac{1}{3}(x - 2)$ **b)** $g^{-1}(x) = -0.5\ln -0.4x$ **c)** $h^{-1}(x) = 2^{\frac{x}{7}} - 9$

d) $i^{-1}(x) = -\log_{0.05}\frac{2x}{3} + 4$ **e)** $j^{-1}(x) = 321e^{\frac{x}{455}}$ **f)** $k^{-1}(x) = 7(10)^{\frac{x}{3}}$

6. a) $x = 5$ **b)** $x = 2$ **c)** $x \approx -1.63$ **d)** $x \approx 3.61$

7. Yes. At an annual interest rate of 4%, the value of the investment after 20 years is $1600(1.04)^{20} = \$3505.80$, whereas if the interest is calculated every 6 months, the amount after 20 years is $1600(1.02)^{40} = \$3532.86$.

8. The value of the critical threshold is approximately 85.43 MW.

Overview (*cont'd*)

9. a) 1) \$7,401.22 **2)** \$7,429.74 **3)** \$7,456.83
b) The more often the interest is compounded, the more the value of the investment increases.

10. a) Approximately 3.46 million visitors.
b) 4 million visitors.
c) Approximately 4.95 million visitors.

11. *Several answers possible. Example:* $y = \frac{9189}{10\ 000}e^{\frac{13x}{2000}}$

Overview *(cont'd)*

12. In approximately 9.63 years.

13. a) $Q = 1000(0.9)^t$ **b)** 810 L **c)** Approximately 35 h after it starts to boil. **d)** 25 L

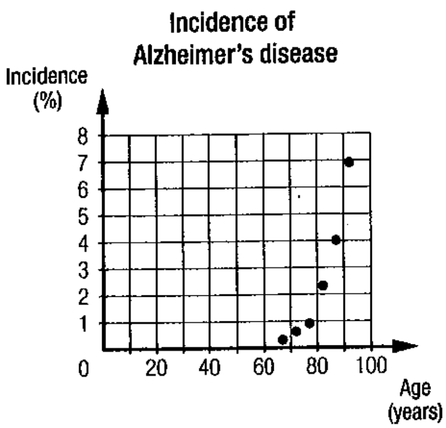
14. a) 1) 450 **2)** 10.7 **3)** 225

b) 1) $225 = 450e^{a10.7}$ **2)** ≈ -0.06 **3)** $M = 450e^{-0.0648t}$

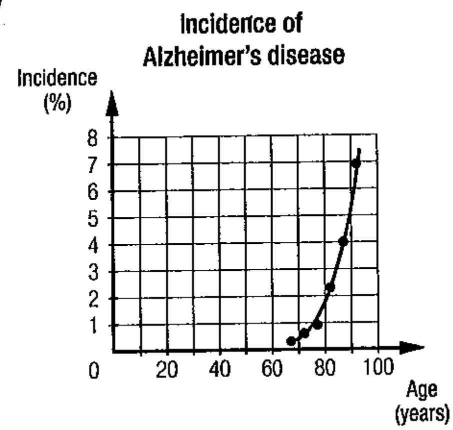
c) 1) $M = 5e^{-0.0001t}$ **2)** $M = 50e^{-0.0564t}$ **3)** $M = M_0e^{\frac{-t}{1\,024\,313\,479}}$

Overview *(cont'd)*

15. a)



b) 1)



2) *Several answers possible. Example:* $I = \frac{e^{0.1275a}}{16\,666.67}$

c) A person must undergo these tests as of the age of 76.

16. a) The rod was expanded approximately 2.54 mm.

b) The rod expands by 2 mm at 20°C.

c) The expansion of the rod is greater than 4 mm for temperatures that are greater than 2000°C.

d) The maximum expansion of the rod is approximately 3.35 mm.

Overview *(cont'd)*

17. Approximately 1.81 minutes after it is activated.

18. a) ≈ 60.65 %

b) The water must remain in the tank for approximately 1 month (approximately 29.96 days).

19. a) The voltage of the battery is decreasing because the base, e-1.2, is less than 1.

b) 1) At 0 h.

2) At 0.24 h (approximately 14 min 23 s).

c) There risks being a fire as of 0.58 h (approximately 34 min 39 s).

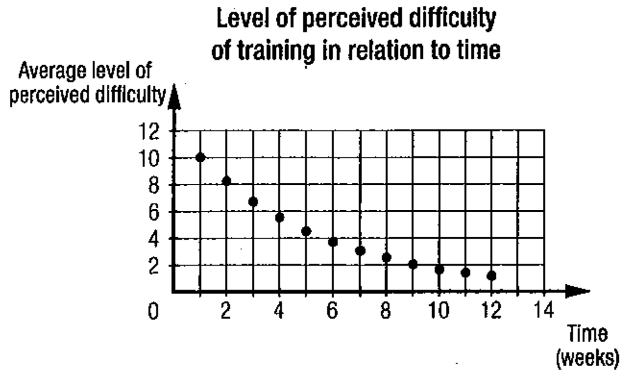
Overview *(cont'd)*

20. a) 250 dandelions.

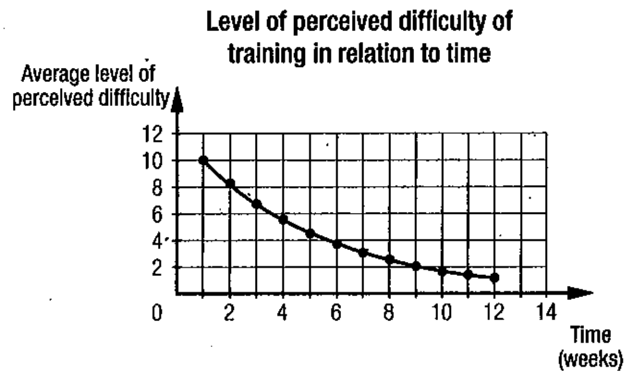
b) 1) Approximately 503 dandelions. **2)** Approximately 1014 dandelions. **3)** Approximately 4111 dandelions.

c) Approximately 26 dandelions.

21. a)



b) 1)



2) *Several possible answers. Example:* $D = 12.342e^{-0.2025x}$

c) 1) At week 2. **2)** At week 3. **3)** At week 4. **4)** At week 6.

Overview (*cont'd*)

22. a) At the start of the aging process, the water counts as 30% of the mass of this cheese.

b) The quantity of water would be 28% of the mass of this cheese in approximately 6.90 years.

23. a) ≈ 99.37 kPa **b)** ≈ 793 m **c)** ≈ 137.38 K

Bank of problems

1. Comparison of the amounts spent based on amount of payment

Payments P of $1,500/month	Payments P of $1,200/month
Value *E* of loan: $200,000 Interest rate i: 6 % $200\,000 = 1500 \times \frac{1-\left(\frac{1}{1+0.005}\right)^n}{0.005}$ $133{,}\bar{3} = \frac{1-\left(\frac{200}{201}\right)^n}{0.005}$ $0.\bar{6} = 1 - \left(\frac{200}{201}\right)^n$ $\left(\frac{200}{201}\right)^n = 1 - 0.\bar{}$ $\left(\frac{200}{201}\right)^n = 0.\bar{}$ $n \approx \log_{\frac{200}{201}} 0.\bar{3}$ $n \approx 220.27$	Value *E* of loan: $200,000 Interest rate i: 6 % $200\,000 = 1200 \times \frac{1-\left(\frac{1}{1+0.005}\right)^n}{0.005}$ $166.\bar{6} = \frac{1-\left(\frac{200}{201}\right)^n}{0.005}$ $0.8\bar{3} = 1 - \left(\frac{200}{201}\right)^n$ $\left(\frac{200}{201}\right)^n = 1 - 0.8\bar{3}$ $\left(\frac{200}{201}\right)^n = 0.1\bar{6}$ $n \approx \log_{\frac{200}{201}} 0.1\bar{6}$ $n \approx 359.25$
If the person makes 220.27 payments of $1,500, the total amount spent would be $330,406.96.	If the person makes 359.25 payments of $1,200, the total amount spent would be $431,096.43.

Making monthly payments of $1,500 rather than $1,200 saves $100,689.47.

2. Situation 1

If the mass increases by 30% every minute, it evolves based on the rule: $M = M_0(1.3)^x$ where M_0 represents the initial mass and x represents time (in min).

The mass would have doubled once $2M_0 = M_0(1.3)^x$. The following equation is solved:

$$2M_0 = M_0(1.3)^x$$
$$2 = (1.3)^x$$
$$x = \log_{1.3}2$$
$$x \approx 2.64 \text{ min or } \approx 2 \text{ min } 39 \text{ s.}$$

Situation 2

If the mass varies based on the rule $M = M_0 \times e^{\frac{3t}{10}}$ where t represents time (in min), the mass would have doubled once $2M_0 = M_0 \times e^{\frac{3t}{10}}$. The following is solved:

$$2M_0 = M_0 \times e^{\frac{3t}{10}}$$
$$2 = e^{\frac{3t}{10}}$$
$$\frac{3t}{10} = \ln 2$$
$$\frac{3t}{10} \approx 0.69$$
$$3t \approx 6.93$$
$$t \approx 2.31 \text{ min or } \approx 2 \text{ min } 19 \text{ s.}$$

Situation 3

If the mass increases by 0.5% every second, it evolves based on the rule: $M = M_0(1.005)^{60x}$ where M_0 represents the initial mass and x represents time (in min).

The mass would have doubled once $2M_0 = M_0(1.005)^{60x}$. The following is solved:

$$2M_0 = M_0(1.005)^{60x}$$
$$2 = (1.005)^{60x}$$
$$60x = \log_{1.005}2$$
$$60x \approx 138.98$$
$$x \approx 2.31 \text{ min or } \approx 2 \text{ min } 19 \text{ s.}$$

The mass of the substance doubles first in Situation **2**, approximately 0.35 s before the mass of the substance in Situation **3** and approximately 19.89 s before the mass of the substance in Situation **1**.

3. *Several answers possible. Example:*

The hero said:

- If the robots continue to reproduce in this manner, their population will increase at an exponential rate! Currently, every 30 h, the number N of robots doubles.
- This situation can be described by the mathematical rule $N = 2^{\frac{x}{30}}$ where x represents time (in h).
- The moment when the population of robots will exceed half the population of people corresponds to the inequality $25\,000\,000\,000 < 2^{\frac{x}{30}}$.
- To find this moment you must solve the equation $25\,000\,000\,000 = 2^{\frac{x}{30}}$:

$$25\,000\,000\,000 = 2^{\frac{x}{30}}$$
$$\frac{x}{30} = \log_2 25\,000\,000\,000$$
$$\frac{x}{30} \approx 34.54$$
$$x \approx 1036.24$$

- The population of robots will exceed half the population of people in approximately 1036.24 h, in other words in a little more than 43 days!
- The technical plan of these robots must be modified! If one robot can definitely be deactivated immediately after building another one, the population of robots will remain constant!

Bank of problems (*cont'd*)

4. • Determine the rule that calculates the minimum number of moves based on the number of discs. The minimum number N of moves based on the number d of discs corresponds to $N = 2^n - 1$.
   • Find the equation to be solved.
   To determine the initial number of discs that the tower has, considering that the minimum number of moves required to rebuild it is 255, you must solve the equation $255 = 2^n - 1$.
   • Solve the equation.
   $255 = 2^n - 1$
   $256 = 2^n$
   $n = \log_2 256$
   $n = 8$

   If the minimum number of moves required to rebuild the tower is 255, it therefore includes 8 discs.

5. *Several possible answers. Example:*

   Multiplication

   By using small numbers, it is possible to notice an equivalence such as$_c MN = \log_c M + \log_c N$.
   For example, $\log_2 4 = 2$, $\log_2 16 = 4$ and $\log_2 64 = 6$.
   Yet, $\log_2(4 \times 16) = \log_2 64$. It can be concluded that $\log_2 64 = \log_2 4 + \log_2 16$.
   Another example produces the same result: $\log 75 \approx 1.8751$, $\log 3 \approx 0.4771$ and $\log 25 \approx 1.3979$.
   Yet, $\log(3 \times 25) = \log 75$. It can be concluded that $\log 75 = \log 3 + \log 25$.
   To multiply two numbers, you must calculate the logarithm of these two numbers, add these logarithms and carry out the reverse operation of the result,that is exponentiation. For example, to multiply 24 531 by 1 596 741:

   $\log 24\,531 \approx 4.3897$
   $\log 1\,596\,741 \approx 6.2032$
   $\log 24\,531 + \log 1\,596\,741 \approx 10.5929$
   $24\,531 \times 1\,596\,741 = 10^{10.5926} = 39\,169\,653\,471$

   Division

   By using small numbers, it is possible to notice equivalence such as: $\log_c \frac{M}{N} = \log_c M - \log_c N$.
   For example, $\log_5 3125 = 5$, $\log_5 25 = 2$ and $\log_5 125 = 3$.
   Yet, $\log_5 \frac{3125}{25} = \log_5 125$. It can be concluded that $\log_5 125 = \log_5 3125 - \log_5 25$.
   Another example that produces the same result: $\log 144 \approx 2.1584$, $\log 6 \approx 0.7782$ and $\log 24 \approx 1.3802$.
   Yet, $\log \frac{144}{6} = \log 24$. It can be concluded that $\log 24 = \log 144 - \log 6$.
   To divide two numbers, you must calculate the logarithm of these two numbers, subtract these logarithms and carry out the reverse operation of the result, in other words exponentiation. For example, to divide 5 808 980 by 9148:

   $\log 5\,808\,980 \approx 6.7641$
   $\log 9148 \approx 3.9613$
   $\log 5\,808\,980 - \log 9148 \approx 2.8028$
   $5\,808\,980 \div 9148 = 10^{2.8028} = 635$

   Exponentiation

   By using the equivalence $\log_c m^n = n\log_c m$, you must multiply the logarithm of m by the exponent of n and carry out the reverse operation, that is exponentiation.
   For example, to find the value of 9^{50}:
   $\log 9^{50} = 50\log 9 \approx 47.7121$
   $9^{50} = 10^{47.7121} \approx 5.15 \times 10^{47}$

6. It is certain that the piece of wood will split when the probability of it splitting is 100% or 1. To determine the number of turns used to tighten a bolt based on where the piece of wood will surely split, the following equation must be solved:
   $1 = 1.0416^t - 1.$
   $1 = 1.0416^t - 1$
   $2 = 1.0416^t$
   $t = \log_{1.0416} 2$
   $t \approx 17$

   It is certain that the piece of wood will split as of 17 turns.

Bank of problems (*cont'd*)

7. • The probability of finding the combination at random if this consists of only one number is $\frac{1}{10}$.

• The probability of finding the combination at random if this consists of two numbers is $\frac{1}{100}$:

Probability of finding the 1st number at random	×	Probability of finding the 2nd number at random	=	Probability of finding the combination at random
$\frac{1}{10}$	×	$\frac{1}{10}$	=	$\frac{1}{100}$

• The probability of finding the combination at random if this consists of three numbers is $\frac{1}{1000}$:

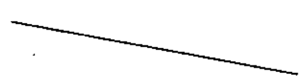
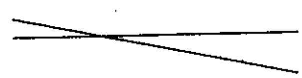
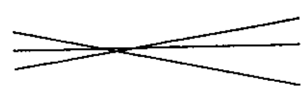
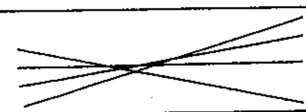
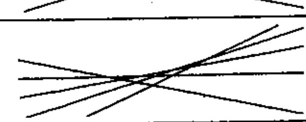
Probability of finding the 1st number at random	×	Probability of finding the 2nd number at random	×	Probability of finding the 3rd number at random	=	Probability of finding the combination at random
$\frac{1}{10}$	×	$\frac{1}{10}$	×	$\frac{1}{10}$	=	$\frac{1}{1000}$

• The probability of finding the combination at random if this consists of four numbers is $\frac{1}{10\,000}$:

Probability of finding the 1st number at random	×	Probability of finding the 2nd number at random	×	Probability of finding the 3rd number at random	×	Probability of finding the 4th number at random	=	Probability of finding the combination at random
$\frac{1}{10}$	×	$\frac{1}{10}$	×	$\frac{1}{10}$	×	$\frac{1}{10}$	=	$\frac{1}{10\,000}$

Probability of finding the 1st number at random	×	Probability of finding the 2nd number at random	×	Probability of finding the 3rd number at random	×	Probability of finding the 4th number at random	×	...	=	Probability of finding the combination at random
$\frac{1}{10}$	×	$\frac{1}{10}$	×	$\frac{1}{10}$	×	$\frac{1}{10}$	=	...	=	$\left(\frac{1}{10}\right)^n$

8.

Number of lines	**Illustration**	**Number of intersection**
1		0
2		1
3		3
4		6
5		10
6		15
...	...	...
D		$\frac{D(D-1)}{2}$ or $0.5D^2 - 0.5D$

The number I of intersecting points, based on the number D of drawn lines, does not vary based on an exponential function but rather on a second-degree polynomial function.

9. You must solve the system of equations associated to the situation where $P = C$.
By using a table to solve this, the following is obtained:

Time t (days)	Population P (millions of individuals)	Capacity C of environment (millions of individuals)
0	1	1
10	1.63	9.78
20	2.65	18.56
30	4.31	27.33
40	7.01	36.11
50	11.41	44.89
60	18.57	53.67
70	30.21	62.44
80	49.16	71.22
90	80	80

This phenomenon would happen after 90 days.

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Bank of problems *(cont'd)*

10. The depth P (in m) varies based on the rule $P = e^{38 - 5\ln R}$ where R corresponds to the rotational speed (in numbers of turns/min).

11. Since during the treatment the population P (in %) of bacteria of the digestive system of a patient evolves based on the rule $P = 100(0.9)^t$ and that the duration of the treatment is 10 days, the population of bacteriaat the end of the treatment is approximately 34.87%. The population would return to its normal level once $P = 100$. You must therefore solve the equation $100 = 34.87e^{0.14(t-10)}$.

$$100 = 34.87e^{0.14(t-10)}$$
$$2.87 = e^{0.14(t-10)}$$
$$0.14(t-10) = \ln 2.87$$
$$0.14(t-10) \approx 1.05$$
$$(t-10) \approx 7.53$$
$$t \approx 17.53$$

The population would return to its normal level approximately 17.53 days after the start of the treatment, in other words approximately 7.53 days after the end of the treatment. The population would therefore return to its normal level faster than the doctor claims.