student book volume 2

Science

VISIONS

MATHEMATICS

Secondary Cycle Two, Year Three

ANSWER KEY

Vision 4

LES ÉDITIONS CEC

A Quebecor Media Company

9001 Louis-H.-La Fontaine, Anjou (Quebec) Canada H1J 2C5
Telephone: 514 351-6010 • Fax: 514 351-3534

PRELIMINARY VERSION

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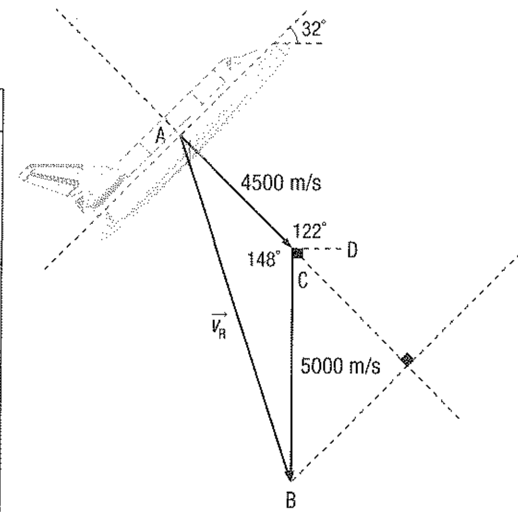
The following is an example of an approach that the students can use to answer the question.

Configuration ①

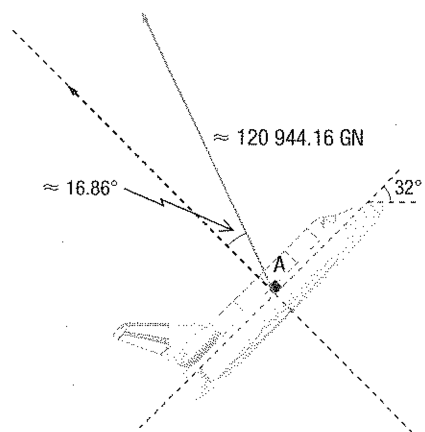
- Calculate speed $\vec{v}$ of the shuttle.



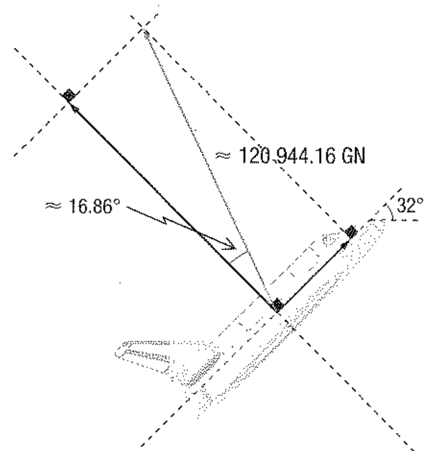
STATEMENT	JUSTIFICATION
m ∠ ACD = 122°	The measure of this angle corresponds to the sum of 32° and the measure of the right angle.
m ∠ ACB = 360° − 90° − 122° = 148°	Angles ACB, ACD and BCD form a straight angle.
$\lVert\vec{v}_R\rVert = \sqrt{4500^2 + 5000^2 - 2(4500)(5000)\cos 148°}$ ≈ 9133.03 m/s	Based on the cosine law.
m ∠ CAB = $\arcsin\left(\frac{5000 \sin 148°}{9133.03}\right) \approx 16.86°$	Based on the sine law.

- Calculate the force of friction $\vec{f}_F$.

The coefficient k is equal to $25 \times \sin 32° \approx 13.25$.
Since $\vec{f}_F = -k\vec{v}$, you can state the following:
– $\vec{f}_F$ is collinear, and $\vec{v}$ (two vectors, in which one corresponds to the product of the other by a scalar, are collinear).
– $\vec{f}_F$ is oriented in the opposite direction of $\vec{v}$.
Moreover, $\lVert\vec{f}_F\rVert = k\lVert\vec{v}\rVert \approx 13.25 \times 9133.03 \approx 20\,944.16$ GN.



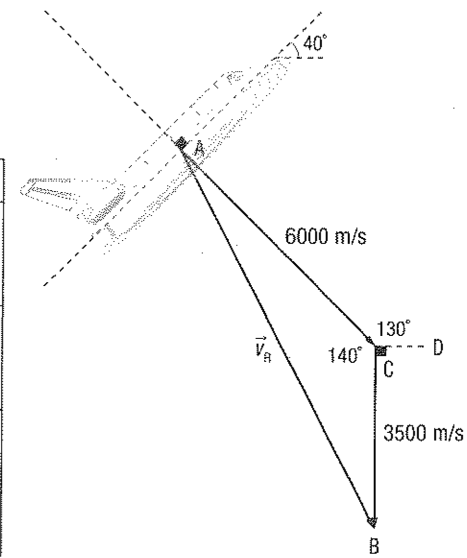
- Calculate the orthogonal projection of $\vec{f}_F$:
  – on the line that is perpendicular to the axis of the shuttle;
  the norm of the projection $\approx \lVert\vec{f}_F\rVert \times \cos 16.86°$
  $\approx 115\,793.38$ GN
  – on the axis of the shuttle;
  the norm of the projection $\approx \lVert\vec{f}_F\rVert \times \sin 19.44°$
  $\approx 35\,101.8$ GN



Configuration ②

- Calculate speed $\vec{v}$ of the shuttle.

  Since $k = 25 \times \sin\theta = 16.07$, it is determined that the angle of entry of the shuttle is equal to $\arcsin\left(\frac{16.07}{25}\right)$, which is approximately 40°.



STATEMENT	JUSTIFICATION
m ∠ ACD = 130°	The measure of this angle is the sum of 40° and the right angle.
m ∠ ACB = 360° − 90° − 130° = 140°	Angles ACB, ACD and BCD form a straight angle.
$\lVert\vec{v}_R\rVert = \sqrt{3500^2 + 6000^2 - 2(3500)(6000)\cos 140°}$ ≈ 8967.94 m/s	Based on the cosine law.
m ∠ BAD = $\arcsin\left(\frac{3500 \sin 140°}{8967.94}\right) \approx 14.53°$	Based on the sine law.

- Calculate the force of friction $\vec{f}_F$.

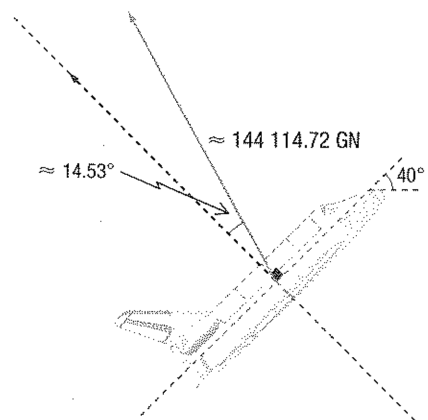
  The coefficient k equal to 16.07.

  Since $\vec{f}_F = -k\vec{v}$, the following is determined:

  – $\vec{f}_F$ is collinear to $\vec{v}$ (two vectors, in which one corresponds to the product of the other by a scalar are collinear).

  – $\vec{f}_F$ is oriented in the opposite direction of $\vec{v}$.

  Moreover, $\lVert\vec{f}_F\rVert = k\lVert\vec{v}\rVert \approx 16.07 \times 8967.94$ m/s ≈ 144 114.72 GN.



- Calculate the orthogonal projection of $\vec{f}_F$:

  – on the line perpendicular to the axis of the shuttle;

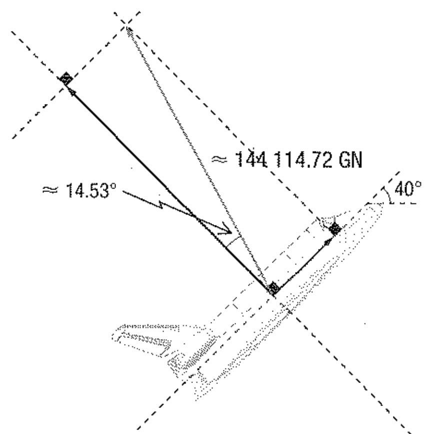
  the norm of the projection $\approx \lVert\vec{f}_F\rVert \times \cos 14.53°$

  ≈ 139 506.17 GN

  – on the axis of the shuttle;

  the norm of the projection $\approx \lVert\vec{f}_F\rVert \times \sin 14.53°$

  ≈ 36 153.59 GN



Conclusion

	Configuration ①	Configuration ②
The angle of entry in the atmosphere is at least 30° and at most 45°.	Yes	Yes
The speed is less than 10 km/s.	Yes	Yes
The orthogonal projection of the force of friction on the axis of the shuttle is less than 40 000 GN.	Yes	Yes
The orthogonal projection of the force of friction on the line perpendicular to the axis of the shuttle is less than 120 000 GN.	Yes	No

The crew of the shuttle should choose Configuration ①, because it respects all the restrictions that were stated.

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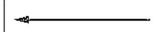
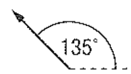
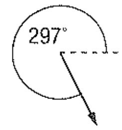
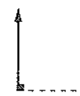
The following is an example of an approach that the students can use to complete the task.

The following are the vectors that come into play in this situation.

Direction of the movement						
Vector associated with this movement	$\vec{u}$	$\vec{v}$	$\vec{w}$	$\vec{r}$	$\vec{s}$	$\vec{t}$

- **From the exit airlock to the solar panel.**

During this step, the movement of the astronaut corresponds to vector SP.

Moreover, the series of manipulations associated with this step correspond to the linear combination $3\vec{v} + 2\vec{s}$ where $\vec{v} = (a, 0)$ and $\vec{s} = (0, b)$. Therefore:

$3\vec{v} + 2\vec{s} = \overrightarrow{SP}$

$3(a, 0) + 2(0, b) = (92 - 50, 68 - 32)$

$3(a, 0) + 2(0, b) = (42, 36)$

$3a = 42$ and $2b = 36$.

$a = 14$, $b = 18$, $\vec{v} = (14, 0)$ and $\vec{s} = (0, 18)$.

It is deduced that each movement is as follows:

– Movement to the right must have a length of 14 m.
– Movement to the front must have a length of 18 m.

- **From the solar panel to the Zvezda module.**

During this step, the movement of the astronaut corresponds to vector PZ.

Moreover, the series of manipulations associated with this step correspond to the linear combination $\vec{s} + 3\vec{u} + \vec{t}$ where $\vec{s} = (0, 18)$, $\vec{u} = (c, 0)$ and $\vec{t} = (0, d)$. Therefore:

$3\vec{u} + \vec{s} + \vec{t} = \overrightarrow{PZ}$

$3(c, 0) + (0, 18) + (0, d) = (55 - 92, 74 - 68)$

$3(c, 0) + (0, 18) + (0, d) = (^-37, 6)$

$3(c, 0) + (0, d) = (^-37, 6) - (0, 18) = (^-37, ^-12)$

$3c = {}^-37$ and $d = {}^-12$.

$c = {}^-\frac{37}{3}$, $d = {}^-12$, $\vec{u} = \left({}^-\frac{37}{3}, 0\right)$ and $\vec{t} = (0, {}^-12)$.

It is deduced that each movement is as follows:

– Movement to the left must have a length of 12.33 m.
– Movement to the back must have a length of 12 m.

- **From the Zvezda module to the airlock exit.**

During this step, the movement of the astronaut corresponds to vector ZS.

Moreover, the series of manipulations associated with this step correspond to the linear combination $\vec{u} + \vec{v} + \vec{w} + \vec{t} + 2\vec{r}$ where $\vec{u} = \left({}^-\frac{37}{3}, 0\right)$, $\vec{v} = (14, 0)$ and $\vec{t} = (0, {}^-12)$. Therefore:

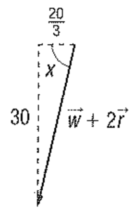
$\vec{u} + \vec{v} + \vec{w} + \vec{t} + 2\vec{r} = \overrightarrow{ZS}$.

$\left({}^-\frac{37}{3}, 0\right) + (14, 0) + (0, {}^-12) + \vec{w} + 2\vec{r} = (50 - 55, 32 - 74)$

$\left(\frac{5}{3}, {}^-12\right) + \vec{w} + 2\vec{r} = ({}^-5, {}^-42)$

$\vec{w} + 2\vec{r} = \left({}^-\frac{20}{3}, {}^-30\right)$

The following is a graphical representation of the vector associated with $\vec{w} + 2\vec{r}$:



You can deduce that:

– $x = \arctan\left(\frac{30}{\frac{20}{3}}\right) \approx 77.5°$

– $\|\vec{w} + 2\vec{r}\| = \sqrt{30^2 + \left(\frac{20}{3}\right)^2} \approx 30.7$ m

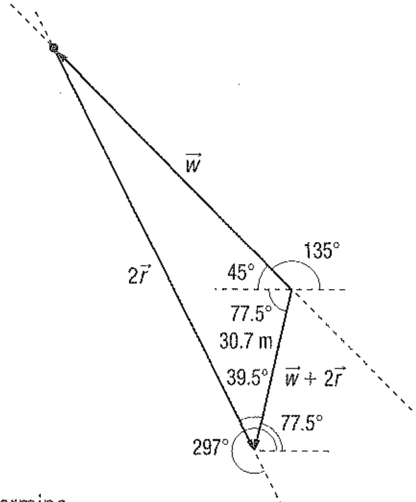
You can then do the following:

– Draw a line parallel to $\vec{w}$ passing through the tail of $\vec{w} + 2\vec{r}$.

– Draw a line parallel to $\vec{r}$ passing through the head of $\vec{w} + 2\vec{r}$.

Vectors $\vec{w}$ and $2\vec{r}$ are obtained.

These constructions determine that:



- $\dfrac{\|\vec{w}\|}{\sin 39.5°} \approx \dfrac{30.7}{\sin 18°}$

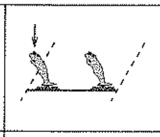
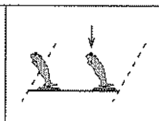
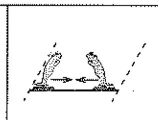
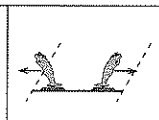
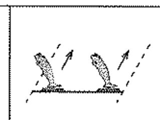
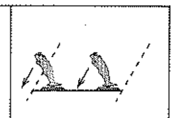
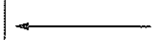
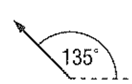
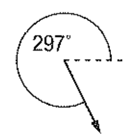
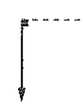
Therefore, $\|\vec{w}\| \approx \dfrac{30.7 \sin 39.5°}{\sin 18°} \approx 62.3$ m.

- $\dfrac{\|2\vec{r}\|}{\sin 122.5°} \approx \dfrac{30.7}{\sin 18°}$

Therefore, $\|2\vec{r}\| \approx \dfrac{30.7 \sin 122.5°}{\sin 18°} \approx 83.8$ m and $\|\vec{r}\| \approx 41.9$.

Since the direction and the norm of vectors w and r are known, it is possible to determine their components. $\vec{w} = (\approx -44.7, \approx 44.7)$ and $\vec{r} = (\approx 19, \approx -37.3)$ are obtained.

- **Summary of the movement generated by each manipulation of the joy sticks.**

Manipulation of the joy sticks						
Direction of the movement generated						
Length of the movement generated	≈ 12.3 m	14 m	≈ 62.3 m	≈ 41.9 m	18 m	12 m

- **Validation.**

By algebraically adding all the vectors associated with the set of manipulations, the following result is obtained:

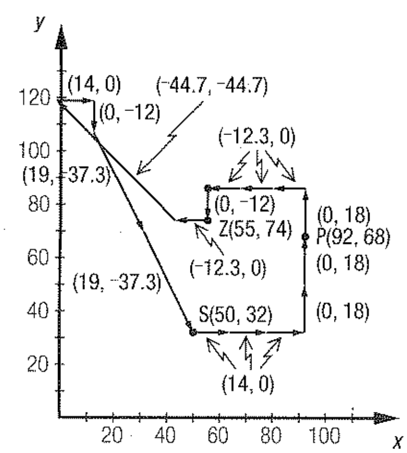
$$\begin{aligned}\text{Resultant movements} &= 3\vec{v} + 2\vec{s} + \vec{s} + 3\vec{u} + \vec{t} + \vec{u} + \vec{v} + \vec{w} + \vec{t} + 2\vec{r}\\ &= 4\vec{v} + 3\vec{s} + 4\vec{u} + 2\vec{t} + \vec{w} + 2\vec{r}\\ &\approx 4(14, 0) + 3(0, 18) + 4(-12.3, 0) + 2(0, -12) + (-44.7, 44.7) + 2(19, -37.3)\\ &\approx (4(14) + 4(-12.3) - 44.7 + 2(19), 3(18) + 2(-12) + 44.7 + 2(-37.3))\\ &\approx (0.1, 0.1)\end{aligned}$$

The norm of this vector is approximately 14 cm, which indicates that the astronaut would arrive approximately 14 cm from the exact position of the airlock.

The following is the graphical representation of this situation:

The resultant vector is a vector that is close to the zero vector.

The discrepancy of 14 cm in relation to the perfect position of the airlock is acceptable since the maximum allowed is 25 cm.



Prior learning 1

Page 4

a. **1)** The sine of this acute angle.
2) The cosine of this acute angle.
3) The tangent of this acute angle.

b. **1)** The distance is approximately 4.76 cm.
2) The distance is approximately 3.1 cm.

c. **1)** The limit of visual perception is approximately 7.09 cm.
2) The limit of visual perception is approximately 8.89 cm.

d. **1)** The measure of the angle of its blind spot is approximately 75.96°.
2) The measure of the angle of its blind spot is approximately 79.11°.

Prior learning 2

Page 5

a. A scalene triangle with an obtuse angle.

b. $(m\ \overline{ND})^2 = (m\ \overline{PN})^2 + (m\ \overline{PD})^2 - 2(m\ \overline{PN})(m\ \overline{PD})\cos P$

c. It is found approximately 124.5 km from its destination.

d. $\frac{m\ \overline{ND}}{\sin P} = \frac{m\ \overline{PN}}{\sin D} = \frac{m\ \overline{PD}}{\sin N}$

e. $\approx 9.6°$

f. The angle of correction would measure approximately 24.6°.

Knowledge in action

Page 8

1. **a)** $m \angle A \approx 44.96°$ **b)** $m \angle A \approx 60.68°$
c) $m \angle A \approx 30.58°$ **d)** $m \angle A \approx 59.92°$

2. **a)** $\angle C \cong 34°$; $m\ \overline{AB} \approx 2.29$ cm; $m\ \overline{BC} \approx 3.40$ cm **b)** $\angle D \cong 57°$; $m\ \overline{DE} \approx 5.37$ cm; $m\ \overline{DF} \approx 2.92$ cm
c) $\angle H \cong 30°$; $m\ \overline{GI} \approx 2.5$ cm; $m\ \overline{IH} \approx 4.33$ cm **d)** $\angle J \cong 20°$; $m\ \overline{JL} \approx 4.40$ cm; $m\ \overline{JK} \approx 4.68$ cm
e) $\angle N \cong 65°$; $m\ \overline{MN} \approx 5.68$ cm; $m\ \overline{MO} \approx 5.15$ cm **f)** $\angle P \cong 45°$; $m\ \overline{PR} \approx 3.3$ cm; $m\ \overline{PQ} \approx 4.67$ cm

3. **a)** 30° **b)** ≈ 78.5° **c)** ≈ 63.4°
d) ≈ 162.5° **e)** ≈ 115.8° **f)** ≈ 130.5°

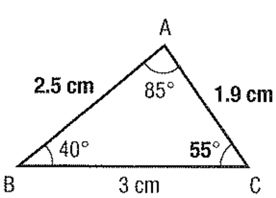
Knowledge in action (*cont'd*)

Page 9

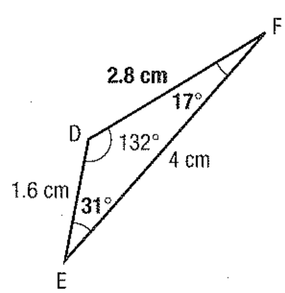
4. **a)** ≈ 11.9 cm **b)** ≈ 12.4 cm **c)** ≈ 8.8 cm
d) ≈ 36.7° **e)** ≈ 54.6° **f)** ≈ 131.4°

5. **a)** ≈ 4.5 cm **b)** ≈ 7.5 cm **c)** ≈ 6.9 cm
d) ≈ 60° **e)** ≈ 117.3° **f)** ≈ 40.9°

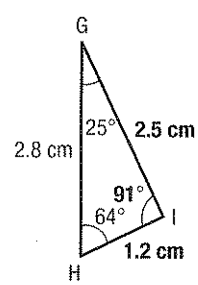
6. **a)**



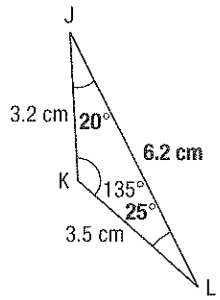
b)



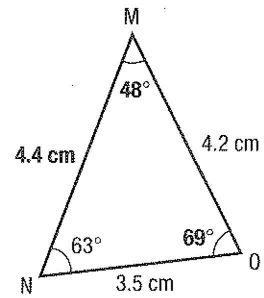
c)



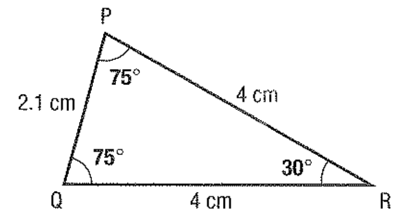
d)



e)



f)



Knowledge in action (*cont'd*)

Page 10

7.

	Slope (%)	Inclination (°)
a)	2	**≈ 1.1**
b)	**≈ 5.2**	3
c)	6	**≈ 3.4**
d)	**≈ 8.7**	5
e)	10	**≈ 5.7**
f)	**≈ 17.6**	10

8. **a) 1)** cos B or sin A. **2)** sin B or cos A. **3)** tan A **4)** tan B

b) 1) True. **2)** True. **3)** False. **4)** False.

9. The length of the cable is approximately 11.76 m.

Knowledge in action (*cont'd*)

Page 11

10. **a)** While completing the calculations, the student forgot to take into consideration the fact that angle B is obtuse.

b) m ∠ B ≈ 129.94°
m ∠ C ≈ 20.06°
m $\overline{AB}$ ≈ 2.06 cm

11. **a)** ≈ 14.97 km **b)** Airplane **A**: ≈ 18.88°, Airplane **B**: 30°. **c)** ≈ 1300.97 m

Knowledge in action (*cont'd*)

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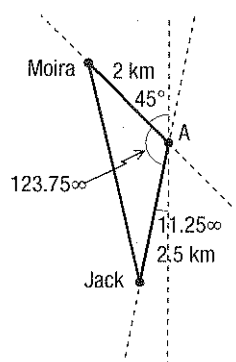
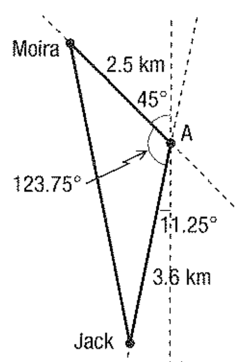
12. **a) 1)** Approximately 4 km separate Moira and Jack.



2) Approximately 5.4 km separate Moira and Jack.



b) The angle measure that Jack's trajectory would have is approximately 11.4°.

13. Itinerary **B** is less expensive. (It costs approximately $180.30 for Itinerary **B** and approximately $221.05 for Itinerary **A**.)

14. **a)** ≈ 2.89 cm **b)** ≈ 3.54 cm **c)** 5 cm **d)** ≈ 6.5 cm

15. a) The total length of the metal rods is approximately 10.89 m.
b) Angle ACB measures approximately 59.48°. **c)** ≈ 50.92°

Knowledge in action (*cont'd*) **Page 13**

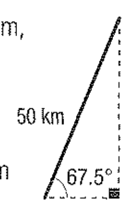
16. a) ≈ 159.66 m **b)** ≈ 19 225.22 m^2

17. a) 1) The size of the smallest discernable detail is approximately 0.15 cm.
2) The size of the smallest discernable detail is approximately 1.5 cm.
3) The size of the smallest discernable detail is approximately 15 cm.
b) 1) The maximum distance is approximately 3.44 m.
2) The maximum distance is approximately 34.4 m.
3) The maximum distance is approximately 6875 m.
c) The pixels are discernable for all the distances that are less than or equal to approximately 24.1 m.
d) The size of the smallest discernable detail is approximately 111.7 km.

SECTION 4.1 The characteristics of a vector

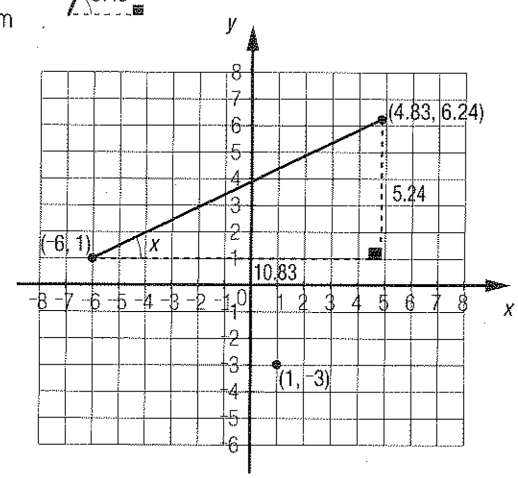
Problem **Page 14**

- Since the floatplane was travelling for 20 min, it covered a distance of $\frac{150}{3}$ km, which is 50 km heading NNE. (67.5° counter-clockwise in relation to the east-west axis.)
- This movement is equivalent to a succession of two movements:
  – a movement to the right of 50 cos 67.5°, which is equivalent to 19.13 km
  – an upward movement of 50 sin 67.5°, which is approximately 46.19 km



In the Cartesian plane, that corresponds to:
– a movement of $\frac{19.13}{5}$ km, which is approximately 3.83 units to the right
– a movement of $\frac{46.19}{5}$ km, which is approximately 9.24 units upwards
The information is deduced from the adjacent diagram.
Therefore, note the following:
- The movement of the helicopter is $\sqrt{10.83^2 + 5.24^2}$, which is approximately 12.03 units in the Cartesian plane and approximately 60.14 km in reality.
- $x \approx \arctan \frac{5.24}{10.83}$, which is approximately 25.82° north of east.

In conclusion, since the helicopter must cover 60 km in 15 min, it must fly at 240 km/h with an direction of 26° measured counter-clockwise in relation to the east-west axis.



Activity 1 **Page 15**

a. This information does not indicate the direction of the movement of each satellite.

b. Although it is known that the satellites move along parallel paths, the direction of the movement of each one is not known.

c. You must also know the direction each satellite is headed on the line.

d. ① Yes, because the satellites are heading toward each other.
② No, because the two satellites have the same speed and are heading in the same direction. The left satellite would never catch up to the right one.
③ Yes, because the left satellite has a greater speed than the right satellite; it would therefore end up catching up to the right satellite and collide with it.

Activity 2

Page 16

a. 1) Equipollent vectors are vectors that are the same size and have the same direction. They are identical vectors.

2) Opposite vectors are vectors that are the same length but whose direction differs by 180°. They are headed along a parallel path but in opposite directions.

3) Collinear vectors are vectors that have the same direction.

b. There are eight different vectors.

Activity 2 (*cont'd*)

Page 17

c. 1) The yellow vector and the grey vector.

2) *Several answers possible. Example:* The yellow vector and the black vector.

3) *Several answers possible. Example:* The grey vector and the green vector.

d. For each vector, the first number of the given ordered pair is obtained by subtracting the *x*-coordinate of the head of the arrow from the *x*-coordinate of the tail of the arrow, and the second number of the ordered pair is obtained by subtracting the *y*-coordinate of the head of the arrow from the *y*-coordinate of the tail of the arrow.

e. 1) (2, 4) **2)** (2, 4) **3)** (-2, -4) **4)** $(x_2 - x_1, y_2 - y_1)$

f. 1) ≈ 6.4 u **2)** ≈ 51.34°

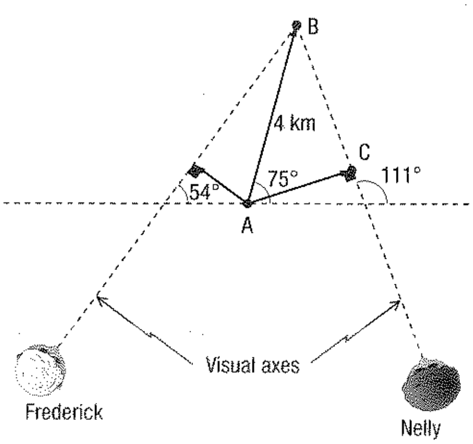
Activity 3

Page 18

a. A vector considers that a movement is defined not only by a length but also by a direction.

b. 1) 54° **2)** ≈ 2.35 km

c. 1)



2) ≈ 1.43 km

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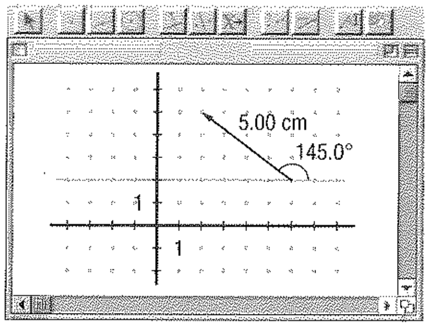
a. The tail is located at the same coordinates.

b. Screen **2**: **1)** ≈ 3 **2)** 3 Screen **3**: **1)** ≈ -4 **2)** -4 Screen **4**: **1)** ≈ 1 **2)** 1

c. If the direction of a vector AB corresponds to the angle measured counter-clockwise that it forms with the horizon, the difference between the *x*-coordinate of points B and A corresponds to the distance between points A and B multiplied by the cosine of the direction of this vector.

d. Screen **2**: **1)** ≈ 2 **2)** 2 Screen **3**: **1)** ≈ 2 **2)** 2 Screen **4**: **1)** ≈ -4 **2)** -4

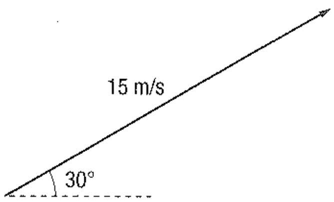
e. If the direction of a vector AB corresponds to the angle measured counter-clockwise that it forms with the horizon, the difference between the *x*-coordinate of points B and A corresponds to the distance between points A and B multiplied by the sine of the direction of this vector.

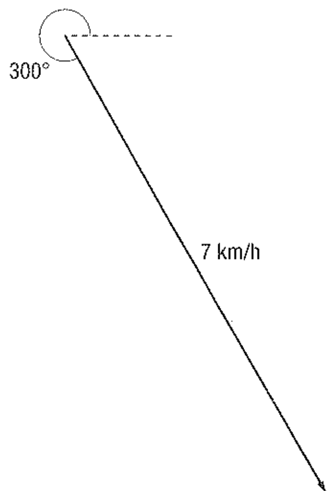
f. 

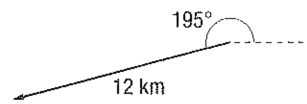
Practice 4.1

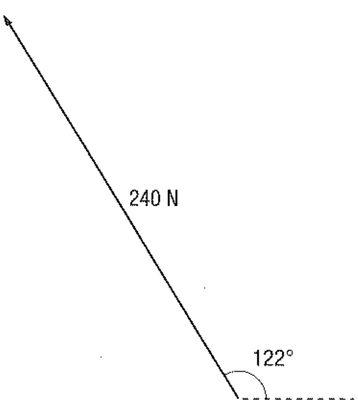
Page 23

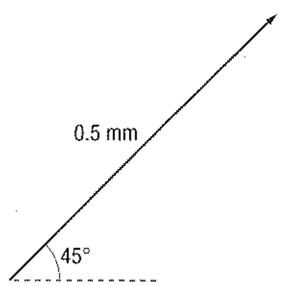
1. **a)** A vector quantity. **b)** A scalar quantity. **c)** A vector quantity.
   d) A vector quantity. **e)** A scalar quantity.

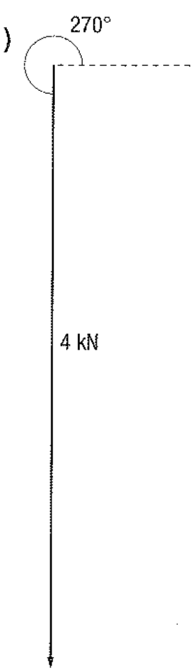
2. **a)** 

   b) 

   c) 

   d) 

   e) 

   f) 

3. **a)** **1)** $\overrightarrow{AB}$ and $\overrightarrow{MN}$.
   2) *Several answers possible. Example:* $\overrightarrow{AB}$ and $\overrightarrow{EF}$.
   b) $\overrightarrow{AB}$, $\overrightarrow{CD}$ and $\overrightarrow{MN}$.
   c) *Several answers possible. Example:* $\overrightarrow{AB}$ and $\overrightarrow{CD}$ as well as $\overrightarrow{KL}$ and $\overrightarrow{EF}$.

Practice 4.1 *(cont'd)*

Page 24

4. **a)** $\vec{v} = (\approx 3.2, \approx 3.8)$ **b)** $\vec{v} = (\approx -35, \approx 60.6)$ **c)** $\vec{v} = (\approx 8.2, \approx -9.7)$ **d)** $\vec{v} = (\approx -786.6, \approx -2161.3)$
   e) $\vec{v} = (0, -0.5)$ **f)** $\vec{v} = (\approx -0.82, \approx -0.57)$ **g)** $\vec{v} = (\approx -8.2, \approx -5.74)$ **h)** $\vec{v} = (\approx 176.8, \approx 176.8)$

5. **a)** $\|\vec{v}\| = \sqrt{2}$; direction: $= 45°$.
b) $\|\vec{w}\| = 3\sqrt{5}$; direction: $\approx 63.43°$.
c) $\|\vec{u}\| = 5\sqrt{5}$; direction: $\approx 26.57°$.
d) $\|\vec{s}\| = 2\sqrt{37}$; direction: $\approx 99.46°$.
e) $\|\vec{t}\| = \sqrt{65}$; direction: $\approx 240.26°$.
f) $\|\vec{m}\| = \sqrt{83.25}$; direction: $\approx 279.46°$.
g) $\|\vec{n}\| = \sqrt{1.01}$; direction: $\approx 95.71°$.
h) $\|\vec{o}\| = 6$; direction: $= 270°$.
i) $\|\vec{p}\| = 3\sqrt{2}$; direction: $= 135°$.
j) $\|\vec{e}\| = \sqrt{10\,361}$; direction: $\approx 349.24°$.
k) $\|\vec{c}\| = 3$; direction: $= 180°$.
l) $\|\vec{h}\| = \sqrt{113}$; direction: $\approx 221.19°$.

6. **a)** These vectors are collinear. **b)** These vectors are equipollent. **c)** These vectors are opposite.

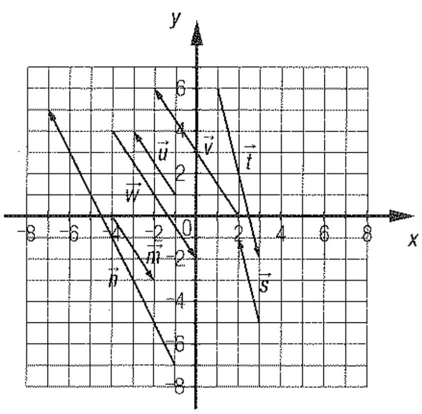
7. **a)** (0, 1) **b)** (-1, 0) **c)** $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ or $(-\sqrt{0.5}, -\sqrt{0.5})$ **d)** $\left(\frac{\sqrt{3}}{2}, 0.5\right)$ or $(\sqrt{0.75}, 0.5)$ **e)** $\left(0.5, \frac{\sqrt{3}}{2}\right)$

8. Ⓐ and Ⓓ.

9. **a)** $\|\overrightarrow{BA}\| \approx 10.63$; direction: $\approx 138.81°$.
b) $\|-\overrightarrow{CD}\| \approx 18.6$; direction: $\approx 306.25°$.

Practice 4.1 (*cont'd*)

10. **a)** *Several answers possible. Example:*



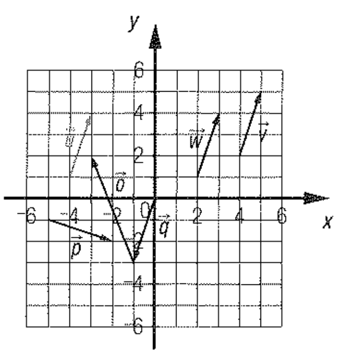
b) $\|\vec{u}\| = \sqrt{13}$; direction: $\approx 123.7°$.
$\|\vec{v}\| = 2\sqrt{13}$; direction: $= 123.7°$.
$\|\vec{w}\| = 2\sqrt{13}$; direction: $\approx 303.7°$.
$\|\vec{t}\| = 2\sqrt{17}$; direction: $\approx 284.04°$.
$\|\vec{m}\| = \sqrt{13}$; direction: $\approx 303.7°$.
$\|\vec{n}\| = 6\sqrt{5}$; direction: $\approx 116.6°$.
$\|\vec{s}\| = \sqrt{17}$; direction: $\approx 104°$.

c) **1)** Vector *u* is opposite to vector *m*, and vector *v* is opposite to vector *w*.
2) Vectors *u, v, w* and *m* are collinear to each other, and vectors *t* and *s* are collinear to each other.

d) **1)** *Several answers possible. Example:* The components of two opposite vectors have opposite signs.
2) *Several answers possible. Example:* The ratio $\frac{\text{vertical component}}{\text{horizontal component}}$ of two collinear vectors is identical.

11. **a)** **1)** $\frac{b}{a}$ **2)** $\frac{-a}{b}$
b) The slope of a line corresponds to the negative reciprocal of the slope of the other line. The product of the two slopes is therefore -1.
c) The two vectors are orthogonal because they are supported by lines whose slopes have a product of -1, which indicates that these lines are perpendicular.

12. *Several answers possible for vectors w, o, p and q. Example:*



13. a) $\approx 63.43°$

b) This vector can be represented by $\vec{u} = (x, 2x)$ where $x \in \mathbb{N}$. Therefore:

$\|\vec{u}\| = \sqrt{(2x)^2 + x^2}\ \sqrt{(2x)^2 + x^2} = \sqrt{4x^2 + x^2} = \sqrt{5x^2} = x\sqrt{5}$

$\|\vec{u}\| = \sqrt{4x^2 + x^2}\ \sqrt{(2x)^2 + x^2} = \sqrt{4x^2 + x^2} = \sqrt{5x^2} = x\sqrt{5}$

$\|\vec{u}\| = \sqrt{5x^2}\ \sqrt{(2x)^2 + x^2} = \sqrt{4x^2 + x^2} = \sqrt{5x^2} = x\sqrt{5}$

$\|\vec{u}\| = x\sqrt{5}$

Since x is a whole number, $x\sqrt{5}$ is a multiple of $\sqrt{5}$.

c) The horizontal component is equal to double the opposite of the vertical component.

Practice 4.1 (*cont'd*) **Page 26**

14. a) ≈ 7.82 **b)** ≈ 35.81 **c)** ≈ 0.71 **d)** ≈ 0.16

15. a) $\overrightarrow{AB} = (\approx -7.41, \approx 12.34)$ **b)** $\overrightarrow{AB} = (\approx 6.43, \approx 13.18)$

16. a) $(\approx 5.61, \approx 3.24)$ **b)** $(\approx 5.54, \approx 3.69)$ **c)** $(\approx 0.8, \approx -0.4)$

Practice 4.1 (*cont'd*) **Page 27**

17. a) 1) Norm: $\approx$ 14.6 million kilometres; direction: $\approx 249.95°$.
2) Norm: $\approx$ 14.6 million kilometres; direction: $\approx 313.06°$.
3) Norm: $\approx$ 14.6 million kilometres; direction: $\approx 55.01°$.
4) Norm: $\approx$ 14.6 million kilometres; direction: $\approx 153.08°$.

b) The coordinates are appoximately $(-12.64, -7.3)$.

18. Particles with the same charge: The force has a norm of 10^{-30} N and direction of approximately 30.96°.
Particles with opposite charges: The force has a norm of 10^{-32} and direction of approximately 153.43°.

Practice 4.1 (*cont'd*) **Page 28**

19. a) B = $(\approx 5.1, \approx 78.7°)$ D = $(\approx 4.12, \approx 166°)$ E = $(\approx 3.61, \approx 213.7°)$ F = $(\approx 5.66, \approx 315°)$

b) H = $(\approx -2.46, \approx 1.72)$

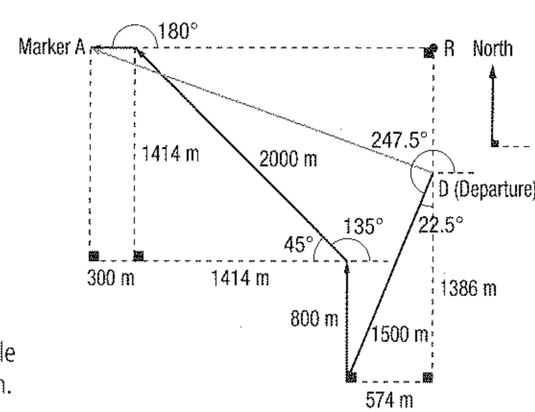
20. a) The force is approximately 139.5 N.

b) The person must place their hands 20 cm from the ground for the rope to be perfectly horizontal. Therefore, $\theta = 0$ and the projection of the force exerted is $\vec{f}_{exerted} \times \cos 0°$, meaning that this is the amount of force exerted.

SECTION 4.2 | Operations on vectors

Problem **Page 29**

The adjacent diagram represents the consecutive displacements defined by the list of instructions as well as the vector displacement that directly links the departure point to Marker **A**. The measures of the horizontal and vertical segments were deduced based on trigonometry.



This diagram also allows you to deduce that:

- m $\overline{DR} \approx 1414 + 800 - 1386 \approx 828$ m
- m $\overline{RA} \approx 300 + 1414 + 574 \approx 2288$ m
- m $\overline{DA} \approx \sqrt{2288^2 + 828^2} \approx 2433$ m
- m $\angle$ ADR $\approx \arctan\frac{2288}{828} \approx 70°$

Amelia must cover approximately 2433 m and place her compass at an angle measuring approximately 290°, which is 360° – 70°, in relation to the north.

Activity 1

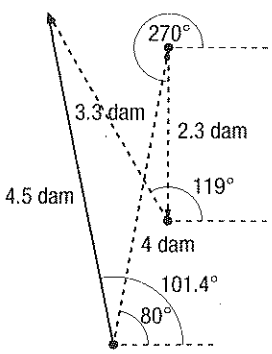
Page 30

a. 1) Since the movements are consecutive, they cannot start at the same point.
   2) The second movement starts where the first movement ended, which corresponds to the definition of two consecutive movements.

b. 1) 135°
   2) By using the cosine law, it is determined that $\|\overrightarrow{DE}\| \approx 5.98$ dam.
   3) By using the sine law, it is determined that angle FDE measures approximately 13.7°. It is deduced that the direction of vector DE is approximately 43.7°.

c. This shows that $\overrightarrow{GI} = \overrightarrow{DE}$.

STATEMENT	JUSTIFICATION
$m\angle GHI = 30° + (180° - 75°) = 135°$ $m\angle GHI = m\angle DFE$	
$\Delta GHI \cong \Delta DEF$	Based on SAS.
$\|\overrightarrow{DE}\| = \|\overrightarrow{GI}\|$	The congruent sides of two congruent triangles are congruent.
$m\angle IGH = \arcsin\frac{4.4 \sin 135°}{\|\overrightarrow{GI}\|} \approx 31.35°$	Based on sine law.
Direction of $\overrightarrow{GI} \approx 75° - 31.35° \approx 43.7°$	
$\overrightarrow{GI} = \overrightarrow{DE}$	Two vectors that have the same norm and direction are equipollent.

d. 

Activity 2

Page 31

a. The horizontal component of $\vec{r}$ is equal to 3.2 cos 22.44°, which is approximately equal to 2.96 kN.
   The vertical component of $\vec{r}$ is equal to 3.2 sin 22.44°, which is approximately equal to 1.22 kN.

b. 1) $0 + {}^-0.5 + {}^-0.9 + 4.3 = 3$ kN.
   2) The sum of the horizontal components of vectors *p, n, t* and *f* is approximately equal to the horizontal component of vector *r*.

c. 1) ${}^-2.1 + 1.4 + {}^-0.3 + 2.2 = 1.2$ kN.
   2) The sum of the vertical components of vectors *p, n, t* and *f* is approximately equal to the vertical component of vector *r*.

d. The components of a vector resulting in the addition of several vectors correspond to the sum of the components of each vector.

Activity 3

Page 32

a. 1) i) The quantity of the total movement of the skiers is equal to $\vec{p} + \vec{p}$.
      ii) The quantity of the total movement of the skiers is equal to $\vec{p} + \vec{p} + \vec{p}$.
   2) i) The quantity of the total movement of the skiers is equal to $2\vec{p}$.
      ii) The quantity of the total movement of the skiers is equal to $3\vec{p}$.
   3) i) The norm is 1400 kgm/s and the direction is 30°.
      ii) The norm is 2100 kgm/s and the direction is 30°.

b. 1) The direction of the resultant vector is identical to the direction of the starting vector.
2) The norm of the resultant vector corresponds to the norm of the starting vector multiplied by the same scalar as the vector.

c. 1) $\vec{p} = (\approx 606.22, \approx 350)$ 2) $2\vec{p} = (\approx 1212.44, \approx 700)$ 3) $3\vec{p} = (\approx 1818.65, \approx 1050)$

d. This conjecture is true because $2\vec{p} = (\approx 1212.44, \approx 700) \approx (2 \times 606.22, 2 \times 350)$ and $3\vec{p} = (\approx 1818.65, \approx 1050) \approx (3 \times 606.22, 3 \times 350)$.

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Page 33

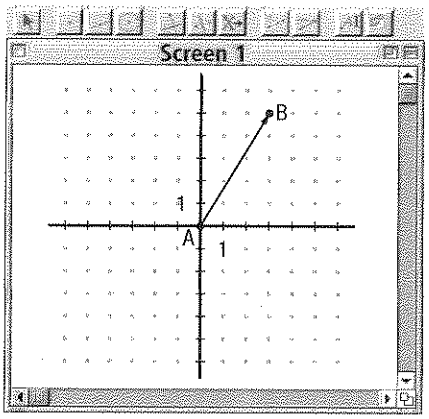
a. 1) (4, 2) 2) (-3, 3) 3) (1, 5)

b. 1) (5, -2) 2) (-3, 3) 3) (2, 1)

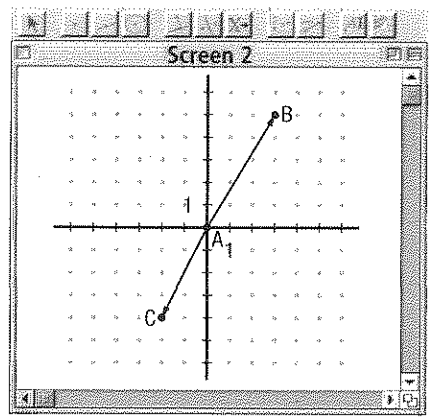
c. The components of a vector resulting from the sum of the two other vectors correspond to the sum of the components of these two vectors.

d. $\overrightarrow{AB} = (6, -1)$, $\overrightarrow{AC} = (1, 7)$ and $\overrightarrow{AD} = (7, 6)$. Yet, since $(6 + 1, -1 + 7) = (7, 6)$, the conjecture is verified.

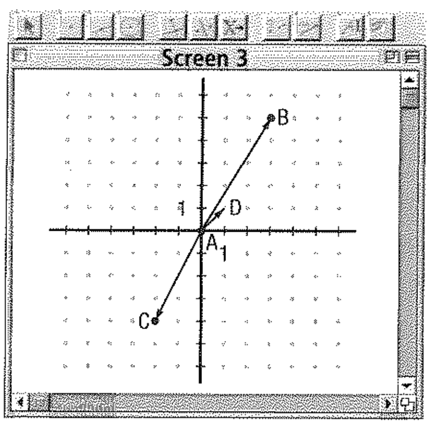
e. 1)



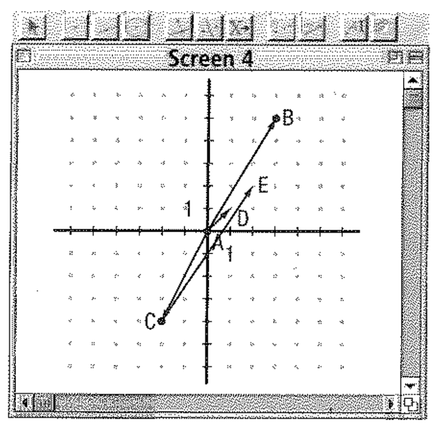
2)



3)



4)

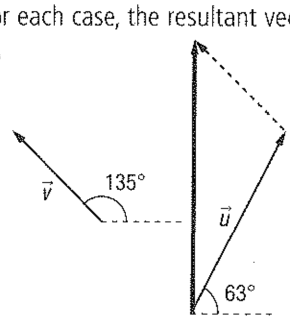


Practice 4.2

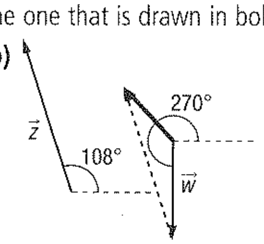
Page 37

1. For each case, the resultant vector is the one that is drawn in bold.

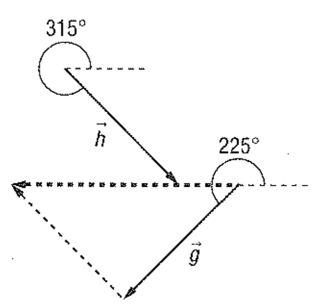
a)



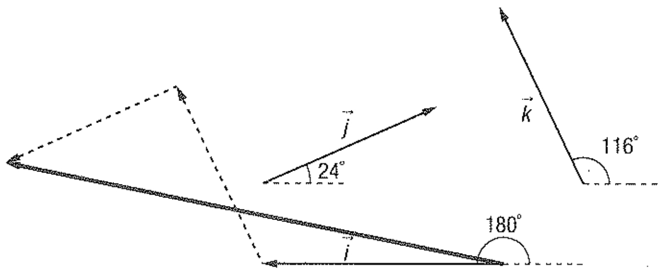
b)



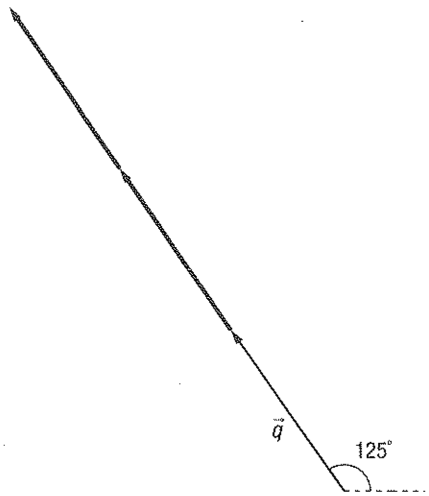
c)



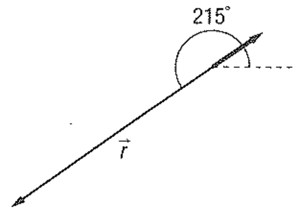
d)



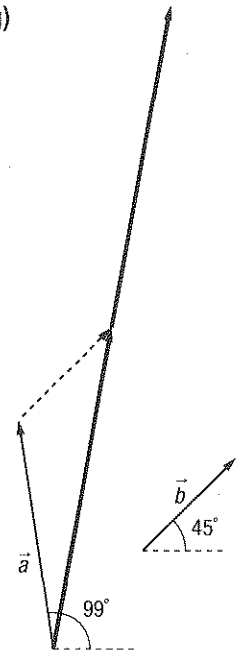
e)



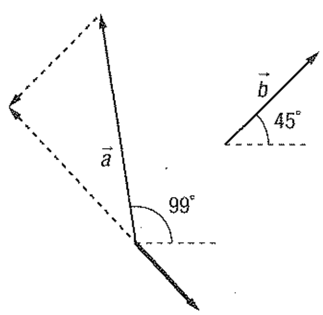
f)



g)



h)



Practice 4.2 (*cont'd*) **Page 38**

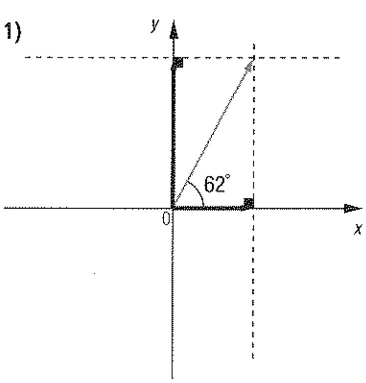
2. a) $\overrightarrow{AC}$ b) $\overrightarrow{BD}$ c) $\overrightarrow{AB}$ d) $\overrightarrow{AA}$ or $\vec{0}$. e) $\overrightarrow{AE}$ f) $\overrightarrow{AB}$

3. a) Norm: ≈ 3.12; direction: ≈ 108°. b) Norm: ≈ 5.07; direction: = 78°.
   c) Norm: ≈ 22.82; direction: ≈ 294°. d) Norm: ≈ 31.82; direction: ≈ 72°.

4. a) (-1, 8) b) (-2, 12) c) (1, -4) d) (-2, 1) e) (6, 9)
   f) (6, -10) g) (-15, -50) h) (8, 56) i) (12, -6)

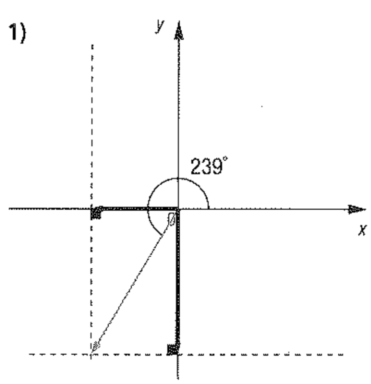
Practice 4.2 (*cont'd*) **Page 39**

5. a) 1)



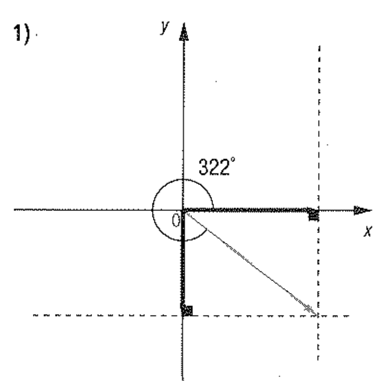
2) $\vec{v}$ = (≈ 1.9, ≈ 3.5)

b) 1)



2) $\vec{v}$ = (≈ -38.6, ≈ -64.3)

c) 1)



2) $\vec{v}$ = (≈ 0.14, ≈ -0.11)

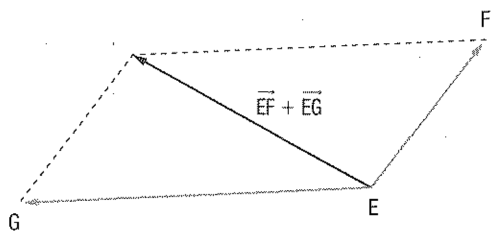
6. **a)** $\vec{s} = (-6, 10)$ **b)** $\vec{s} = (-46, -33)$ **c)** $\vec{s} = (-10, 3)$
d) $\vec{s} = (7, -21)$ **e)** $\vec{s} = (-18, -18)$ **f)** $\vec{s} = (-a - c, -b - d)$

7. **a)** ① Segments AC and BD are opposite sides of a parallelogram and as a result are parallel and congruent. Vectors AC and BD therefore have the same norm, direction and are equipollent.
② It is a direct application of the Chasles relation.
③ In the equality $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$, $\overrightarrow{BD}$ has been replaced by $\overrightarrow{AC}$ which is equipollent to it as explained in **a)** ①. Yet, replacing a term by an equivalent term conserves the equality.

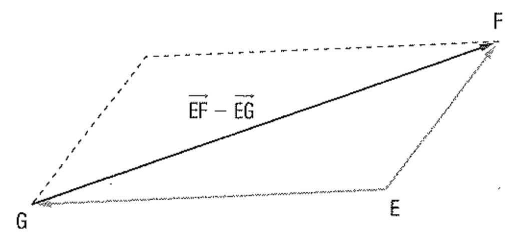
b)

Statement	Justification
$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{CA}$	Since $-\overrightarrow{AC} = \overrightarrow{CA}$.
$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CA} + \overrightarrow{AB}$	Since the addition of vectors is commutative.
$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$	Based on the Chasles relation.

c) 1)



2)



Practice 4.2 (*cont'd*)

8. **a)** $\overrightarrow{DE} + \overrightarrow{EB}$
b) $\overrightarrow{AB} - \overrightarrow{AD}$
c) $\overrightarrow{EB} + \overrightarrow{BC} + \overrightarrow{CA}$
d) *Several answers possible. Example:* $\overrightarrow{EB} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AE}$

9. **a)** $\overrightarrow{CA}$ **b)** $\overrightarrow{BD}$ **c)** $\overrightarrow{BD}$ **d)** $\overrightarrow{BD}$

10. **a)**

STATEMENT	JUSTIFICATION
$\overrightarrow{AB} + \overrightarrow{BC} + \vec{v} = \vec{0}$	
$\overrightarrow{AC} + \vec{v} = \vec{0}$	Based on the Chasles relation.
$\overrightarrow{AC} + \vec{v} = \overrightarrow{AA}$	Since $\overrightarrow{AA} = \vec{0}$.
$\overrightarrow{AC} + \overrightarrow{CA} = \overrightarrow{AA}$	Based on the Chasles relation.
$\vec{v} = \overrightarrow{CA}$	$\overrightarrow{CA}$ has been substituted by $\vec{v}$ while conserving its equality.
$\vec{v} = \overrightarrow{AC}$	The opposite of a vector is obtained by inversing its tail and head.

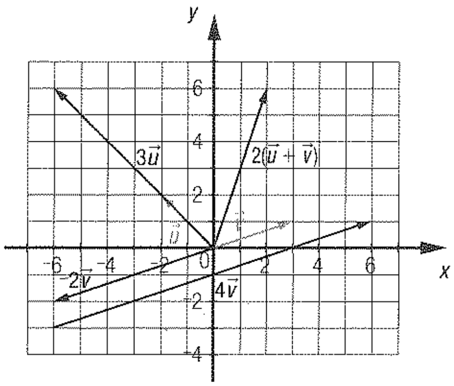
b) 1) $\vec{v} = \overrightarrow{AC}$ 2) $\vec{v} = \overrightarrow{AC}$ 3) $\vec{v} = \overrightarrow{DB}$

11. **a)** $\vec{v} = (6, 4)$, $\vec{p} = (3, -12)$ and $\vec{t} = (-0.4, -1)$.
b) 1) $\frac{2}{3}$ 2) $\frac{2}{3}$ 3) -4 4) -4 5) 2.5 6) 2.5
c) 1) Since two vectors in which one of them corresponds to the product of the other by a scalar are supported by a line with the same slope, you can deduce that they are parallel and have a direction that differs by 180°.
2) The slope of the line that supports $\vec{u}$ is $\frac{b}{a}$ and the slope of the line that supports $\vec{v}$ is $\frac{kb}{ka}$, which is equivalent to $\frac{b}{a}$. The conjecture is confirmed.

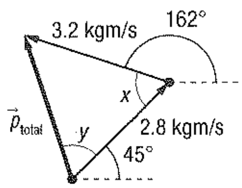
Practice 4.2 (*cont'd*)

12. a) $\vec{v} = 4\vec{u}$ **b)** $\vec{v} = \frac{5}{21}\vec{u}$ **c)** $\vec{v} = -2.5\vec{u}$

13.



14. a)



$x = (180° - 162°) + 45 = 63°$

$\|\vec{p}_{total}\| = \sqrt{2.8^2 + 3.2^2 - 2(2.8)(3.2)\cos 63°} \approx 3.15$ kgm/s

$y \approx \arcsin\frac{3.2\sin 63°}{3.15} \approx 65°$ and a direction of $\vec{p}_{total} \approx 110°$.

The components of $\vec{p}_{total}$ are therefore approximately (-1.06, 2.97).

b) You have $\vec{p}_{final\,A} + \vec{p}_{final\,B} = \vec{p}_{total}$, $\vec{p}_{final\,A} = (\approx 2.98, \approx -2.01)$ and $\vec{p}_{total} = (\approx -1.06, \approx 2.97)$.

You can deduce that:

$(\approx 2.98, \approx -2.01) + \vec{p}_{final\,B} = (\approx -1.06, \approx 2.97)$

$\vec{p}_{final\,B} = (\approx -1.06, \approx 2.97) - (\approx 2.98, \approx -2.01) = (\approx -4.05, \approx 4.98)$

The components of the vector that represents the quantity of movement of Object **B** after the collision are approximately (-1.9, 4.97).

c) The norm of $\vec{v}_{final\,B}$ is approximately 2.15 m/s.

The direction of $\vec{p}_{final\,B}$ is approximately $180° - \arctan\frac{4.98}{4.05}$, which is equal to approximately 129.1°.

Since $\vec{p}_{final\,B} = m_B\vec{v}_{final\,B} = 3\vec{v}_{final\,B}$, it is determined that the direction of $\vec{v}_{final\,B}$ is identical to the direction of $\vec{p}_{final\,B}$, which is equal to approximately 129.1°.

Practice 4.2 (*cont'd*)

15. a) 1) The direction of $\overrightarrow{OA}$ is 180°.

2) Since $\overrightarrow{OA} = -\frac{m_2}{m_1 + m_2}\overrightarrow{AB}$, then:

$\|\overrightarrow{OA}\| = \frac{m_2}{m_1 + m_2}\|\overrightarrow{AB}\|$

$\|\overrightarrow{OA}\| = \frac{3}{2 + 3} \times 3$ m

$\|\overrightarrow{OA}\| = 1.8$ m

3) The barycentre is placed at 1.8 m from point A or at 1.2 m from point B.

b) $\|\overrightarrow{OA}\| = \frac{m_2}{m_1 + m_2}\|\overrightarrow{AB}\|$

10 cm $= \frac{1}{1000 + 1}\|\overrightarrow{AB}\|$

$\|\overrightarrow{AB}\| = 10\ 010$ cm or 10.01 m.

This lever must have a length of 10.01 m.

16. a) You have $\vec{f_1} = (\approx -120.4, \approx 159.7)$ and $\vec{f_2} = (\approx 133.65, \approx 68.1)$. It is determined that:

$\vec{f_1} + \vec{f_2} = (\approx -120.4, \approx 159.7) + (\approx 133.65, \approx 68.1)$

$\vec{f_1} + \vec{f_2} = (\approx 13.25, \approx 227.8)$

The components of the force that corresponds to the sum of the $\vec{f_1}$ and $\vec{f_2}$ are approximately (13.29, 227.83).

b) No, because the only horizontal components of the force corresponds to the horizontal movement of the concrete block. The contender that exerts a total force that is weaker than the other contender but whose direction is closer to the horizon can win. In fact, this is the case in the situation illustrated in the problem.

c) The disadvantaged contender is the one on the left because the horizontal component of the resultant force is directed toward the right. In order for this contender to win this match, they must:

$\|\vec{f_1}\| \cos 53° > 150 \cos 27°$

$\|\vec{f_1}\| > \frac{150 \cos 27°}{\cos 53°}$

$\|\vec{f_1}\| > 222.08 \text{ N}$

The disadvantaged contender must exert a force of at least 223 N.

SECTION 4.3 Linear combinations and scalar products

Problem

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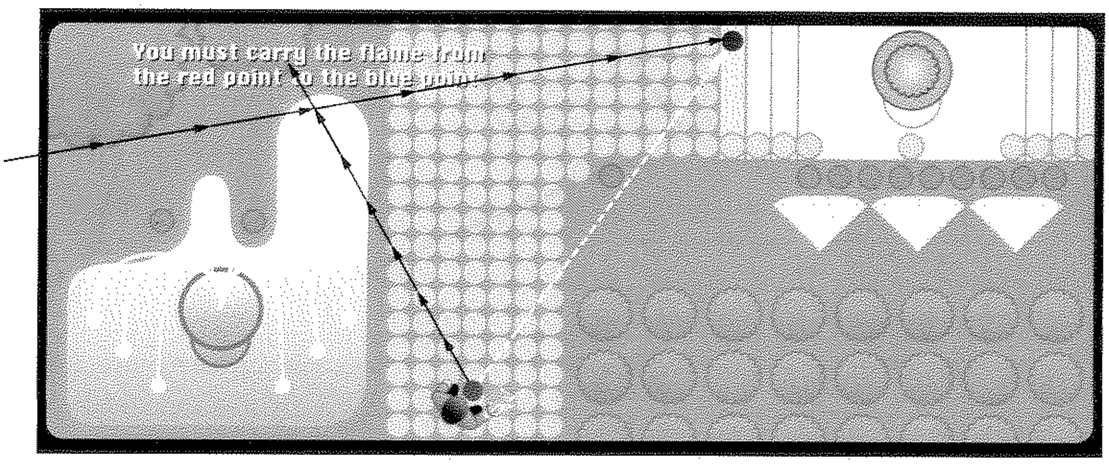
- By placing the vector associated with the key end-to-end several times and by placing the vector associated with the key end-to-end several times, two chains of vectors are obtained.
- By placing these chains of vectors so that the tail of a chain corresponds to the departure point and the head of the other point corresponds to the arrival point, you can determine the number of times you must press on each key.



Karim must press the key 6 times and the key 4 times.

Activity 1

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a. 1) These vectors are collinear.

2) These vectors are collinear.

b. 1) $m\angle ACB = 45° + (180° - 117°) = 108°$

2) $m\angle ABC = 180° - (70° - 45°) - 108° = 47°$

3) $\|\overrightarrow{AC}\| \approx \frac{14.87 \sin 47°}{\sin 108°} \approx 11.43 \text{ cm}$

4) $\|\overrightarrow{CB}\| \approx \frac{14.87 \sin 25°}{\sin 108°} \approx 6.61 \text{ cm}$

c. 1) $\overrightarrow{AC} \approx 4\vec{u}$ **2)** $\overrightarrow{CB} \approx 3\vec{v}$

d. $\overrightarrow{AB} \approx \mathbf{4}\vec{u} + \mathbf{3}\vec{v}$

e. 1) $29 = \mathbf{2}k_1 + \mathbf{3}k_2$
$-4 = \mathbf{3}k_1 + \mathbf{5}k_2$

2) $k_1 = 7$ and $k_2 = -5$.

3) $\overrightarrow{EF} \approx \mathbf{7}\vec{s} + \mathbf{5}\vec{t}$

Activity 2

a. **1)** The force is approximately 281.91 N. **2)** The force is approximately 198.51 N.

b. **1)** The work performed is approximately 1409.54 J. **2)** The work performed is approximately 1985.09 J.

c. $W = \|\vec{f}\| \times \cos\theta \times \|\vec{d}\|$ or $\|\vec{f}\| \times \|\vec{d}\| \times \cos\theta$.

d. **1)** The length of the movement is 6 m. **2)** The length of the movement is approximately 6.1 m. **3)** The length of the movement is approximately 8.49 m. **4)** The length of the movement is 12 m.

Practice 4.3

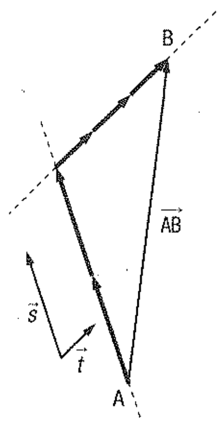
1. a) ≈ 18.53 **b)** ≈ 2.91 **c)** ≈ 2.96 **d)** ≈ -1.98 **e)** ≈ 9.27
f) ≈ -6.07 **g)** 0 **h)** 15 **i)** -5.4

2. a) $\vec{w} = 2\vec{u} + 3\vec{v}$ **b)** $\vec{s} = -1\vec{u} + 2\vec{v}$ **c)** $\vec{t} = \frac{2}{3}\vec{u} - \frac{65}{3}\vec{v}$ **d)** $\vec{r} = -\frac{3}{5}\vec{u} + \frac{59}{5}\vec{v}$
e) $\vec{p} = -1\vec{u} + 3\vec{v}$ **f)** $\vec{q} = \frac{2}{3}\vec{u} - \frac{14}{3}\vec{v}$ **g)** $\vec{m} = \frac{107}{150}\vec{u} + \frac{19}{150}\vec{v}$ **h)** $\vec{n} = 0\vec{u} + 0\vec{v}$

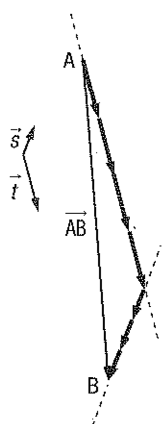
3. a) 3 **b)** 41 **c)** 1.7 **d)** 313 **e)** -4.5 **f)** 100

Practice 4.3 *(cont'd)*

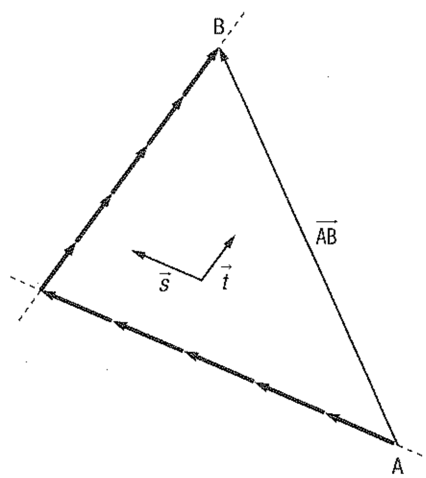
4. a)



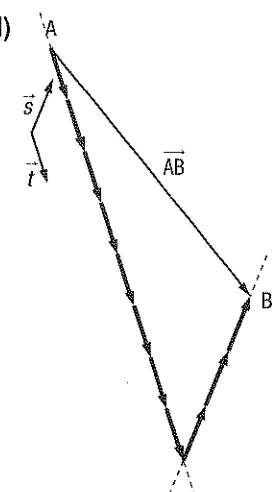
b)



c)



d)



5. a) (3, 23) **b)** 17 **c)** 153 **d)** (6, 8)
e) -34 **f)** (-40, -9) **g)** (15, 1) **h)** 119

6. Solve the following system:

$k_1 a + k_2 c = 0$
$k_1 b + k_2 d = 0$

By isolating k_1 in each equation, the system can be solved by using the comparison method. You obtain:

$-k_2\frac{c}{a} = -k_2\frac{d}{b}$

$k_2\left(\frac{d}{b} - \frac{c}{a}\right) = 0$

It is determined that there are two possible cases.

- You have $k_2 = 0$, which implies that $k_1 = 0$ and the linear combination that obtains $\vec{0}$ and $\vec{0} = 0\vec{u} + 0\vec{v}$.
- You have $\frac{d}{b} = \frac{c}{a}$, which implies that vectors $\vec{u}$ and $\vec{v}$ are collinear $\left(\frac{d}{b} = \frac{c}{a}\right.$ equals $\frac{a}{b} = \frac{c}{d}$, which indicates that the vectors are supported by the lines with the same slope), and there are an infinite number of linear combinations that can obtain $\vec{0}$. Only coefficients k_1 and $k_2 = 0$ are such that $k_2 = \pm k_1\frac{\|\vec{u}\|}{\|\vec{v}\|}$, such that the vectors are either headed in the same or opposite directions.

7. **a)** 1) 0 2) ≈ −7.88 3) ≈ −7.46 4) 0 5) ≈ −3.94 6) 0
   b) The scalar product of two vectors that form an obtuse angle is negative.
   c) E and H.
   d) 1) *Several answers possible. Example:* (6, −4) 2) *Several answers possible. Example:* (2, 5)
   3) *Several answers possible. Example:* (8, −30)

8. **a)** 1) *Several answers possible. Example:* $\vec{u} = (2, 4)$ 2) *Several answers possible. Example:* $\vec{v} = (2, 7)$
   b) 1) *Several answers possible, based on the vectors mentioned in* **a)**. *Example:*
   $\|\vec{u}\| \approx 4.47$ and $\|\vec{v}\| \approx 7.28$.
   2) *Several answers possible, based on the vectors mentioned in* **a)**. *Example:*
   $\vec{u} \cdot \vec{v} = 2 \times 2 + 4 \times 7 = 32$
   c) Since $\vec{u} \cdot \vec{v} = 32$ and $\vec{u} \cdot \vec{v} = \|\vec{u}\| \times \|\vec{v}\| \times \cos\theta$, you have:
   $$\theta = \arccos\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \times \|\vec{v}\|} \approx \arccos\frac{32}{(4.47)(7.28)} \approx 10.47°$$
   The measure of the acute angle formed by lines l_1 and l_2 is approximately 10.47°.

9. **a)**
$$\vec{u} \cdot \vec{v} = (a, b) \cdot (c, d)$$
$$\vec{u} \cdot \vec{v} = ac + bd$$
$$\vec{u} \cdot \vec{v} = ca + db$$
$$\vec{u} \cdot \vec{v} = (c, d) \cdot (a, b)$$
$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

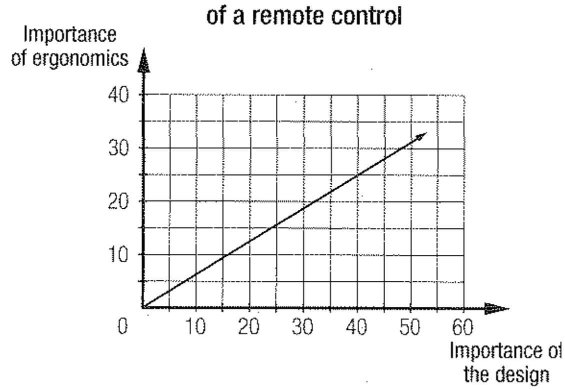
b)
$$k_1\vec{u} \cdot k_2\vec{v} = k_1(a, b) \cdot k_2(c, d)$$
$$k_1\vec{u} \cdot k_2\vec{v} = (k_1a, k_1b) \cdot (k_2c, k_2d)$$
$$k_1\vec{u} \cdot k_2\vec{v} = k_1ak_2c + k_1bk_2d$$
$$k_1\vec{u} \cdot k_2\vec{v} = k_1k_2ac + k_1k_2bd$$
$$k_1\vec{u} \cdot k_2\vec{v} = k_1k_2(ac + bd)$$
$$k_1\vec{u} \cdot k_2\vec{v} = k_1k_2((a, b) \cdot (c, d))$$
$$k_1\vec{u} \cdot k_2\vec{v} = k_1k_2(\vec{u} \cdot \vec{v})$$

c)
$$\vec{u} \cdot (\vec{v} + \vec{w}) = (a, b) \cdot ((c, d) + (e, f))$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = (a, b) \cdot ((c + e, d + f))$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = a(c + e) + b(d + f)$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = ac + ae + bd + bf$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = ac + bd + ae + bf$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = (a, b) \cdot (c, d) + (a, b) \cdot (e, f)$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

d)
$$\vec{u} \cdot \vec{u} = (a, b) \cdot (a, b)$$
$$\vec{u} \cdot \vec{u} = a^2 + b^2$$
$$\vec{u} \cdot \vec{u} = (\sqrt{a^2 + b^2})^2$$
$$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$$

10. **a)** 2(1, 3) + 3(4, 2) + 4(4, 1) + 4(5, 3) + 1(3, 5)
    b) The resultant vector is (53, 33).

Design and ergonomics of a remote control

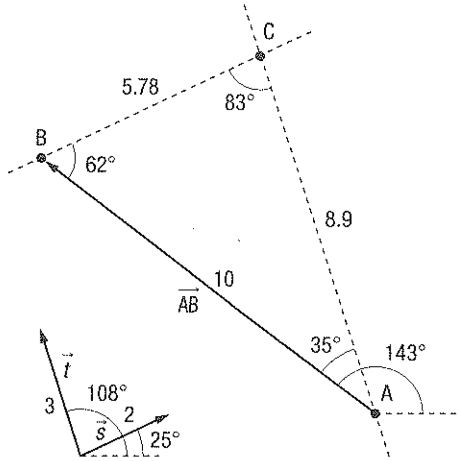


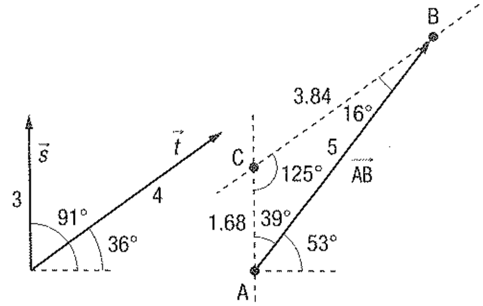
   c) The design influences the consumers more because the vector has a horizontal component greater than its vertical component.
   d) The vector would have a direction of 45°.

Practice 4.3 (*cont'd*)

11. a) $\overrightarrow{AB} \approx 2.97\vec{t} - 2.89\vec{s}$

b) $\overrightarrow{AB} \approx 0.56\vec{s} + 0.96\vec{t}$





12. a) If each worker found at point A generates a force $\vec{f_1}$ and each worker found at point B generates a force $\vec{f_2}$, you have $\vec{f_1} \approx (-153.21, 128.56)$ and $\vec{f_2} \approx (58.61, 138.08)$. The resultant force is therefore $\vec{f_r} = 5\vec{f_1} + 2\vec{f_2} \approx (-648.83, 918.84)$.

The following is determined:

- $\|\vec{f_r}\| \approx 1124.91$ N
- The direction of $\vec{f_r}$ is based on an angle measuring approximately $180° - \arctan\frac{919.2}{649.3}$, which is approximately equal to 125°.

b) You have $\vec{f_r} \approx (-809.15, 2002.72)$. You must therefore determine which linear combination of $\vec{f_1}$ and $\vec{f_2}$ can generate $\vec{f_r}$, in other words, solve the following system where k_1 and k_2 correspond to the sought-after number of workers:

$-153.21k_1 + 58.61k_2 = -809.15$
$128.56k_1 + 138.08k_2 = 2002.72$

You obtain $k_1 \approx 8$ and $k_2 \approx 7$.

Eight workers must pull the cord at point A and seven workers must pull the cord at point B.

Practice 4.3 (*cont'd*)

13. a) 1) ≈ 1223.52 J   2) ≈ 281.58 J   3) ≈ 1151.88 J   4) ≈ 2656.98 J

b) If the three movements correspond respectively to vectors $\vec{d_1}$, $\vec{d_2}$ and $\vec{d_3}$, the sum of the work W_{total} can be expressed as follows:

STATEMENT	JUSTIFICATION
$W_{total} = W_1 + W_2 + W_3$	
$W_{total} = \vec{f} \cdot \vec{d_1} + \vec{f} \cdot \vec{d_2} + \vec{f} \cdot \vec{d_3}$	
$W_{total} = \vec{f} \cdot (\vec{d_1} + \vec{d_2} + \vec{d_3})$	The scalar product is distributive over a vector quantity.
$W_{total} = \vec{f} \cdot \overrightarrow{AB}$	Based on the Chasles relation. Since $\vec{d_1}$, $\vec{d_2}$ and $\vec{d_3}$ are placed end-to-end point A corresponds to the origin of $\vec{d_1}$ and point B corresponds to the extremity of $\vec{d_3}$.

14. If $\vec{u} = (a, b)$ and $\vec{v} = (c, d)$, the slope of the line that supports $\vec{u}$ is $m_1 = \frac{b}{a}$ and the slope of the line that supports $\vec{v}$ is $m_2 = \frac{d}{c}$. Moreover, $\vec{u}$ and $\vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$. Therefore:

$\vec{u} \cdot \vec{v} = 0$
$(a, b) \cdot (c, d) = 0$
$ac + bd = 0$
$ac = -bd$

By dividing each side of the equality by bc, you obtain:

$\frac{a}{b} = \frac{-d}{c} \Rightarrow \frac{a}{b} = -\frac{1}{\frac{c}{d}} \Rightarrow m_1 = -\frac{1}{m_2}$

15. a) *Several answers possible. Example:*

The vector (-5, 3) is orthogonal to $\vec{v} = (3, 5)$. The following system must therefore be solved.

$3k_1 - 2k_2 = -5$
$5k_1 + 3k_2 = 3$

You obtain $k_1 = -\frac{9}{19}$ and $k_2 = \frac{34}{19}$, and the linear combination is $-\frac{9}{19}\vec{v} + \frac{34}{19}\vec{w}$.

b) *Several answers possible. Example:*

The vector (3, 2) is orthogonal to $\vec{w} = (-2, 3)$. The following system must therefore be solved.

$3k_1 - 2k_2 = 3$
$5k_1 + 3k_2 = 2$

You obtain $k_1 = \frac{13}{19}$ and $k_2 = -\frac{9}{19}$, the linear combination is $\frac{13}{19}\vec{v} - \frac{9}{19}\vec{w}$.

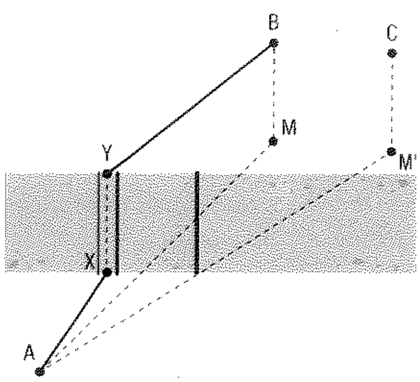
SPECIAL FEATURES 4

Chronicle of the past

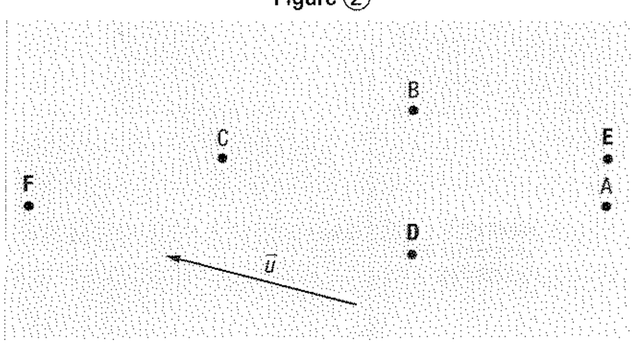
Page 55

1. (A, X) + (X, Y) + (Y, B) = (A, X) + (X, **M**) + (M, **B**)
(A, X) + (X, Y) + (Y, B) = (A, X) + (X, M) + (X, Y)
(A, X) + (Y, B) = (A, X) + (X, M) = (A, M)
It is determined that (Y, B) = **(X, M)**.
Since the straight line consists of the shortest trajectory between points A and M, the trajectory (A, X) + (X, M) is minimal if points A, X and M are aligned. Yet, this trajectory is the same length as (A, X) + (**Y, B**) because (X, M) = (Y, B).

2. Figure ①



3. Figure ②



In the workplace

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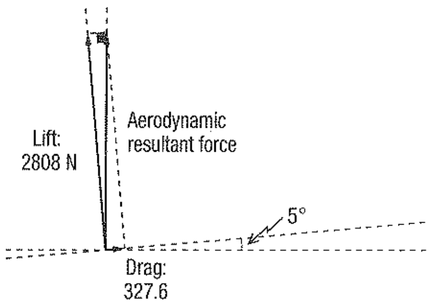
1. **a)** The glider's drag is $C_t \times r \times S \times \frac{\|\vec{v}\|^2}{2} = 0.07 \times 1.3 \times 8 \times \frac{30^2}{2} = 327.6$ N.

b) The glider's lift is $C_p \times r \times S \times \frac{\|\vec{v}\|^2}{2} = 0.6 \times 1.3 \times 8 \times \frac{30^2}{2} = 2808$ N.

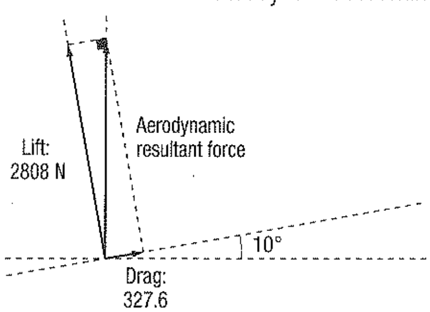
2. a) The norm of the aerodynamic resultant force is $\sqrt{327.6^2 + 2808^2}$, which is approximately 2827.05 N.

The direction of the aerodynamic resultant force is $95° - \arctan\frac{327.6}{2808}$, which is approximately 88.35°.

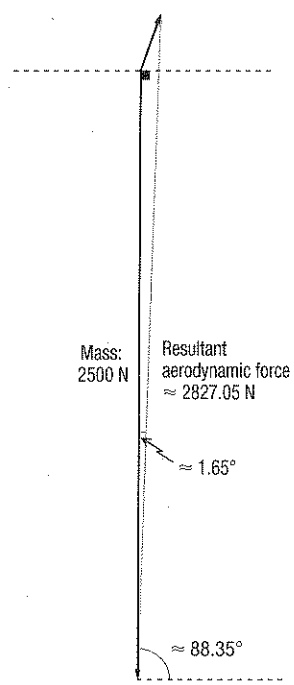


b) The norm of the aerodynamic resultant force is $\sqrt{327.6^2 + 2808^2}$, which is approximately 2827.05 N.

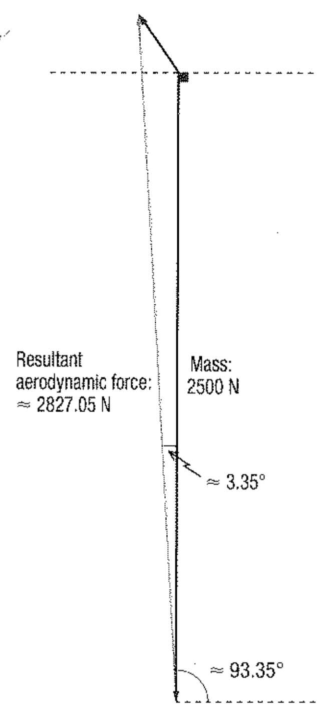
The direction of the aerodynamic resultant force is $100° - \arctan\frac{327.6}{2808}$, which is approximately 93.35°.



3. a) The norm of the vector associated with the sum of the mass and the resultant force is $\sqrt{2827.105^2 + 2500^2 - 2(2827.05)(2500)\cos 1.65°}$, which is approximately equal to 336.5 N.
The direction of this vector corresponds to an angle of $90° - \arcsin\frac{2827.05 \sin 1.65°}{335.93}$, which is approximately equal to 75.94°.



b) The norm of the vector associated with the sum of the mass and the resultant force is $\sqrt{2827.105^2 + 2500^2 - 2(2827.05)(2500)\cos 3.35°}$, which is approximately 362.04 N.
The direction of this vector corresponds to an angle of $90° + \arcsin\frac{2827.05 \sin 3.35°}{362.01}$, which is approximately 117.11°.



Overview

1. a) $\vec{v} \approx (-13.3, -8.9)$ b) $\vec{v} \approx (-83.5, 217.5)$ c) $\vec{v} \approx (0.5, -0.4)$
   d) $\vec{v} \approx (-0.7, -1.9)$ e) $\vec{v} \approx (46.8, 14.3)$ f) $\vec{v} \approx (559.2, 829)$

2. a) $\|\vec{v}\| \approx 18$; direction: $\approx 56.3°$. b) $\|\vec{w}\| \approx 9.5$; direction: $\approx 251.6°$.
   c) $\|\vec{u}\| \approx 17$; direction: $= 45°$. d) $\|\vec{t}\| \approx 107.7$; direction: $\approx 164.4°$.
   e) $\|\vec{m}\| \approx 1.4$; direction: $\approx 135°$. f) $\|\vec{n}\| \approx 0.6$; direction: $\approx 303.7°$.

3. a) $\overrightarrow{AB}$ b) $\overrightarrow{AC}$ c) $\overrightarrow{AB}$
   d) $\overrightarrow{AA}$ or $\vec{0}$. e) $\overrightarrow{CC}$ or $\overrightarrow{CO}$. f) $\overrightarrow{AD}$

4. a) $\vec{v} = \frac{54}{139}\vec{u}$ b) $\vec{v} = \frac{331}{184}\vec{u}$ c) $\vec{v} = -\frac{12}{7}\vec{u}$
   d) $\vec{v} = -10\vec{u}$ e) $\vec{v} = 0.75\vec{u}$ f) $\vec{v} = \frac{1}{m+n}\vec{u}$

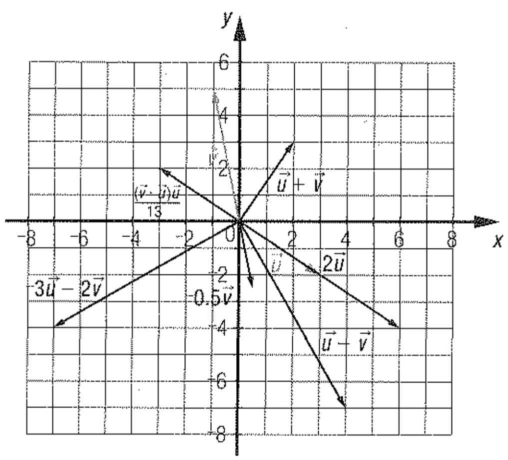
Overview (*cont'd*)

5. a) ≈ 13.4 b) ≈ 710.9 c) ≈ 16.67 d) ≈ 212.78

6. a) $\|\vec{u} + \vec{v}\| \approx 5.4$; direction: $\approx 52.2°$. b) $\|\vec{w} - \vec{z}\| \approx 8.6$; direction: $\approx 254.5°$.
   c) $\|\vec{g} - \vec{h}\| \approx 15.2$; direction: $\approx 66.8°$. d) $\|\vec{i} + \vec{k} - \vec{j}\| \approx 21.4$; direction: $\approx 127.4°$.

Overview (*cont'd*)

7.



8. a) ≈ -15.4 b) ≈ 3.6 c) ≈ 27.9 d) -109 e) -23.76 f) $-13\,858$

9. a) $(\approx -0.82, \approx 0.57)$ or $(\approx 0.82, \approx -0.57)$. b) $(\approx 3.44, \approx -4.91)$ c) $(\approx 2.4, \approx -2.9)$ or $(\approx 3.7, \approx -0.6)$.

Overview (*cont'd*)

10. a) A, E and H. b) C, D and I. c) A and H. d) F

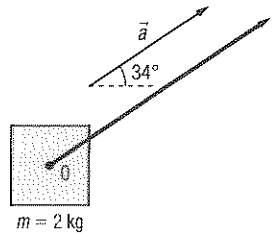
11. a) $\overrightarrow{AB} \approx 2.6\vec{s} + 17.8\vec{t}$ b) $\overrightarrow{AB} \approx 2.5\vec{s} + 1.4\vec{t}$ c) $\overrightarrow{AB} = \frac{35}{12}\vec{s} + 7.55\vec{t}$ d) $\overrightarrow{AB} = -2\vec{s} + \frac{1}{3}\vec{t}$

12. a) $\approx 63°$ b) $90°$ c) $\approx 64°$
    d) $0°$ e) $\approx 26°$ f) $\approx 79°$

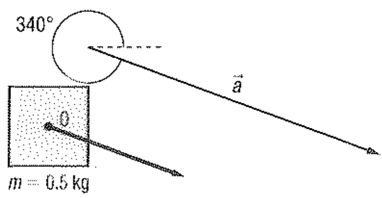
Overview (*cont'd*)

13. a) $(7, -3)$ or $(-7, 3)$. b) $\vec{u} = (3, 7)$ c) $\vec{u} = \left(-1, -\frac{14}{6}\right)$

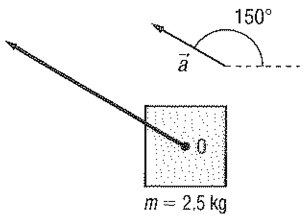
14. a)



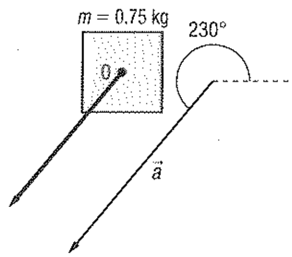
b)



c)



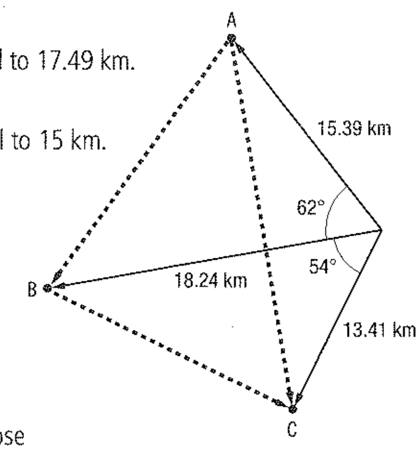
d)



15. a) The resultant vector that is associated with the outward trip is (16, 8). The return trip can therefore be represented by the vector (-16, -8). Its direction would be approximately 180° + arctan 0.5, which is approximately equal to 207°.

b) The distance corresponds to $\sqrt{16^2 + 8^2}$ and is approximately 17.9 km.

Overview *(cont'd)* **Page 63**



16. a) 1) The distance that separates Fragment **A** from Fragment **B** is $\sqrt{15.39^2 + 18.24^2 - 2(15.39)(18.24)\cos 62°}$, which is approximately equal to 17.49 km.

2) The distance that separates Fragment **B** from Fragment **C** is $\sqrt{13.41^2 + 18.24^2 - 2(13.41)(18.24)\cos 54°}$, which is approximately equal to 15 km.

3) The distance that separates Fragment **A** from Fragment **C** is $\sqrt{13.41^2 + 15.39^2 - 2(13.41)(15.39)\cos 116°}$, which is approximately equal to 24.45 km.

b) 3 s after the explosion, the quantity of the movement:

- of Fragment **A** is represented by $\vec{p_A} = m_A\vec{v_A}$, in a direction of 128° and whose norm is 0.5 kg × 5.13 km/s, which is 2.565 kg km/s. Therefore $\vec{p_A} = (\approx -1.58, \approx 2.02)$,
- of Fragment **B** is represented by $\vec{p_B} = m_B\vec{v_B}$, in a direction of 190° and whose norm is 0.3 kg × 6.08 km/s, which is 1.824 kg km/s. Therefore $\vec{p_B} = (\approx -1.8, \approx -0.32)$,
- of Fragment **C** is represented by $\vec{p_C} = m_C\vec{v_C}$, in a direction of 244° and whose norm is 0.4 kg × 4.47 km/s, which is 1.788 kg km/s. Therefore $\vec{p_C} = (\approx -0.78, \approx -1.61)$.

It is determined that the quantity of movement of the asteroid is:

$\vec{p}_{asteroid} = m_{asteroid}\vec{v}_{asteroid} = \vec{p_A} + \vec{p_B} + \vec{p_C}$

$1.2 \text{ kg} \times \vec{v}_{asteroid} \approx (-1.58, 2.02) + (-1.8, -0.32) + (-0.78, -1.61) \approx (-4.16, -0.9)$

$\vec{v}_{asteroid} \approx \frac{1}{1.2 \text{ kg}}(-4.16, -0.9)$

$\vec{v}_{asteroid} \approx (-3.47, -0.75)$

$\|\vec{v}_{asteroid}\| \approx \sqrt{3.47^2 + 0.75^2} \approx 3.55$ km/s and $\arctan\frac{0.75}{3.47} \approx 12.2°$.

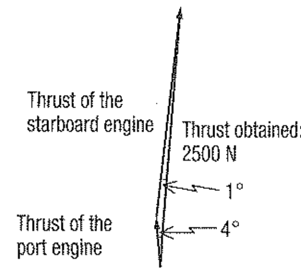
The speed of the asteroid before the explosion was approximately 3.55 km/s, in a direction of 192.2°.

17. a) By using the sine law, you discover the following:

- The thrust of the port engine must be approximately 550.61 N.
- The thrust of the starboard engine must be approximately 2000.91 N.

The following is determined:

- The port engine turns at a speed of $1000 \times \frac{550.61}{200}$ turns/min, meaning approximately 2503.05 turns/min counter-clockwise (due to the direction of the force of the engine).
- The starboard engine turns at a speed of $1000 \times \frac{2000.91}{300}$ turns/min, meaning approximately 6669.71 turns/min clockwise.

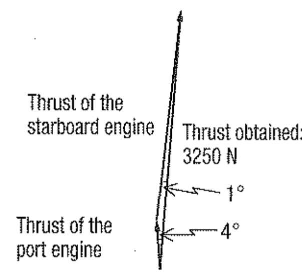


b) By using the sine law, you discover the following:

- The thrust of the port engine must be approximately 650.79 N.
- The thrust of the starboard engine must be approximately 2601.19 N.

The following is determined:

- The port engine turns at a speed of $1000 \times \frac{650.79}{200}$ turns/min, meaning approximately 3253.96 turns/min clockwise.
- The starboard engine turns at a speed of $1000 \times \frac{2601.19}{300}$ turns/min, meaning approximately 8670.63 turns/min counter-clockwise.



Overview (*cont'd*)

Page 64

18. a)

STATEMENT	JUSTIFICATION
$\overrightarrow{AB} = \overrightarrow{CB} - \overrightarrow{CA}$	
$\overrightarrow{AB} = \overrightarrow{CB} + \overrightarrow{AC}$	By inversing the tail and the head of a vector, you obtain a vector that is opposite to it.
$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$	The addition of vectors is commutative.
$\overrightarrow{AB} = \overrightarrow{AB}$	Based on the Chasles relation.

b) **1)** The scalar products of two identical pairs of vectors are equal.

2) The scalar product of vectors is distributive. Therefore:

$$\begin{aligned}(\overrightarrow{CB} - \overrightarrow{CA}) \cdot (\overrightarrow{CB} - \overrightarrow{CA}) &= \overrightarrow{CB} \cdot (\overrightarrow{CB} - \overrightarrow{CA}) - \overrightarrow{CA} \cdot (\overrightarrow{CB} - \overrightarrow{CA}) \\ &= \overrightarrow{CB} \cdot \overrightarrow{CB} - \overrightarrow{CB} \cdot \overrightarrow{CA} - \overrightarrow{CA} \cdot \overrightarrow{CB} + \overrightarrow{CA} \cdot \overrightarrow{CA} \\ &= \overrightarrow{CB} \cdot \overrightarrow{CB} + \overrightarrow{CA} \cdot \overrightarrow{CA} - 2\overrightarrow{CB} \cdot \overrightarrow{CA}\end{aligned}$$

c) $\|\overrightarrow{AB}\|^2 = \|\overrightarrow{CB}\|^2 + \|\overrightarrow{CA}\|^2 - 2\|\overrightarrow{CB}\| \times \|\overrightarrow{CA}\| \cos\theta$

19. Solve the following system:

$k_1a + k_2c = ka$
$k_1b + k_2d = kc$

By isolating k_1 in each equation and by using the comparison method, it is established that $\vec{z}$ and $\vec{v}$ are collinear if $\frac{ka - k_2c}{a} = \frac{kc - k_2d}{b}$ or $\frac{ka - k_2c}{kc - k_2d} = \frac{a}{b}$.

According to one of the properties of the proportions, this equality is true only if $\frac{k_2c}{k_2d} = \frac{a}{b}$, in other words if $\frac{c}{d} = \frac{a}{b}$.

Yet, since $\vec{v}$ and $\vec{w}$ are not collinear, this proportion is not confirmed. In order for proportion $\frac{ka - k_2c}{kc - k_2d} = \frac{a}{b}$ to be true, $k_2 = 0$.

20. Since the variation rate between points B and C must be the same as the one between points A and B, $\frac{y - 4}{x - 3} = 0.5$ and y must therefore be 2.5 units more than half of x ($y = 0.5x + 2.5$)

21. a) The angle between vectors (1, 2) and (2, 1) measures $\arctan\left(\frac{1 \times 2 + 2 \times 1}{2\sqrt{5}}\right)$, in other words $\approx 42°$.

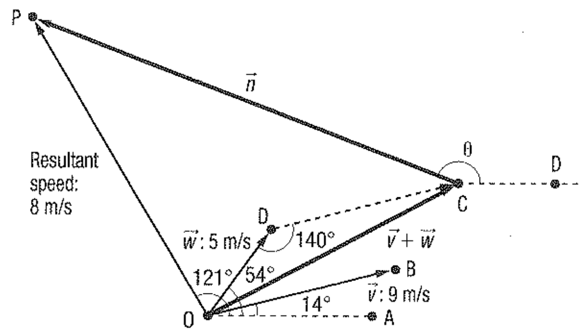
b) The scalar product of these vectors is $4a^2$, and each of these vectors has a norm of $\sqrt{5}a$. Therefore:

$$4a^2 = (\sqrt{5}a)(\sqrt{5}a)\cos\theta$$
$$4a^2 = (5a^2)\cos\theta$$
$$4 = 5\cos\theta$$
$$\cos\theta = \frac{4}{5}$$

22. a) 1) $2 + 2i = (\sqrt{8}, 45°)$
2) $1 + 3i \approx (\sqrt{10}, 71.6°)$
3) $6 - 5i \approx (\sqrt{61}, 320.2°)$
4) $7 + 0i = (7, 0°)$

b) 1) $(4, 35°) \approx 3.28 + 2.29i$
2) $(7, 123°) \approx -3.81 + 5.87i$
3) $(\sqrt{2}, 225°) = -1 - i$
4) $(12, 150°) \approx -10.4 + 6i$

1. In order for the ship to arrive at the port in 25 min, the norm of the resultant speed must be $\frac{\|\overrightarrow{OP}\|}{1500\text{ s}} = \frac{12\,000\text{ m}}{1500\text{ s}}$, which is 8 m/s. Its direction must be 121°. Since $\vec{n}$ is the speed that must be added to $\vec{v}$ and $\vec{w}$ to obtain the desired resultant speed, it is possible to graphically represent this situation as follows:



It is deduced that:

$\|\vec{v} + \vec{w}\| = \sqrt{9^2 + 5^2 - 2(5)(9)\cos 140°} \approx 13.2$ m/s	Based on the cosine law.
$m\angle COD \approx \arcsin\frac{9\sin 140°}{13.2} \approx 26°$	Based on the sine law.
$m\angle AOC \approx 54° - 26° \approx 28°$	
$m\angle COP \approx 121° - 26° \approx 95°$	
$\|\vec{n}\| = \sqrt{13.2^2 + 8^2 - 2(13.2)(8)\cos 95°} \approx 16$ m/s	Based on the cosine law.
$m\angle OCP \approx \arcsin\frac{8\sin 95°}{16} \approx 30°$	Based on the sine law.
$\theta = 360° - m\angle OCP - (180° - m\angle AOC)$	Angles AOC and OCF are supplementary.
$\theta \approx 360° - 30° - (180° - 28°) \approx 178°$	

The pilot must apply a speed of approximately 16 m/s to the ship, in a direction of approximately 178°.

2. Since the sought-after vector (c, d) is a unit vector and orthogonal to (a, b), the two following equations can be proposed:

① $(a, b) \cdot (c, d) = 0$. ② $\sqrt{c^2 + d^2} = 1$, or $c^2 + d^2 = 1$.

By isolating d in Equation ①, you obtain $d = -\frac{ac}{b}$.

After having substituted this expression by d in Equation ②, the following process can be made.

$$c^2 + \frac{a^2c^2}{b^2} = 1$$
$$c^2\left(\frac{a^2}{b^2} + 1\right) = 1 \Rightarrow c^2\left(\frac{a^2 + b^2}{b^2}\right) = 1$$
$$c = \pm\frac{b}{\sqrt{a^2 + b^2}}$$

If $c = \frac{b}{\sqrt{a^2 + b^2}}$, then $d = -\frac{ab}{b\sqrt{a^2 + b^2}} = \frac{-a}{\sqrt{a^2 + b^2}}$

If $c = \frac{-b}{\sqrt{a^2 + b^2}}$, then $d = -\frac{-ab}{b\sqrt{a^2 + b^2}} = \frac{a}{\sqrt{a^2 + b^2}}$

The vectors obtained are therefore $\left(\frac{-b}{\sqrt{a^2 + b^2}}, \frac{a}{\sqrt{a^2 + b^2}}\right)$ and $\left(\frac{b}{\sqrt{a^2 + b^2}}, \frac{-a}{\sqrt{a^2 + b^2}}\right)$.

Bank of problems (*cont'd*)

3. By following the first series of instructions by the Internet user ~Einstein~, you can state:

$$\vec{z} = k_1(a, b) + k_2(c, d) = (k_1a + k_2c, k_1b + k_2d)$$
$$\vec{z} \cdot \vec{w} = (k_1a + k_2c, k_1b + k_2d) \cdot (c, d)$$
$$\vec{z} \cdot \vec{w} = (k_1a + k_2c)c + (k_1b + k_2d)d$$
$$\vec{z} \cdot \vec{w} = k_1ac + k_2c^2 + k_1bd + k_2d^2$$

By following the second series of instructions by the Internet user ~Einstein~, you can state:

$$k_1ac + k_2c^2 + k_1bd + k_2d^2 = 0$$
$$k_1ac + k_1bd + k_2c^2 + k_2d^2 = 0$$
$$k_1(ac + bd) + k_2(c^2 + d^2) = 0$$

By following the last series of the instructions by the Internet user ~Einstein~, you can state:

$$\frac{k_1(ac + bd) + k_2(c^2 + d^2)}{k_1} = \frac{0}{k_1}$$
$$(ac + bd) + \frac{k_2}{k_1}(c^2 + d^2) = 0$$
$$\frac{k_2}{k_1}(c^2 + d^2) = -(ac + bd)$$
$$\frac{k_2}{k_1} = \frac{-(ac + bd)}{c^2 + d^2}$$

Yet, $\vec{v} \cdot \vec{w} = (a, b) \cdot (c, d) = ac + bd$ and $\|\vec{w}\|^2 = (\sqrt{c^2 + d^2})^2 = c^2 + d^2$.

It can be concluded that $\frac{k_2}{k_1} = -\frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2}$.

4. If point C divides segment AB in the ratio $\frac{m}{n}$, this indicates that $\frac{\|\overrightarrow{AC}\|}{\|\overrightarrow{CB}\|} = \frac{m}{n}$. You must therefore show that this proportion is true.

STATEMENT	JUSTIFICATION
$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$	Based on the Chasles relation.
$\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$	Based on the Chasles relation.
$\overrightarrow{CB} = \overrightarrow{AB} - \frac{m}{m+n}\overrightarrow{AB}$	Since $\overrightarrow{AC} = \frac{m}{m+n}\overrightarrow{AB}$.
$\overrightarrow{CB} = \overrightarrow{AB}\left(1 - \frac{m}{m+n}\right)$	Since the multiplication of a vector by a scalar is distributive over the addition of scalars.
$\overrightarrow{CB} = \overrightarrow{AB}\left(\frac{m+n}{m+n} - \frac{m}{m+n}\right) = \overrightarrow{AB}\left(\frac{m+n-m}{m+n}\right) = \frac{n}{m+n}\overrightarrow{AB}$	
$\frac{\|\overrightarrow{AC}\|}{\|\overrightarrow{CB}\|} = \frac{\frac{m}{m+n}\|\overrightarrow{AB}\|}{\frac{n}{m+n}\|\overrightarrow{AB}\|}$	The norm of a resultant vector of the multiplication of a first vector by a scalar is the norm of this vector multiplied by this scalar.
$\frac{\|\overrightarrow{AC}\|}{\|\overrightarrow{CB}\|} = \frac{\frac{m}{m+n}}{\frac{n}{m+n}} = \left(\frac{m}{m+n}\right)\left(\frac{m+n}{n}\right) = \frac{m}{n}$	

Conclusion

Since $\frac{\|\overrightarrow{AC}\|}{\|\overrightarrow{CB}\|} = \frac{m}{n}$, then point C shares segment AB in the ratio $\frac{m}{n}$.

5. The information on the drawing shows the following:

- The area of ADEF is given by m m $\overline{AF} \times$ m $\overline{AD}$.
- m $\overline{AD}$ = m $\overline{AC}$ = $\|\overrightarrow{AC}\|$.
- m $\overline{AF}$ corresponds to the norm of the orthogonal projection of $\overrightarrow{AB}$ on the line that supports $\overrightarrow{AC}$.

Calculating the area of ADEF therefore comes from multiplying the norm $\overrightarrow{AC}$ by the norm of the projection $\overrightarrow{AB}$ on the line that supports $\overrightarrow{AC}$. This is the definition of the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

The information on the drawing shows the following:

- The area of AGHI is given by m $\overline{AG} \times$ m $\overline{AI}$.
- m $\overline{AI}$ = m $\overline{AB}$ = $\|\overrightarrow{AB}\|$.
- m $\overline{AG}$ corresponds to the norm of the orthogonal projection $\overrightarrow{AC}$ on the line that supports $\overrightarrow{AB}$.

Calculating the area of AGHI therefore comes from multiplying the norm $\overrightarrow{AB}$ by the norm of the projection $\overrightarrow{AC}$ on the line that supports $\overrightarrow{AB}$. This is the definition of the scalar product $\overrightarrow{AC} \cdot \overrightarrow{AB}$.

Since the scalar product is commutative, you have $\overrightarrow{AB} \cdot \overrightarrow{AC} = \overrightarrow{AC} \cdot \overrightarrow{AB}$. It is deduced that the quadrilaterals ADEF and AGHI are equivalent.

Bank of problems (*cont'd*) Page 68

6.
- The components of the vector associated with Joanne's second movement are ($\approx$ 4.33, -2.5).
- The resultant vector of the set of Joanne's movements is $\vec{u}$ = (-3, -2) + ($\approx$ 4.33, -2.5) = ($\approx$ 1.33, -4.5).
- In relation to Joanne's initial position, Alfred's initial position is obtained following a movement whose components are ($\approx$ -0.35, $\approx$ 0.35).
- The resultant vector of Alfred's set of movements corresponds to $-3\vec{u}$: $-3\vec{u}$ = -3($\approx$ 1.33, -4.5) = ($\approx$ -3.99, 13.5).
- In order to join Alfred, Joanne must make a resultant movement of the inverse of her movements added to Alfred's movements and to the movement that separated them at the beginning. The resultant vector of this trajectory is therefore:

  $-\vec{u} - 3\vec{u}$ + ($\approx$ -0.35, $\approx$ 0.35) = -($\approx$ 1.33, -4.5) + ($\approx$ -3.99, 13.5) + ($\approx$ -0.35, $\approx$ 0.35)
  = ($\approx$ -5.32, 18.35)
- The norm of this vector is approximately $\sqrt{(-5.32)^2 + (18.35)^2}$, which is approximately equal to 19.11 km and the direction of this vector is approximately $180° - \arctan\frac{18.35}{5.32}$, which is approximately equal to 106.17°.

7.
- The components of $a+b$ correspond to the components of the projection of $\vec{v}$ in the xy plan.
  This norm equals $\|\vec{v}\| \cos 30° = 13 \cos 30° \approx 11.3$.
  Therefore you have $a \approx 11.3 \cos 50° \approx 7.3$, $b \approx 11.3 \sin 50° \approx 8.7$ and $c = \|\vec{v}\| \sin 30° = 13 \sin 30° = 6.5$.
  It can be concluded that $\vec{v} \approx (7.3, 8.7, 6.5)$.
- The components d and e correspond to the components of the projection of $\vec{w}$ in the xy plan.
  This norm equals $\|\vec{w}\| \cos 25° = 13 \cos 30° \approx 10.9$.
  Since the projection of $\vec{w}$ in the xy plan is located in the 4th quadrant, you have:
  $d \approx -10.9 \sin 20° \approx -3.7$, $e \approx -10.9 \cos 20° \approx -10.2$ and $f = \|\vec{w}\| \sin 25° = 12 \sin 25° \approx 5.1$.
  It can be concluded that $\vec{w} \approx (-3.7, -10.2, 10.9)$.
- Therefore:
  $\vec{v} \cdot \vec{w} = (a, b, c) \cdot (d, e, f)$
  $\vec{v} \cdot \vec{w} \approx (7.3, 8.7, 6.5) \cdot (-3.7, -10.2, 10.9)$
  $\vec{v} \cdot \vec{w} \approx (7.3)(-3.7) + (8.7)(-10.2) + (6.5)(10.9)$
  $\vec{v} \cdot \vec{w} \approx -44.9$

8. The movement that allows the actress to get from point to point is the following:

- From point A to point B is represented by a vector d_1 in a direction of 36° and whose norm is approximately 11 m.
- From point B to point C is represented by a vector d_2 in a direction of 234° and whose norm is 12 m.

The movement that allows the cameraman to follow the actress is the following:

- From point A to point B is represented by a vector that corresponds to the projection of $\vec{d_1}$ on a line inclined at 23° in relation to the horizontal. The norm of this vector is $d_1 \cos(36° - 23°) = 11 \cos 13° \approx 10.8$ m. Since this movement must be done in 3 s, the cameraman must move at an approximate speed of 3.6 m/s.
- From point C to point D is represented by a vector that corresponds to the projection of $\vec{d_2}$ on a line inclined at 23° in relation to the horizon. The norm of this vector is $d_2 \cos(234° - 180° - 23°) = 12 \cos 31° \approx 10.2$ m. Since this movement must be done in 4 s, the cameraman must move at an approximate speed of 2.55 m/s.

The cameraman must make a first movement of 10.8 m on the rails to the right, at a constant speed of 3.6 m/s for 3 s. He must then stay still for 15 s then move 10.8 m to his left for 4 s at an approximate speed of 2.55 m/s.

9. The Chasles relation states that $\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$ and $\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BC}$. Therefore:

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$$

$$\overrightarrow{PA} + (\overrightarrow{PA} + \overrightarrow{AB}) + (\overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BC}) = \vec{0}$$

$$3\overrightarrow{PA} + 2\overrightarrow{AB} + \overrightarrow{BC} = 0$$

$$\overrightarrow{PA} = \frac{-2\overrightarrow{AB} - \overrightarrow{BC}}{3}$$

$$\overrightarrow{PA} = \frac{-2(6, -8) - (-10, 2)}{3}$$

$$\overrightarrow{PA} = \left(-\frac{2}{3}, \frac{14}{3}\right)$$

If the triangle is placed on the Cartesian plane so that the centre of the gravity is located at the origin, the coordinates of vertices A, B and C can be deduced. You obtain:

A: $\left(-\frac{2}{3}, \frac{14}{3}\right)$, B: $\left(\frac{16}{3}, -\frac{10}{3}\right)$ and C: $\left(-\frac{14}{3}, -\frac{4}{3}\right)$.

Therefore:

- $d(A, P) = \sqrt{\left(\frac{-2}{3}\right)^2 + \left(\frac{14}{3}\right)^2} \approx 4.7$
- $d(B, P) = \sqrt{\left(\frac{16}{3}\right)^2 + \left(\frac{-10}{3}\right)^2} \approx 6.3$
- $d(C, P) = \sqrt{\left(\frac{-14}{3}\right)^2 + \left(\frac{-4}{3}\right)^2} \approx 4.9$