student book volume 2

Science

VISIONS

MATHEMATICS

Secondary Cycle Two, Year Three

ANSWER KEY

Vision 5

LES ÉDITIONS CEC
A Quebecor Media Company

9001 Louis-H.-La Fontaine, Anjou (Québec) Canada H1J 2C5
Telephone: 514 351-6010 • Fax: 514 351-3534

PRELIMINARY VERSION

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LES 9 | The seeding machine

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The following example of a process using an asparagus seeding machine can help the students complete the task.

Seed	Diameter of the circular mechanism (cm)	Rule, where H represents the depth (in cm) in relation to the ground and x represents time (in min)
Asparagus	$\frac{60}{\pi} \approx 19.1$	$H(x) \approx 9.55 \sin \frac{3\pi}{5}x + 7.05$
Broccoli	$\frac{60}{\pi} \approx 19.1$	$H(x) \approx 9.55 \sin \frac{3\pi}{5}x + 8.95$
Pumpkin	$\frac{200}{\pi} \approx 63.66$	$H(x) \approx 31.83 \sin 2\pi x + 29.33$
Cucumber	$\frac{25}{\pi} \approx 7.96$	$H(x) \approx 3.98 \sin \frac{\pi}{4}x + 1.48$
Melon	$\frac{30}{\pi} \approx 9.55$	$H(x) \approx 4.77 \sin \frac{3\pi}{10}x + 2.27$
Leek	$\frac{15}{\pi} \approx 4.77$	$H(x) \approx 2.39 \sin \frac{3\pi}{20}x + 1.39$
Tomato	$\frac{50}{\pi} \approx 15.92$	$H(x) \approx 7.96 \sin \frac{\pi}{2}x + 7.36$

- Calculation of the diameter of the circular mechanism.
  $C = \pi d$
  $C = 60$ cm; therefore $d = \frac{60}{\pi}$ cm
- Finding the rule of the sinusoidal function.
  $A = \frac{60}{\pi}$ cm $\div 2$
  $A = \frac{30}{\pi}$ cm ≈ 9.55 and $A = |a|$, therefore $a \approx 9.55$
  200 cm/min ÷ 60 cm/turn = $\frac{10}{3}$ turn/min
  $P = \frac{2\pi}{|b|}$ therefore $b = \frac{2\pi}{\frac{10}{3}}$
  $b = \frac{3\pi}{5}$
  $h = 0 \quad k = \frac{30}{\pi}$ cm $-$ 2.5 cm
  $H(x) \approx 9.55 \sin \frac{3\pi}{5}x + 7.05$

LES 10 | The adjustable incline

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The following example of a process can help the students complete the requested part of the Student Book.

To calculate the length of the hydraulic cylinder AG, you must calculate the cosine of angle DAF. To calculate the length of the hydraulic cylinder DG, you must calculate the sine of angle DAF.

Since Δ ACD ~ Δ ADG based on the minimum condition of similar triangles AA, and $\angle$ ACD $\cong$ $\angle$ DAG, the length of the steel rod AC is obtained by: $m\,\overline{AC} = \frac{1}{\sin DAG} = \operatorname{cosec} DAG$

The length of the steel rod AE is obtained by: $m\,\overline{AE} = \frac{1}{\cos DAG} = \sec DAG$.

Since Δ ACE is a right triangle whose right angle is located at vertex A, the length of the steel rod CE, is obtained by:

$m\,\overline{CE} = \sqrt{(m\,\overline{AC})^2 + (m\,\overline{AE})^2}$

Angle of inclination of the plane (°)	Angle DAF (°)	Angle DAF (rad)	Hydraulic cylinder AG (m)	Hydraulic cylinder DG (m)	Rod AE (m)	Rod AC (m)	Plan CE (m)
80	10	$\frac{\pi}{18}$	0.9848	0.1736	1.0154	5.7588	5.8476
70	20	$\frac{\pi}{9}$	0.9397	0.3420	1.0642	2.9238	3.1114
60	30	$\frac{\pi}{6}$	0.8660	0.5	1.1547	2	2.3094
50	40	$\frac{2\pi}{9}$	0.7660	0.6428	1.3054	1.5557	2.0309
40	50	$\frac{5\pi}{18}$	0.6428	0.7660	1.5557	1.3054	2.0309
30	60	$\frac{\pi}{3}$	0.5	0.8660	2	1.1547	2.3094
20	70	$\frac{7\pi}{18}$	0.3420	0.9397	2.9238	1.0642	3.1114
10	80	$\frac{4\pi}{9}$	0.1736	0.9848	5.7588	1.0154	5.8476

Prior learning 1

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a. ≈ 145.77 m   b. ≈ 64.31 m   c. ≈ 38.59 m   d. ≈ 107.19 m

Prior learning 2

Page 81

a. 1) 30°   2) 60°   3) 60°   4) 60°

b. 1) 10 m   2) $10\sqrt{3}$ m   3) 10 m   4) 10 m

Knowledge in action

Page 84

1. a) 1) 40 cm
   2) The Pythagorean theorem $(\text{m } \overline{AB})^2 = (\text{m } \overline{AC})^2 + (\text{m } \overline{CB})^2$

   b) 1) 40.5 cm
   2) In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse: $(\text{m } \overline{CD})^2 = \text{m } \overline{AD} \times \text{m } \overline{BD}$

   c) 1) ≈ 1.98 cm
   2) In a right triangle, the length of a leg of a right triangle is the geometric mean of the length of its projection on the hypotenuse and the length of the hypotenuse: $(\text{m } \overline{AB})^2 = \text{m } \overline{AD} \times \text{m } \overline{AC}$

   d) 1) 12 cm
   2) In a right triangle, the length of a leg of a right triangle is the geometric mean of the length of its projection on the hypotenuse and the length of the hypotenuse: $(\text{m } \overline{BC})^2 = \text{m } \overline{CD} \times \text{m } \overline{AC}$

2. a) 24 cm   b) $\frac{120}{13}$ or ≈ 9.23 cm   c) $\frac{50}{13}$ or ≈ 3.85 cm

3. **Length of segments**

	a	*b*	*c*	*m*	*n*	*h*
a)	9	12	**15**	**5.4**	**9.6**	**7.2**
b)	$4\sqrt{5}$	$8\sqrt{5}$	**20**	4	**16**	8
c)	10	**7.5**	**12.5**	**8**	**4.5**	6

Knowledge in action (*cont'd*)

Page 85

4. The perimeter is approximately 24.07 cm.
5. $\frac{3136}{1077}$ cm or approximately 2.91 cm.
6. The costs of this repair are $233.66.
7. ≈ 57.74 cm^2

Knowledge in action (*cont'd*)

Page 86

8. ≈ 2.67 cm
9. ≈ 327.02 cm^2

10. a)

Calculation of lengths	Geometric statement
$(m\overline{AD})^2 + (m\overline{DC})^2 = (m\overline{AC})^2$ $(m\overline{AD})^2 + 5^2 = 8^2$ $(m\overline{AD})^2 + 25 = 64$ $(m\overline{AD})^2 = 39$ $m\overline{AD} = \sqrt{39}$	The Pythagorean theorem.
$(m\overline{AC})^2 = m\overline{AD} \times m\overline{AB}$ $8^2 = \sqrt{39} \times m\overline{AB}$ $\frac{64\sqrt{39}}{39} = m\overline{AB}$	In a right triangle, the length of a leg of a right triangle is the geometric mean of the length of its projection on the hypotenuse and the length of the hypotenuse.
$m\overline{AD} + m\overline{BD} = m\overline{AB}$ $\sqrt{39} + m\overline{BD} = \frac{64\sqrt{39}}{39}$ $m\overline{BD} = \frac{64\sqrt{39}}{39} - \sqrt{39}$ $m\overline{BD} = \frac{25\sqrt{39}}{39}$	The length of a segment is the sum of the lengths of the segments that define this statement.

b)

Calculation of lengths	Geometric statement
$(m\overline{BD})^2 + (m\overline{CD})^2 = (m\overline{BC})^2$ $\left(\frac{25\sqrt{39}}{39}\right)^2 + 5^2 = (m\overline{BC})^2$ $\frac{625}{39} + 25 = (m\overline{BC})^2$ $\frac{1600}{39} = (m\overline{BC})^2$ $\frac{40\sqrt{39}}{39} = m\overline{BC}$	The Pythagorean theorem.

11. ≈ 0.5865 m

Knowledge in action (*cont'd*) Page 87

12. 12.99 cm

13. a) ≈ 315.67 cm^2 b) ≈ 74.95 cm^2 c) ≈ 167.14 cm^2 d) ≈ 65.12 cm^2

14. 23.31 cm

Knowledge in action (*cont'd*) Page 88

15. a) $x \approx 2.54$ cm, $y \approx 6.52$ cm, $z \approx 16.78$ cm
b) $x \approx 3$ cm, $y \approx 5.2$ cm, $z \approx 2.6$ cm
c) $x \approx 6.24$ cm, $y \approx 6.41$ cm, $z \approx 10.25$ cm
d) $x = 5$ cm, $y \approx 6.67$ cm, $z \approx 5.33$ cm
e) $x \approx 9.75$ cm, $y \approx 16.31$ cm, $z \approx 8.37$ cm
f) $x \approx 5.2$ cm, $y \approx 10.39$ cm, $z \approx 20.78$ cm
g) $x = 1$ cm, $y \approx 8.49$ cm, $z \approx 2.83$ cm
h) $x \approx 5.42$ cm, $y \approx 2.08$ cm, $z = 5$ cm
i) $x \approx 8.66$ cm, $y \approx 12.25$ cm, $z \approx 7.07$ cm
j) $x = 4$ cm, $y = 16$ cm, $z = 8\sqrt{5}$ cm

Knowledge in action (*cont'd*) Page 89

16. 3079.49 cm^2

17. a) ≈ 46.95 m b) ≈ 57.77 m c) ≈ 10.82 m d) ≈ 27.73 m

SECTION 5.1 The trigonometric circle

Problem Page 90

$C = 2\pi r$ where $r = 1$ km

$C = 2\pi$ km

$\frac{1 \text{ km}}{2\pi \text{ km}} = \frac{m \angle AOB}{360°}$

$m \angle AOB = \left(\frac{180}{\pi}\right)°$

$B\left(1 \cos\left(\frac{180}{\pi}\right)°, 1 \sin\left(\frac{180}{\pi}\right)°\right)$

The coordinates of the ship at point B are approximately (0.54, 0.84).

Activity 1

Page 91

a. 1) 360° 2) $2\pi r$ units.

b. 1) 2π times 2) $\left(\frac{180}{\pi}\right)°$

c. The length of the arc is equal to the length of the radius.

d. 1) 2π radians. 2) π radians. 3) $\frac{\pi}{2}$ radian. 4) $\frac{n\pi}{180}$ radians.

e. 1) The length of the arc is the same as the radius.
2) The length of the arc is double the length of the radius.
3) The length of the arc is 4.5 times the length of the radius.
4) The length of the arc is 8.71 times the length of the radius.

f. $L = r\theta$

Activity 2

Page 92

a. 1) (1, 0) 2) (0, 1) 3) (-1, 0) 4) (0, -1)

b. 1) Triangle BOP_5 is a right-isosceles triangle. 2) $\frac{\pi}{4}$ radian.

c. Let $m\overline{OB} = m\overline{BP_5} = x$.
The coordinates of P_5 are therefore (x, x)
Based on the Pythagorean theorem, you have $x^2 + x^2 = 1$.
Therefore, $2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

$x = \frac{\sqrt{2}}{2}$, therefore the coordinates of P_5 are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

d. 1) $\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 2) $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ 3) $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

e. 1) It is a right triangle that has a 30° angle.
2) $\frac{\pi}{6}$ radian.

f. 1) In a right triangle, the length of the side opposite to a 30° angle is equal to half the length of the hypotenuse.

2) $y^2 + \left(\frac{1}{2}\right)^2 = 1^2$ where y represents the length of $\overline{OB}$.

$y^2 = 1^2 - \left(\frac{1}{2}\right)^2$

$y^2 = 1 - \frac{1}{4}$

$y^2 = \frac{3}{4}$

$y = \sqrt{\frac{3}{4}}$

$y = \frac{\sqrt{3}}{2}$

g. 1) $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ 2) $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$ 3) $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

Activity 2 (*cont'd*) Page 93

h. **1)** It is a right triangle with a 30° angle.

2) $\frac{\pi}{3}$ radian.

i. **1)** In a right triangle, the length of the side opposite to a 30° angle is equal to half the length of the hypotenuse.

2) $y^2 + \left(\frac{1}{2}\right)^2 = 1^2$ where y represents the length of $\overline{BP_{13}}$.

$$y^2 = 1^2 - \left(\frac{1}{2}\right)^2$$
$$y^2 = 1 - \frac{1}{4}$$
$$y^2 = \frac{3}{4}$$
$$y = \sqrt{\frac{3}{4}}$$
$$y = \frac{\sqrt{3}}{2}$$

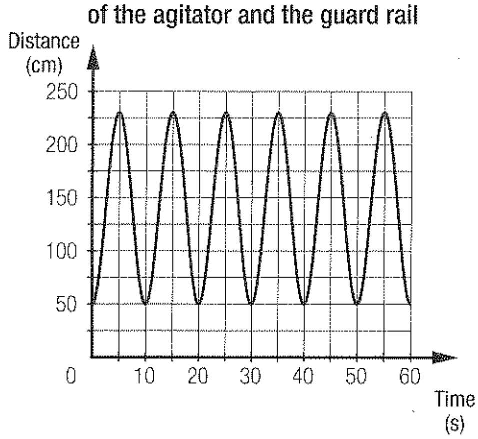
j. **1)** $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ **2)** $\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$ **3)** $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

k. **1)** $\cos\theta$ **2)** $\sin\theta$

Activity 3 Page 94

a.

Distance between the end of the agitator and the guard rail



b. Domain: [0, 60] s, range: [50, 230] cm.

c. The agitator is at its initial position at 10 s, 20 s, 30 s, 40 s, 50 s, 60 s, 70 s, 80 s, 90 s, 100 s, 110 s, 120 s, 130 s, 140 s, 150 s, 160 s, 170 s, 180 s, 190 s, 200 s, 210 s, 220 s, 230 s, 240 s, 250 s, 260 s, 270 s, 280 s, 290 s, 300 s.

Technomath Page 95

a. **1)** m ∠ AOB = 1 radian on Screen **1** and m ∠ AOB = 1 radian on Screen **2**.

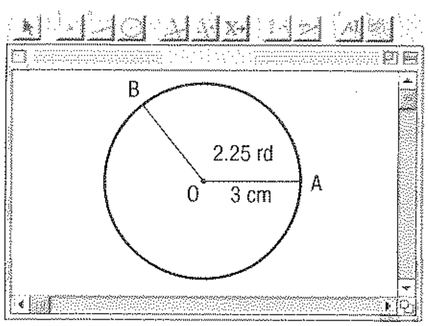
2) m ∠ AOB ≈ 57.3° on Screen **1** and m ∠ AOB ≈ 57.3° on Screen **2**.

b. This length is 1 radian.

c. **1)** m ∠ AOB ≈ 68.75° on Screen **3** and m ∠ AOB ≈ 166.16° on Screen **4**.

2) Arc AB has a length of 3.192 cm on Screen **3** and a length of 5.568 cm on Screen **4**.

d. 1)



2) The intercepted arc has a length of 15.75 cm.

Practice 5.1

Page 99

1. a) $\frac{35\pi}{18}$ rad   b) $\frac{\pi}{36}$ rad   c) $\frac{7\pi}{9}$ rad   d) $\frac{\pi}{18}$ rad
   e) $\frac{5\pi}{36}$ rad   f) $\frac{7\pi}{18}$ rad   g) $\frac{35\pi}{18}$ rad or $\frac{-1\pi}{18}$ rad.   h) $\frac{25\pi}{18}$ rad or $\frac{-11\pi}{18}$ rad.

2. a) 30°   b) 75°   c) 27°   d) 540°
   e) $\approx$ 401.07° or $\frac{1260°}{\pi}$.   f) -36°   g) $\frac{-360°}{\pi}$ h) $\approx$ 42.97° or $\frac{135°}{\pi}$.

3. a) In the 3rd quadrant.   b) In the 2nd quadrant.   c) In the 3rd quadrant.   d) In the 1st quadrant.
   e) In the 3rd quadrant.   f) In the 3rd quadrant.   g) In the 3rd quadrant.   h) In the 2nd quadrant.

4. a) 0   b) $\frac{1}{\sqrt{3}}$ c) 1   d) $\sqrt{3}$ e) Not defined.   f) $-\sqrt{3}$
   g) -1   h) $\frac{-1}{\sqrt{3}}$ i) Not defined.   j) $-\sqrt{3}$ k) -1   l) $\frac{-1}{\sqrt{3}}$

5. a) 6   b) [-1, 2]   c) 1) 1   2) 2   3) 1

Practice 5.1 *(cont'd)*

Page 100

6. A 5, B 1, C 2, D 6, E 4, F 3

Practice 5.1 *(cont'd)*

Page 101

7. a) 1) Maximum: 1   2) Minimum: -1   3) Period: 2π
   b) 1) Maximum: 1   2) Minimum: -1   3) Period: 2π

8. a) 2π rad   b) $\frac{3\pi}{2}$ rad   c) π rad   d) $\frac{4\pi}{3}$ rad
   e) $\frac{\pi}{6}$ rad   f) $\frac{3\pi}{4}$ rad   g) $\frac{3\pi}{2}$ rad   h) $\approx$ 2.69 rad

9.

L	r	θ
$\frac{6\pi}{5}$	6	$\frac{\pi}{5}$
3	**$\frac{5}{3}$**	1.8
16	4	**4**
37.8	18	2.1
9	**≈ 1.97**	4.56
1	9	**$\frac{1}{9}$**

10. No, since the nature of the periodic function is that a value of the dependent variable can be associated with more than one value of the independent variable.

11. **a)** $(-a, -b)$ **b)** $(-a, -b)$ **c)** $(-b, a)$
d) $(b, -a)$ **e)** $(b, -a)$ **f)** $(-b, a)$

Practice 5.1 *(cont'd)* **Page 102**

12. **a) 1)** In the 2nd quadrant. **2)** $\frac{3\pi}{4}$ rad
b) 1) On the *y*-axis, between the 3rd and 4th quadrant. **2)** $\frac{3\pi}{2}$ rad
c) 1) In the 2nd quadrant. **2)** $\frac{2\pi}{3}$ rad
d) 1) On the *y*-axis, between the 3rd and 4th quadrant. **2)** $\frac{3\pi}{2}$ rad
e) 1) In the 1st quadrant. **2)** $\frac{\pi}{6}$ rad
f) 1) In the 4th quadrant. **2)** $\frac{5\pi}{3}$ rad

13. **a)** $\pm\frac{1}{2}$ **b)** $\pm\frac{\sqrt{3}}{2}$ **c)** ± 1 **d)** $\pm\frac{4}{5}$ **e)** $\pm\frac{\sqrt{11}}{6}$ **f)** $\pm\frac{\sqrt{5}}{3}$

14. $\tan\frac{3\pi}{2} = \frac{\sin\frac{3\pi}{2}}{\cos\frac{3\pi}{2}} = \frac{1}{0}$, which does not exist in the set of real numbers.

15. A F, B C, D G, E H

16. **a)** The length of this hedge is approximately 33.16 m.
b) $\frac{33.16}{0.3} \approx 110.53$ cedars, 111 cedars × \$4.50 = \$499.50. The landscaping of this hedge costs \$499.50.

Practice 5.1 *(cont'd)* **Page 103**

17. **a)** 10 **b) 1)** 1 **2)** 1 **3)** 2

18. **a)** The mean radius of the ISS orbit is 6718 km.
b) 1) The ISS moves at approximately 0.0012 rad/s. **2)** The ISS moves at approximately 7730.85 m/s.
3) The ISS moves at approximately 27 831.06 km/h.

19. The rotational velocity of Drum **B** is 4.8 rad/s.

Practice 5.1 *(cont'd)* **Page 104**

20. $m\overset{\frown}{AB} = 639.163$ km $= m\overset{\frown}{EB}$
$m\overset{\frown}{BC} = 218.127$ km $= m\overset{\frown}{BD}$
$m\overset{\frown}{CD} = 543.056$ km
The space probe has therefore covered a distance approximately 2257.64 km.

21. **a)** 26.25 cm **b)** 70 cm **c)** 105 cm

22. The minimum radius of the torus is 245.25 m.

SECTION 5.2 | Trigonometric functions

Problem **Page 105**

On July 19, it is preferable for a sailboat to leave the port between 6:30 a.m. and 11:30 a.m. or between 5:30 p.m. and 10:30 p.m., once the tide lowers and moves toward the ocean.

Activity 1

Page 106

a. 1) 2π rad 2) π rad 3) $\frac{\pi}{2}$ rad 4) $\frac{\pi}{4}$ rad

b. 1) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 2) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 3) $(0, 1)$ 4) $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

5) $(-1, 0)$ 6) $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$ 7) $(0, -1)$ 8) $(1, 0)$

c. 1) $f(\theta) = \sin\theta$ 2) $f(\theta) = \cos\theta$

d. A horizontal translation of $\frac{\pi}{2}$ in one direction or another based on the curve that is considered as the initial curve.

Activity 2

Page 107

a. 1) 50 000 people. 2) 10 000 people. 3) 30 000 people.

b. 12 years.

Activity 2 (*cont'd*)

Page 108

c. 1) 20 000 2) $\frac{\pi}{6}$ 3) 3 4) 30 000

d. 1) Both expressions have the same value, which is 20 000.

2) Both expressions have the same value, which is $\frac{\pi}{6}$.

3) The coordinates (h, k + a) correspond to a maximum of the function.

e. For function *f*, cosine is used; however, for function *g* sine is used. The parameters are identical, with the exception of parameter **h**.

f. The values of **k** are identical; however, the values of **h** differ by 3, which corresponds to one-quarter of the period.

Activity 3

Page 109

a. 1) The zeros of the sine function coincide with the vertices of the cosine function.

2) The zeros of the sine function coincide with the zeros of the tangent function.

b. 1) To the zeros. 2) To the vertices.

c. 1) The tangent function is not defined for the values of *x* which correspond to the zeros of the cosine function.

2) The period of the tangent function is half the period of the cosine function.

d. *Several answers possible. Example:*

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\tan x$	0	1	undefined	−1	0
$\frac{\sin x}{\cos x}$	0	1	undefined	−1	0

e. The zeros.

Technomath

Page 110

a. 1) Parameter **h**. 2) Parameter **k**.

b. 1) One of the curves underwent a horizontal translation in comparison to the other.

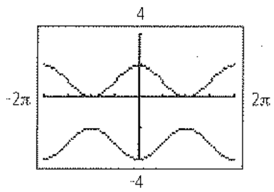
2) One of the curves underwent a vertical translation in comparison to the other.

c. 1) The parameter **h** of the equation associated with Y_1 in Screen **5** equals 2π more than parameter **h** of the equation associated with Y_1 in Screen **6**.

2) Both curves are identical.

3) $(h, k) = (0.5\pi, 1)$ in Screen **6** and $(h, k) = (2.5\pi, 1)$ in Screen **5**. Both points have the same y-coordinate, but their x-coordinate differs by 2π.

d. 1)



2) In comparison with the curve associated with Y_1, the curve associated with Y_2 is translated π units to the left and 4 units downwards.

3) *Several answers possible. For example:* $y = \sin\left(x + \frac{\pi}{2}\right) - 3$.

Practice 5.2

Page 115

1.

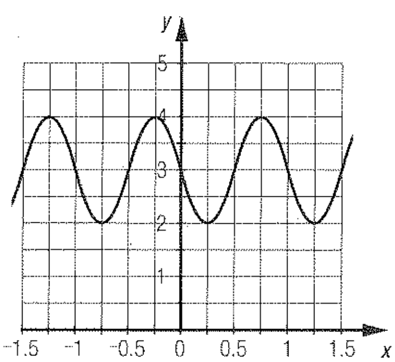
	Rule	Amplitude	Period	Maximum	Minimum
a)	$f(x) = \sin 2\left(x - \frac{\pi}{4}\right) + 3$	2	π	5	-1
b)	$f(x) = 1.5 \cos\pi(x + 3) - 5$	1.5	2	-3.5	-6.5
c)	$f(x) = -3 \sin 3.5\left(x - \frac{\pi}{6}\right) + 6$	3	$\frac{4\pi}{7}$	9	3
d)	$f(x) = 6 \cos\frac{\pi}{5}(x - 8) + 7$	6	10	13	1
e)	$f(x) = 10 \sin 0.5\left(x + \frac{\pi}{8}\right) - 4$	10	4π	6	-14

2. a) 0 b) $\frac{-\pi}{4}$ c) $\frac{\pi}{6}$

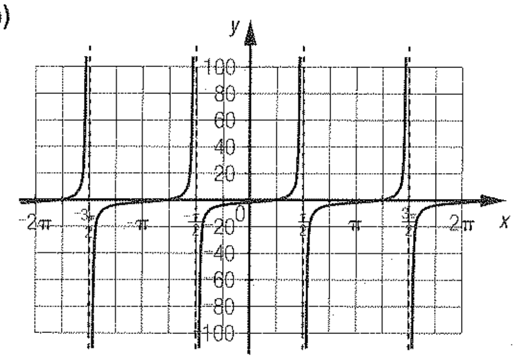
d) π e) 0 f) $\frac{\pi}{4}$

g) $\frac{-\pi}{6}$ h) $\frac{\pi}{2}$ i) $\frac{5\pi}{6}$

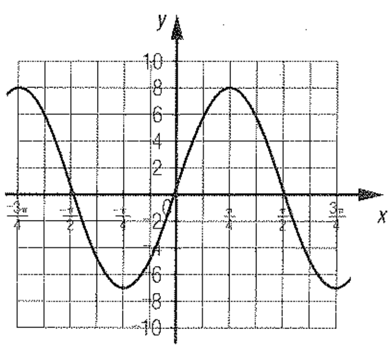
3. a)



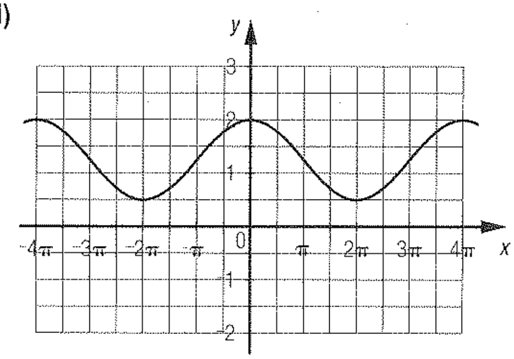
b)



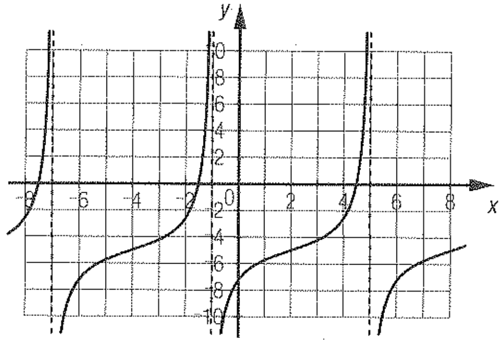
c)



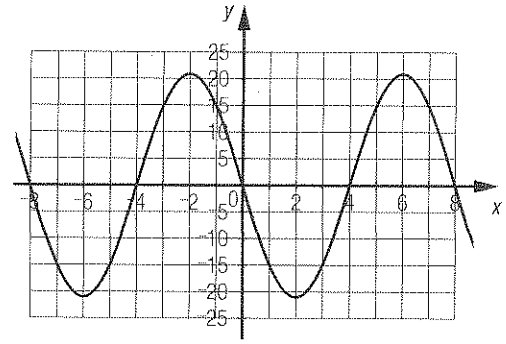
d)



e)



f)



4. a) 1) π 2) π 3) π
   b) 1) 10 and 0 2) 7 and 5 3) 3 and -5
   c) *Several answers possible. Example:* $i(x) = -2\sin 2(x - 3) - 8$

Practice 5.2 (*cont'd*) — Page 116

5. A 2, B 4, C 3, D 5, E 1

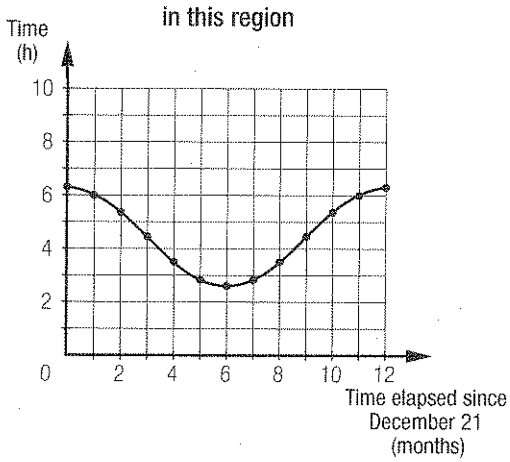
Practice 5.2 (*cont'd*) — Page 117

6. a) 1) $f(x) = \sin 2\left(x - \frac{\pi}{8}\right) + 0.5$ 2) $f(x) = \cos 2\left(x - \frac{3\pi}{8}\right) + 0.5$
   b) 1) $f(x) = 3\sin\pi(x) - 1$ 2) $f(x) = 3\cos\pi(x + 0.5) - 1$
   c) 1) $f(x) = 6\sin\pi\left(x + \frac{1}{4}\right)$ 2) $f(x) = 6\cos\pi\left(x - \frac{1}{4}\right)$
   d) 1) $f(x) = -0.75\sin\frac{\pi}{2}x + 0.05$ 2) $f(x) = -0.75\cos\frac{\pi}{2}(x - 1) + 0.05$
   e) 1) $f(x) = -2\sin 4x - 1$ 2) $f(x) = 2\cos 4\left(x + \frac{\pi}{8}\right) - 1$
   f) 1) $f(x) = 7\sin\frac{\pi}{6}(x - 1) - 7$ 2) $f(x) = 7\cos\frac{\pi}{6}(x - 4) - 7$
7. $T(x) = 120\sin 10\pi x$

Practice 5.2 (*cont'd*) — Page 118

8. a) $y = \tan\frac{\pi}{2}x$
   b) $y \approx 7.83\tan(x - 0.5) + 0.5$
   c) $y = -0.5\tan\left(x + \frac{\pi}{4}\right)$
   d) $y = -2\tan x - 1$
9. a)

Time of nautical dawn in this region



   b) $f(x) = -1.85\sin\frac{\pi}{6}(x - 3) + 4.5$ where $f(x)$ represents the time of the nautical dawn (in h) and x represents the time elapsed since December 21 (in months).

Practice 5.2 *(cont'd)*

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10. a) 1)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$f(x)$	0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$\frac{\sqrt{2}}{2}$	-1	$\frac{-1}{2}$	0
$g(x)$	0	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{2}$	0	$\frac{-\sqrt{2}}{2}$	1	$\frac{1}{2}$	0
$f(x) + g(x)$	0	0	0	0	0	0	0	0	0	0	0

2) It can be noted that $g(x) = \sin(x - \pi)$ always results in the opposite of $f(x) = \sin x$.

b) $i(x) = \cos(x - \pi)$

11. The bell can be heard 12 times every minute.

12. The daily variation of the angle of oscillation of a Foucault pendulum located in the city of Québec is approximately -4.58 rad.

Practice 5.2 *(cont'd)*

Page 120

13. $f(x) = 45\sin\frac{2\pi}{11}(x)$ where x represents time (in years) and $f(x)$ represents the number of sunspots noticed.

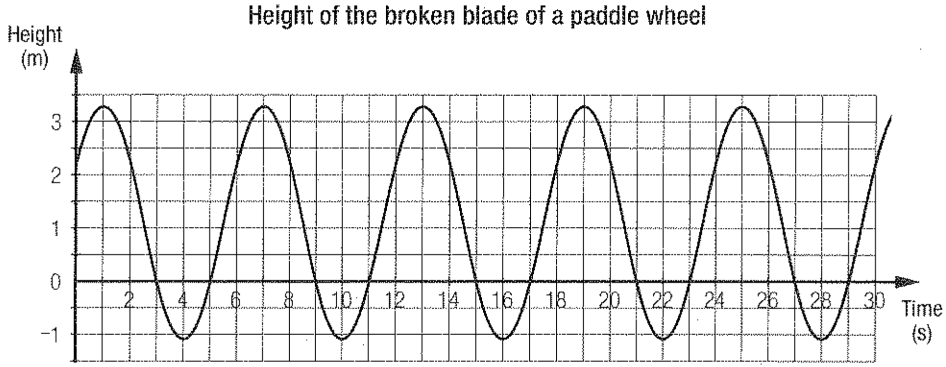
14.
- 10 rpm = $0.1\overline{6}$ rpm
- Each second, the wheel does a $0.1\overline{6}$ turn $\Rightarrow 60° \Rightarrow \frac{\pi}{3}$ rad.

Let h be the height (in m) of the broken blade in relation to the surface of the water and t, the time (in s).

- At $t = 0$, $h = 2.2$ m.
- At $t = 1$, $h = 3.3$ m.
- At $t = 2$, $h = 2.2$ m.
- At $t = 3$, $h = 0$ m.
- At $t = 4$, $h = -1.1$ m.
- At $t = 5$, $h = 0$ m.

a)

Height of the broken blade of a paddle wheel



b) $h(t) = 2.2\sin\frac{\pi}{3}(t + 0.5) + 1.1$

SECTION 5.3 | Solving trigonometric equations and inequalities

Problem

Page 121

The flash lights up at 1 s, 4 s, 7 s, 10 s, 13 s, 16 s, 19 s, 22 s, 25 s, 28 s, 31 s, 34 s, 37 s, 40 s, 43 s, 46 s, 49 s, 52 s, 55 s and 58 s.

Activity 1

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a. 1) 3 °C **2)** -1 °C

b. After 6 h, 18 h, 30 h and 42 h.

c. The solution to this equation allows you to determine at which moments the temperature is 2°C.

d. $\frac{\pi}{6}$ rad or $\frac{5\pi}{6}$ rad.

e. **1)** To go from:

- Step ① to Step ②, you subtract 1 from both sides of the equality and then divide by 2.
- Step ② to Step ③, you apply arcsin to both sides of the equality to remove the sine found on the left side.
- Step ③ to Step ④, you determine the values of θ for which $\sin\theta = \frac{1}{2}$, which is $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.
- Step ④ to Step ⑤, you multiply by 12 and divide both sides of the equality by π.
- Step ⑤ to Step ⑥, you add 6 to both sides of the equality.

2) These values represent the moments in a period where the temperature reached is 2°C.

f. **1)** 24 h **2)** These two expressions allow you to determine the moments of the second day when it is 2°C.

g. **1)** $2\sin\frac{\pi}{12}(x - 6) + 1 \leq 2$ **2)** During the intervals [0, 8] and [16, 32] and [40, 48].

Practice 5.3

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1. a) $\left\{\frac{-8\pi}{3}, \frac{-5\pi}{2}, \frac{-5\pi}{3}, \frac{-3\pi}{2}, \frac{-2\pi}{3}, \frac{-\pi}{2}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{7\pi}{3}, \frac{5\pi}{2}\right\}$

b) {0, 1.5, 2, 3.5, 4}

c) $\left\{\frac{-25}{4}, \frac{-5}{4}, \frac{15}{4}, \frac{35}{4}\right\}$

d) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}\right\}$

e) There are no solutions.

f) $\left\{\frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{11}{9}, \frac{14}{9}, \frac{17}{9}, \frac{20}{9}, \frac{23}{9}, \frac{26}{9}, \frac{29}{9}, \frac{32}{9}, \frac{35}{9}, \frac{38}{9}, \frac{41}{9}, \frac{44}{9}\right\}$

2. a) $\{x \in \mathbb{R} \mid x = 5\pi + 16n\pi \vee x = 9\pi + 16n\pi, n \in \mathbb{Z}\}$

b) $\left\{x \in \mathbb{R} \mid x = \frac{11}{3} + 4n \vee x = \frac{13}{3} + 4n, n \in \mathbb{Z}\right\}$

c) $\left\{x \in \mathbb{R} \mid x = \frac{7}{3} + 4n, n \in \mathbb{Z}\right\}$

d) $\left\{x \in \mathbb{R} \mid x = \frac{20\pi}{3} + 16n\pi \vee x = \frac{28\pi}{3} + 16n\pi, n \in \mathbb{Z}\right\}$

e) $\left\{x \in \mathbb{R} \mid x = \frac{61\pi}{84} + n\pi \vee x = \frac{89\pi}{84} + n\pi, n \in \mathbb{Z}\right\}$

f) $\left\{x \in \mathbb{R} \mid x = \frac{7\pi}{12} + n\pi, n \in \mathbb{Z}\right\}$

3. a) The function is:

- positive over $\left[-2\pi, \frac{-17\pi}{12}\right] \cup \left[\frac{-13\pi}{12}, \frac{-5\pi}{12}\right] \cup \left[\frac{-\pi}{12}, \frac{7\pi}{12}\right] \cup \left[\frac{11\pi}{12}, \frac{19\pi}{12}\right] \cup \left[\frac{23\pi}{12}, 2\pi\right]$,
- negative over $\left[\frac{-17\pi}{12}, \frac{-13\pi}{12}\right] \cup \left[\frac{-5\pi}{12}, \frac{-\pi}{12}\right] \cup \left[\frac{7\pi}{12}, \frac{11\pi}{12}\right] \cup \left[\frac{19\pi}{12}, \frac{23\pi}{12}\right]$.

b) The function is:

- positive over $\left[\frac{-\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{11\pi}{6}, \frac{13\pi}{6}\right]$,
- negative over $\left[-\pi, \frac{-\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{11\pi}{6}\right] \cup \left[\frac{13\pi}{6}, 3\pi\right]$.

c) The function is:

- positive over $\left[-3\pi, \frac{-5\pi}{2}\right[\cup \left[\frac{-27\pi}{12}, \frac{-3\pi}{2}\right[\cup \left[\frac{-15\pi}{12}, \frac{-\pi}{2}\right[\cup \left[\frac{-\pi}{4}, \frac{\pi}{2}\right[\cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right[\cup \left[\frac{21\pi}{12}, \frac{5\pi}{2}\right[\cup \left[\frac{33\pi}{12}, 3\pi\right]$,
- negative over $\left]\frac{-5\pi}{2}, \frac{-27\pi}{12}\right] \cup \left]\frac{-3\pi}{2}, \frac{-15\pi}{12}\right] \cup \left]\frac{-\pi}{2}, \frac{-\pi}{4}\right] \cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left]\frac{3\pi}{2}, \frac{21\pi}{12}\right] \cup \left]\frac{5\pi}{2}, \frac{33\pi}{12}\right]$.

d) The function is strictly negative.

e) The function is strictly negative.

f) The function is:

- positive over $\left[\frac{3}{4}, 1\right[\cup \left[\frac{7}{4}, 2\right[\cup \left[\frac{11}{4}, 3\right[\cup \left[\frac{15}{4}, 4\right[$,
- negative over $\left]0, \frac{3}{4}\right] \cup \left]1, \frac{7}{4}\right] \cup \left]2, \frac{11}{4}\right] \cup \left]3, \frac{15}{4}\right]$.

4. a) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{3} + n\pi \vee x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}\right\}$

b) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}\right\}$

c) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{3} + n\pi \vee x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}\right\}$

d) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

e) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{3} + n\pi \vee x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}\right\}$

f) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{6} + n\pi \vee x = \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}\right\}$

Practice 5.3 (*cont'd*) **Page 127**

5. a) $\{-3.61, -1.61, 0.39, 2.39\}$

b) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

c) $\{-1.75, -1.25, -0.75, -0.25, 0.25, 0.75, 1.25, 1.75\}$

d) $0.91\overline{6}$ and $1.58\overline{3}$.

e) $\left\{-\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}\right\}$

f) $\left\{\frac{-5\pi}{8}, \frac{-\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}\right\}$

6. a) $\frac{\pi}{4}$ and $\frac{5\pi}{4}$. **b)** $0, \pi$ and 2π. **c)** $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ and 2π. **d)** $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. **e)** $0, \pi$ and 2π.

f) $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. **g)** $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$. **h)** $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. **i)** $\frac{\pi}{3}$ and $\frac{4\pi}{3}$.

7. a) $\left[-\pi, \frac{-17\pi}{24}\right[\cup \left]\frac{-13\pi}{24}, \frac{-5\pi}{24}\right[\cup \left]\frac{-\pi}{24}, \frac{7\pi}{24}\right[\cup \left]\frac{11\pi}{24}, \frac{19\pi}{24}\right[\cup \left]\frac{23\pi}{24}, \pi\right]$

b) $\left[\frac{-27\pi}{16}, \frac{-19\pi}{16}\right] \cup \left[\frac{-11\pi}{16}, \frac{-3\pi}{16}\right] \cup \left[\frac{5\pi}{16}, \frac{13\pi}{16}\right]$

c) $\left]\frac{3\pi}{8}, \frac{5\pi}{8}\right[\cup \left]\frac{11\pi}{8}, \frac{13\pi}{8}\right[$

d) $[0, 4]$

e) $\left[-3\pi, \frac{-8\pi}{3}\right[\cup \left]\frac{-7\pi}{3}, \frac{-2\pi}{3}\right[\cup \left]\frac{-\pi}{3}, \frac{4\pi}{3}\right[\cup \left]\frac{5\pi}{3}, 3\pi\right]$

f) $\left]\frac{-\pi}{2}, 0\right[\cup \left]\frac{\pi}{2}, \pi\right[$

Practice 5.3 (*cont'd*) **Page 128**

8. a) $\ldots \cup \left]\frac{-\pi}{3}, \frac{-\pi}{6}\right[\cup \left]\frac{-\pi}{12}, \frac{\pi}{12}\right[\cup \left]\frac{\pi}{6}, \frac{\pi}{3}\right[\cup \ldots$

b) $\ldots \cup \left[\frac{-2\pi}{3}, \frac{-\pi}{3}\right] \cup \left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \pi\right] \cup \ldots$

c) $\ldots \cup \left]\frac{\pi}{12}, \frac{\pi}{3}\right[\cup \left]\frac{13\pi}{12}, \frac{4\pi}{3}\right[\cup \left]\frac{25\pi}{12}, \frac{7\pi}{3}\right[\cup \ldots$

d) $x = 2 + 4n$ where $n \in \mathbb{Z}$.

e) $\ldots \cup \left]\frac{-3\pi}{4}, \frac{-7\pi}{12}\right[\cup \left]\frac{-\pi}{12}, \frac{\pi}{12}\right[\cup \left]\frac{7\pi}{12}, \frac{3\pi}{4}\right[\cup \ldots$

f) $\ldots \cup \left]\frac{-2\pi}{3}, \frac{-\pi}{2}\right[\cup \left]\frac{\pi}{3}, \frac{\pi}{2}\right[\cup \left]\frac{4\pi}{3}, \frac{3\pi}{2}\right[\cup \ldots$

9. The parachute deploys after 15 s.

10. Leona's feet touch the bottom for 400 s.

11. a) $\ldots \cup \left]\frac{-7\pi}{4}, \frac{-3\pi}{4}\right[\cup \left]\frac{\pi}{4}, \frac{5\pi}{4}\right[\cup \left]\frac{9\pi}{4}, \frac{13\pi}{4}\right[\cup \ldots$

b) $\ldots \cup \left]0, \frac{\pi}{2}\right[\cup \left]\pi, \frac{3\pi}{2}\right[\cup \left]2\pi, \frac{5\pi}{2}\right[\cup \ldots$

12. During the first 10 seconds, the eye of the needle goes through the material 100 times.

Practice 5.3 (*cont'd*) **Page 129**

13. a) This person jumps 400 times throughout this workout session.

b) Each jump lasts 0.125 s; therefore this person's feet do not touch the ground for 50 s.

14. a) 1) ≈ 0.2679 cm^2 **2)** ≈ 0.364 cm^2 **b) 1)** $\frac{\pi}{6}$ rad **2)** $\frac{\pi}{4}$ rad

15. a) 1) 600 times.
2) 1200 times.
3) 1200 times.

b) 1) At $0.008\overline{3}$ s, $0.041\overline{6}$ s, $0.108\overline{3}$ s, $0.141\overline{6}$ s, …
2) At 0.05 s, 0.1 s, 0.15 s, 0.2 s, …
3) At 0.075 s, 0.175 s, 0.275 s, 0.375 s, …

c) 1) 300 times.
2) 300 times.
3) 150 times.

d) 1) At 0.35 s, 0.75 s, 1.15 s, 1.55 s, …
2) At $0.01\overline{6}$ s, $0.28\overline{3}$ s, $0.41\overline{6}$ s, $0.68\overline{3}$ s, …
3) At 0.05 s, 0.25 s, 0.45 s, 0.65 s, …

Practice 5.3 (*cont'd*) **Page 130**

16. a) 1) -5 m **2)** -5 m

b) 1) At $0.1\overline{6}$ s, 0.5 s, $0.8\overline{3}$ s …
2) At 0 s, $0.\overline{3}$ s, $0.\overline{6}$ s, 1 s, …
3) At $0.\overline{2}$ s, $0.\overline{4}$ s, $0.\overline{5}$ s, $0.\overline{7}$ s, …
4) At $0.2\overline{7}$ s, $0.3\overline{8}$ s, $0.6\overline{1}$ s, $0.7\overline{2}$ s, …
5) At $0.08\overline{3}$ s, 0.25 s, $0.41\overline{6}$ s, $0.58\overline{3}$ s, …

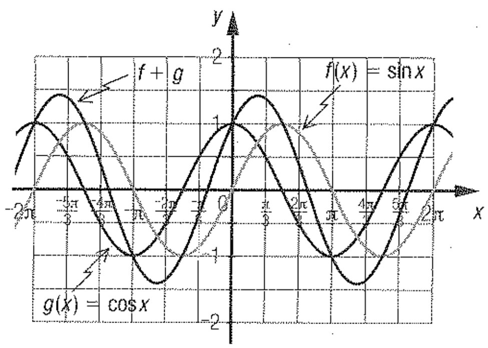
17. Yes, Joseph is right because $-\frac{\pi}{2}$ represents one-quarter of the circle, which is 90°. Once you move away from the quarter of the circle, the value of the sine becomes the value of the cosine.

18. During one minute, the sailboat is found in this situation for 20 s.

Practice 5.3 (*cont'd*) **Page 131**

19. a) 2 cm   b) 6 cm

20. a)



b) 1) $y = \frac{2}{\sqrt{2}}\sin\left(x + \frac{\pi}{4}\right)$ 2) $y = \frac{2}{\sqrt{2}}\cos\left(x - \frac{\pi}{4}\right)$

c) 1) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + 2n\pi \vee x = \frac{5\pi}{4} + 2n\pi,\ n \in \mathbb{Z}\right\}$

2) These values are associated with the maximum of the function $f + g$.

d) 1) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{2} + 2n\pi \vee x = \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\right\}$

2) These values are associated with the maximum of the function f.

e) 1) $\{x \in \mathbb{R} \mid x = n\pi \text{ where } n \in \mathbb{Z}\}$

2) These values are associated with the maximum of the function g.

21. a) The mean temperature of this room is 20°C.

b) The person is uncomfortable for approximately 13.33 min for every 20-min period. She is therefore uncomfortable for approximately 960 min or 16 h.

c) The mean temperature of this room is 20°C.

d) The person is comfortable the whole time.

SECTION 5.4 Trigonometric identities

Problem **Page 132**

Several answers possible. Example:

The exact value of $\tan\frac{\pi}{6}$ is $\frac{\sqrt{3}}{3}$, which is approximately 0.58. Based on the strategy suggested by this student, the exact value of $\tan\frac{\pi}{12}$ would be $\frac{\sqrt{3}}{6}$ which is approximately 0.29. Yet, if you calculate $\tan\frac{\pi}{12}$ with a calculator, you obtain approximately 0.27. The strategy suggested by this student is therefore not correct.

Activity 1 **Page 133**

a. 1) The length of the beam AE corresponds to the cosine.
   2) The length of the beam CE corresponds to the sine.

b. $(m\,\overline{AE})^2 + (m\,\overline{CE})^2 = (m\,\overline{AC})^2$, which is $(m\,\overline{AE})^2 + (m\,\overline{CE})^2 = 1$.

c. 1)
- ∠ CEA ≅ ∠ CED, because both angles are right angles.
- ∠ ACE ≅ ∠ CDE, because both angles are complementary to angle CAE.
- Δ ACE ~ Δ CDE, two triangles that have two corresponding congruent angles are similar (AA).

2)
- ∠ ACB ≅ ∠ ACD, because both angles are right angles.
- ∠ BAC ≅ ∠ ADC, because both angles are complementary to angle ABC.
- Δ ABC ~ Δ ACD, two triangles that have two corresponding congruent angles are similar (AA).

3)
- ∠ ACD ≅ ∠ AEC, because both angles are right angles.
- ∠ CAE ≅ ∠ CAD, based on reflexivity.
- Δ ACD ~ Δ ACE, two triangles that have two corresponding congruent angles are similar (AA).

4)
- ∠ BAD ≅ ∠ AEC, because both angles are right angles.
- ∠ ABC ≅ ∠ CAE, because both angles are complementary to angle CAB.
- Δ ABD ~ Δ ACE, both triangles that have two corresponding congruent angles are similar (AA).

d. The length of the beam CD corresponds to $\frac{\sin\theta}{\cos\theta}$, which is $\tan\theta$.

Activity 1 *(cont'd)*

Page 134

e. 1) $\frac{m\,\overline{BC}}{1} = \frac{1}{\tan\theta}$ 2) $m\,\overline{BC} = \frac{1}{\tan\theta}$ 3) They are multiplicative inverses.

f. 1) $\frac{m\,\overline{AD}}{1} = \frac{1}{\cos\theta}$ 2) $m\,\overline{AD} = \frac{1}{\cos\theta}$ 3) They are multiplicative inverses.

g. $(m\,\overline{AC})^2 + (m\,\overline{CD})^2 = (m\,\overline{AD})^2$

h. 1) $\frac{\sin\theta}{1} = \frac{1}{m\,\overline{AB}}$ 2) $m\,\overline{AB} = \frac{1}{\sin\theta}$ 3) They are multiplicative inverses.

i. $(m\,\overline{BC})^2 + (m\,\overline{AC})^2 = (m\,\overline{AB})^2$

Activity 2

Page 135

a. 1) i) False, because $\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \approx 0.96$ and $\sin\frac{\pi}{4} + \sin\frac{\pi}{3} \approx 1.57$.

ii) False, because $\cos\left(\frac{\pi}{2} - \pi\right) = 0$ and $\cos\frac{\pi}{2} - \cos\pi = -1$.

iii) False, because $\tan\left(\frac{\pi}{6} + \frac{\pi}{3}\right)$ is undefined and $\tan\frac{\pi}{6} + \tan\frac{\pi}{3} = \frac{4\sqrt{3}}{3}$.

iv) False, because $\sin\left(\pi - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\sin\pi - \sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$.

2) The sine, cosine and tangent trigonometric functions are not distributive over addition, nor over subtraction.

b. ①: Sides CD and EF are parallel because they are both perpendicular to side AD.
$\angle$ ACD $\cong$ $\angle$ AFE because if a segment intersects two parallel segments, then the corresponding angles are congruent.
$\angle$ AFE $\cong$ $\angle$ BFC because two angles opposite the vertex are congruent.
$\angle$ ACD $\cong$ $\angle$ BFC, based on the transitivity of both previous expressions.

②: $\angle$ CAD $\cong$ $\angle$ CBF because both angles are respectively complementary to angles AFE and BFC, both of them are congruent, because they are opposite the vertex.

③: $\angle$ ADC $\cong$ $\angle$ BGC because both angles are right angles.

④: Δ ACD ~ Δ BCG because two triangles that have two corresponding congruent angles are similar (AA).

c. You can go from:

- Step ① to Step ② because $m\,\overline{BE} = m\,\overline{GE} + m\,\overline{BG}$ and $m\,\overline{GE} = m\,\overline{CD}$.
- Step ② to Step ③ by adding two fractions.
- Step ③ to Step ④ by multiplying the expressions $\frac{m\,\overline{CD}}{m\,\overline{AB}}$ and $\frac{m\,\overline{BG}}{m\,\overline{AB}}$ by a unit fraction.
- Step ④ to Step ⑤ by commutativity over the multiplication.
- Step ⑤ to Step ⑥ since $\sin CAD = \frac{m\,\overline{CD}}{m\,\overline{AC}}$, $\cos BAC = \frac{m\,\overline{AC}}{m\,\overline{AB}}$, $\cos CBG = \frac{m\,\overline{BG}}{m\,\overline{BC}}$ and $\sin BAC = \frac{m\,\overline{BC}}{m\,\overline{AB}}$.
- Step ⑥ to Step ⑦ since Δ ACD ~ Δ BCG based on the minimum condition of similar triangles AA, $\angle$ CBG $\cong$ $\angle$ CAD.

Activity 2 *(cont'd)*

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d. You can go from:

- Step ① to Step ② because $m\,\overline{BE} = m\,\overline{GE} + m\,\overline{BG}$ and $m\,\overline{GE} = m\,\overline{CD}$.
- Step ② to Step ③ by subtracting two fractions.
- Step ③ to Step ④ by multiplying the expressions $\frac{m\,\overline{AD}}{m\,\overline{AB}}$ and $\frac{m\,\overline{CG}}{m\,\overline{AB}}$ by a unit fraction.
- Step ④ to Step ⑤ by commutativity over the multiplication.
- Step ⑤ to Step ⑥ since $\cos CAD = \frac{m\,\overline{AD}}{m\,\overline{AC}}$, $\cos BAC = \frac{m\,\overline{AC}}{m\,\overline{AB}}$, $\sin CBG = \frac{m\,\overline{CG}}{m\,\overline{BC}}$ and $\sin BAC = \frac{m\,\overline{BC}}{m\,\overline{AB}}$.
- Step ⑥ to Step ⑦ since Δ ACD ~ Δ BCG based on the minimum condition of similar triangles AA, $\angle$ CBG $\cong$ $\angle$ CAD.

e. 1) $\tan(a+b) = \frac{\sin(a+b)}{\cos(a+b)}$

$$= \frac{\sin a\cos b + \sin b\cos a}{\cos a\cos b - \sin a\sin b}$$

$$= \frac{\frac{\sin a\cos b + \sin b\cos a}{\cos a\cos b}}{\frac{\cos a\cos b - \sin a\sin b}{\cos a\cos b}}$$

$$= \frac{\frac{\sin a\cos b}{\cos a\cos b} + \frac{\sin b\cos a}{\cos a\cos b}}{\frac{\cos a\cos b}{\cos a\cos b} - \frac{\sin a\sin b}{\cos a\cos b}}$$

$$= \frac{\frac{\sin a}{\cos a} + \frac{\sin b}{\cos b}}{1 - \frac{\sin a}{\cos a} \times \frac{\sin b}{\cos b}}$$

$$= \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

2) $\tan(a-b) = \frac{\sin(a-b)}{\cos(a-b)}$

$$= \frac{\sin a\cos b - \sin b\cos a}{\cos a\cos b + \sin a\sin b}$$

$$= \frac{\frac{\sin a\cos b - \sin b\cos a}{\cos a\cos b}}{\frac{\cos a\cos b + \sin a\sin b}{\cos a\cos b}}$$

$$= \frac{\frac{\sin a\cos b}{\cos a\cos b} - \frac{\sin b\cos a}{\cos a\cos b}}{\frac{\cos a\cos b}{\cos a\cos b} + \frac{\sin a\sin b}{\cos a\cos b}}$$

$$= \frac{\frac{\sin a}{\cos a} - \frac{\sin b}{\cos b}}{1 + \frac{\sin a}{\cos a} \times \frac{\sin b}{\cos b}}$$

$$= \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

Practice 5.4

1. a) $\cos^2 x$ b) $\sin^2 x$ c) $\cos^2 x$ d) 1
   e) 1 f) $\tan^2 x$ g) $\cot x$ h) $\tan^2 x$
2. a) $\frac{3\sqrt{7}}{8}$ b) $3\sqrt{7}$ c) 8 d) $\frac{8\sqrt{7}}{21}$
3. a) $\frac{3\sqrt{159}}{40}$ b) $\frac{40\sqrt{159}}{477}$ c) $-\frac{40}{13}$ d) $-\frac{3\sqrt{159}}{13}$
4. a) $\frac{2\sqrt{2}}{3}$ b) $-\frac{1}{3}$ c) $-2\sqrt{2}$ d) $-\frac{\sqrt{2}}{4}$
5. a) $-\frac{3}{5}$ b) $-\frac{4}{5}$ c) $-\frac{5}{3}$ d) $-\frac{4}{3}$
6. a) $\frac{3}{5}$ b) $-\frac{3}{4}$ c) $-\frac{5}{4}$ d) $-\frac{4}{3}$
7. a) $\{-2\pi, 0, 2\pi\}$
   b) $\left\{-\frac{8\pi}{3}, -2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3}\right\}$
   c) $\left\{-3\pi, -\frac{8\pi}{3}, -\frac{4\pi}{3}, -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{8\pi}{3}, 3\pi\right\}$
   d) $\left\{-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}\right\}$
   e) No solution.
   f) $\left\{-\frac{5\pi}{2}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}$

Practice 5.4 (*cont'd*)

8. a) 1 b) $\sin^2 x$ c) $\text{cosec}^2 x$ d) $\sin^2 x$ e) $\sec x$ f) 1
9.
$$\frac{\cos^2 x - \cos^4 x}{\sin^2 x - \sin^4 x} = 1$$
$$\frac{\cos^2 x(1 - \cos^2 x)}{\sin^2 x(1 - \sin^2 x)} = 1$$
$$\frac{\cos^2 x \times \sin^2 x}{\sin^2 x \times \cos^2 x} = 1$$
$$1 = 1$$
10. a) $\frac{\sqrt{2}+\sqrt{6}}{4}$ b) $\sqrt{3} - 2$ c) $\sqrt{6} - \sqrt{2}$ d) $\frac{\sqrt{2}-\sqrt{6}}{4}$ e) $2 - \sqrt{3}$ f) $-\frac{\sqrt{2}+\sqrt{6}}{4}$
11. a)
$$\sin^2 x = 1 - \cot^2 x\sin^2 x$$
$$1 - \cos^2 x = 1 - \cot^2 x\sin^2 x$$
$$1 - \cos^2 x\frac{\sin^2 x}{\sin^2 x} = 1 - \cot^2 x\sin^2 x$$
$$1 - \frac{\cos^2 x}{\sin^2 x}\sin^2 x = 1 - \cot^2 x\sin^2 x$$
$$1 - \cot^2 x\sin^2 x = 1 - \cot^2 x\sin^2 x$$
   b)
$$\sin x\cot x = \cos x$$
$$\sin x\frac{\cos x}{\sin x} = \cos x$$
$$\frac{\sin x}{\sin x}\cos x = \cos x$$
$$\cos x = \cos x$$

c)

$$\frac{\cot x - \tan x}{\cot x + \tan x} = 2\cos^2 x - 1$$

$$\frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} = 2\cos^2 x - 1$$

$$\frac{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = 2\cos^2 x - 1$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \times \frac{\sin x \cos x}{\cos^2 x + \sin^2 x} = 2\cos^2 x - 1$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = 2\cos^2 x - 1$$

$$\frac{\cos^2 x - \sin^2 x}{1} = 2\cos^2 x - 1$$

$$\cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$2\cos^2 x - 1 = 2\cos^2 x - 1$$

d)

$$\tan x(\sin x + \cot x \cos x) = \sec x$$

$$\frac{\sin x}{\cos x}\left(\sin x + \frac{\cos x}{\sin x}\cos x\right) = \sec x$$

$$\frac{\sin^2 x}{\cos x} + \cos x = \sec x$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x} = \sec x$$

$$\frac{1}{\cos x} = \sec x$$

$$\sec x = \sec x$$

e)

$$(1 - \sin^2 x)(1 + \cot^2 x) = \cot^2 x$$

$$\cos^2 x \operatorname{cosec}^2 x = \cot^2 x$$

$$\cos^2 x \frac{1}{\sin^2 x} = \cot^2 x$$

$$\frac{\cos^2 x}{\sin^2 x} = \cot^2 x$$

$$\cot^2 x = \cot^2 x$$

f)

$$\sin^2 x \cot^2 x \sec x = \cos x$$

$$\sin^2 \frac{\cos^2 x}{\sin^2 x}\,\frac{1}{\cos x} = \cos x$$

$$\cos x = \cos x$$

g)

$$\cos x\sqrt{\sec^2 x - 1} = \sin x$$

$$\cos x\sqrt{\tan^2 x} = \sin x$$

$$\cos x \tan x = \sin x$$

$$\cos x \frac{\sin x}{\cos x} = \sin x$$

$$\sin x = \sin x$$

h)

$$\tan^2 x + \cos^2 x - 1 = \sin^2 x \tan^2 x$$

$$\tan^2 x + \cos^2 x - (\cos^2 x + \sin^2 x) = \sin^2 x \tan^2 x$$

$$\tan^2 x + \cos^2 x - \cos^2 x - \sin^2 x = \sin^2 x \tan^2 x$$

$$\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \sin^2 x \tan^2 x$$

$$\sin^2 x\left(\frac{1}{\cos^2 x} - 1\right) = \sin^2 x \tan^2 x$$

$$\sin^2 x(\sec^2 x - 1) = \sin^2 x \tan^2 x$$

$$\sin^2 x \tan^2 x = \sin^2 x \tan^2 x$$

12. **a)** $-\sin x$ **b)** $\cos x$ **c)** $\sin x$ **d)** $\cos x$ **e)** $\sin x$
f) $-\cos x$ **g)** $-\tan x$ **h)** $-\cot x$ **i)** $-\tan x$

Practice 5.4 *(cont'd)*

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13. **a)**

$$(\operatorname{cosec} x - \cot x)^2 = \frac{1 - \cos x}{1 + \cos x}$$

$$\operatorname{cosec}^2 x - 2\operatorname{cosec} x \cot x + \cot^2 x = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{1}{\sin^2 x} - 2\frac{1}{\sin x}\,\frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{\cos^2 x - 2\cos x + 1}{1 - \cos^2 x} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{(1 - \cos x)^2}{(1 - \cos x)(1 + \cos x)} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{1 - \cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x}$$

b)

$$\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$$

$$\frac{\frac{1}{\cos x}}{\cos x} - \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = 1$$

$$\frac{1}{\cos x} \times \frac{1}{\cos x} - \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = 1$$

$$\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = 1$$

$$\sec^2 x - \tan^2 x = 1$$

$$1 = 1$$

c)

$$\frac{1 + \tan^2 x}{\operatorname{cosec}^2 x} = \tan^2 x$$
$$\frac{\sec^2 x}{\operatorname{cosec}^2 x} = \tan^2 x$$
$$\frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} = \tan^2 x$$
$$\frac{1}{\cos^2 x} \times \sin^2 x = \tan^2 x$$
$$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$
$$\tan^2 x = \tan^2 x$$

d)

$$\frac{(2 \sin x \cos x - 1)(2 \sin x \cos x + 1)}{(1 + \sin x)(1 - \sin x)} = 4 \sin^2 x - \sec^2 x$$
$$\frac{4 \sin^2 x \cos^2 x - 1}{1 - \sin^2 x} = 4 \sin^2 x - \sec^2 x$$
$$\frac{4 \sin^2 x \cos^2 x - 1}{\cos^2 x} = 4 \sin^2 x - \sec^2 x$$
$$\frac{4 \sin^2 x \cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = 4 \sin^2 x - \sec^2 x$$
$$4 \sin^2 x - \sec^2 x = 4 \sin^2 x - \sec^2 x$$

e)

$$\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$
$$\frac{1 - \cos^2 x}{1 - \cos x} = 1 + \cos x$$
$$\frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x} = 1 + \cos x$$
$$1 + \cos x = 1 + \cos x$$

f)

$$\sec^2 x(1 - \sin^2 x \cos^2 x - \cos^4 x) = \tan^2 x$$
$$\sec^2 x - \sec^2 x \sin^2 x \cos^2 x - \sec^2 x \cos^4 x = \tan^2 x$$
$$\frac{1}{\cos^2 x} - \frac{1}{\cos^2 x} \sin^2 x \cos^2 x - \frac{1}{\cos^2 x} \cos^4 x = \tan^2 x$$
$$\frac{1}{\cos^2 x} - \sin^2 x - \cos^2 x = \tan^2 x$$
$$\sec^2 x - (\sin^2 x + \cos^2 x) = \tan^2 x$$
$$\sec^2 x - 1 = \tan^2 x$$
$$\tan^2 x = \tan^2 x$$

g)

$$\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$$
$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \sin^2 x \tan^2 x$$
$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$$
$$\frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$$
$$\frac{\sin^2 x(1 - \cos^2 x)}{\cos^2 x} = \sin^2 x \tan^2 x$$
$$\frac{\sin^2 x \sin^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$$
$$\sin^2 x \frac{\sin^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$$
$$\sin^2 x \tan^2 x = \sin^2 x \tan^2 x$$

h)

$$\sec^2 x + \operatorname{cosec}^2 x = \frac{1}{\cos^2 x \sin^2 x}$$
$$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$
$$\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$
$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$
$$\frac{1}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$

14. Demonstration **1**

- from Step ① to Step ② since $\cos^2 a = 1 - \sin^2 a$
- from Step ② to Step ③ since $-\sin^2 a - \sin^2 a = -2 \sin^2 a$
- from Step ③ to Step ④ by adding $2 \sin^2 a$ and subtracting $\cos 2a$ to both sides of the equation
- from Step ④ to Step ⑤ by dividing both sides of the equation by 2
- from Step ⑤ to Step ⑥ by completing a square root on both sides of the equation
- from Step ⑥ to Step ⑦ by applying the value $\frac{b}{2}$ to the variable a
- from Step ⑦ to Step ⑧ since $2\left(\frac{b}{2}\right) = b$

Demonstration **2**

- from Step ① to Step ② since $\sin^2 a = 1 - \cos^2 a$
- from Step ② to Step ③ since $\cos^2 a - 1 + \cos^2 a = 2 \cos^2 a - 1$
- from Step ③ to Step ④ by adding 1 to both sides of the equation and inverting both sides
- from Step ④ to Step ⑤ by dividing both sides of the equation by 2
- from Step ⑤ to Step ⑥ by completing a square root on both sides of the equation
- from Step ⑥ to Step ⑦ by applying the value $\frac{b}{2}$ to the variable a
- from Step ⑦ to Step ⑧ since $2\left(\frac{b}{2}\right) = b$

15. a)
$$\begin{aligned}\sin 3x &= \sin(x+2x)\\ &= \sin x\cos 2x + \cos x\sin 2x\\ &= \sin x\cos(x+x) + \cos x\sin(x+x)\\ &= \sin x(\cos x\cdot\cos x - \sin x\sin x) + \cos x(\sin x\cos x + \sin x\cos x)\\ &= \sin x(\cos^2 x - \sin^2 x) + 2\cos x\sin x\cos x\\ &= \sin x(1 - \sin^2 x - \sin^2 x) + 2\sin x\cos^2 x\\ &= \sin x(1 - 2\sin^2 x) + 2\sin x(1 - \sin^2 x)\\ &= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x\\ &= 3\sin x - 4\sin^3 x\end{aligned}$$

b)
$$\begin{aligned}\sin 4x &= \sin 2(2x)\\ &= \sin(2x+2x)\\ &= \sin 2x\cos 2x + \sin 2x\cos 2x\\ &= 2\sin 2x\cos 2x\\ &= 2\sin(x+x)\cos(x+x)\\ &= 2(\sin x\cos x + \sin x\cos x)(\cos x\cos x - \sin x\sin x)\\ &= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x)\\ &= 4\sin x\cos x(\cos^2 x - \sin^2 x)\\ &= 4\sin x\cos^3 x - 4\sin^3 x\cos x\end{aligned}$$

c)
$$\begin{aligned}\sin 6x &= \sin 2(3x)\\ &= \sin(3x+3x)\\ &= \sin 3x\cos 3x + \sin 3x\cos 3x\\ &= 2\sin 3x\cos 3x\end{aligned}$$

16. a) $\frac{\sqrt{2-\sqrt{2}}}{2}$ **b)** $\frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{2}$ **c)** $-1-\sqrt{2}$ **d)** $\sqrt{2}-1$

e) $\frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2}$ **f)** $2-\sqrt{3}$ **g)** $\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$ **h)** $-\frac{\sqrt{2-\sqrt{2}}}{2}$

i) $-\frac{\sqrt{2-\sqrt{2}}}{2}$ **j)** $-\frac{\sqrt{2+\sqrt{3}}}{2}$ **k)** $\sqrt{\frac{2-\sqrt{2+\sqrt{3}}}{2+\sqrt{2+\sqrt{3}}}}$ **l)** $-\frac{\sqrt{2+\sqrt{3}}}{2}$

17. a)
$$\begin{aligned}\cos(1000\pi - x) &= \cos x\\ \cos 1000\pi\cos x + \sin 1000\pi\sin x &= \cos x\\ 1\cos x + 0\sin x &= \cos x\\ \cos x &= \cos x\end{aligned}$$

b)
$$\begin{aligned}\tan(x + 51\pi) &= \tan x\\ \frac{\tan x + \tan 51\pi}{1 - \tan x\tan\pi} &= \tan x\\ \frac{\tan x + 0}{1 - \tan x\cdot 0} &= \tan x\\ \tan x &= \tan x\end{aligned}$$

c)
$$\begin{aligned}\sin\left(\frac{\pi}{2} + x\right) &= \cos x\\ \sin\frac{\pi}{2}\cos x + \sin x\cos\frac{\pi}{2} &= \cos x\\ 1\cos x + \sin x\cdot 0 &= \cos x\\ \cos x &= \cos x\end{aligned}$$

d)
$$\begin{aligned}\tan(211\pi - x) &= -\tan x\\ \frac{\tan 211\pi - \tan x}{1 + \tan 211\pi\tan x} &= -\tan x\\ \frac{0 - \tan x}{1 + 0} &= -\tan x\\ -\tan x &= -\tan x\end{aligned}$$

e)
$$\begin{aligned}\cos\left(\frac{3\pi}{2} + x\right) &= \sin x\\ \cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x &= \sin x\\ 0\cos x - -1\sin x &= \sin x\\ \sin x &= \sin x\end{aligned}$$

f)
$$\begin{aligned}\sin(x - 101\pi) &= -\sin x\\ \sin x\cos 101\pi - \sin 101\pi\cos x &= -\sin x\\ -1\sin x - 0\cos x &= -\sin x\\ -\sin x &= -\sin x\end{aligned}$$

18. a)
$$\frac{\cot x}{\operatorname{cosec} x - \sin x} = \sec x$$
$$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \sin x} = \sec x$$
$$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}} = \sec x$$
$$\frac{\frac{\cos x}{\sin x}}{\frac{1 - \sin^2 x}{\sin x}} = \sec x$$
$$\frac{\frac{\cos x}{\sin x}}{\frac{\cos^2 x}{\sin x}} = \sec x$$
$$\frac{\cos x}{\sin x} \times \frac{\sin x}{\cos^2 x} = \sec x$$
$$\frac{1}{\cos x} = \sec x$$
$$\sec x = \sec x$$

b)
$$\frac{\cos^2 x}{1 - \cos^2 x} + \sin^2 x + \cos^2 x = \operatorname{cosec}^2 x$$
$$\frac{\cos^2 x}{\sin^2 x} + \sin^2 x + \cos^2 x = \operatorname{cosec}^2 x$$
$$\cot^2 x + \sin^2 x + \cos^2 x = \operatorname{cosec}^2 x$$
$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$
$$\operatorname{cosec}^2 x = \operatorname{cosec}^2 x$$

c)
$$\frac{\sin x \sec x}{\operatorname{cosec} x \sqrt{1 - \sin^2 x}} = \tan^2 x$$
$$\frac{\sin x \sec x}{\operatorname{cosec} x \sqrt{\cos^2 x}} = \tan^2 x$$
$$\frac{\sin x \sec x}{\operatorname{cosec} x \cos x} = \tan^2 x$$
$$\frac{\sin x \frac{1}{\cos x}}{\frac{1}{\sin x} \cos x} = \tan^2 x$$
$$\frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \tan^2 x$$
$$\frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \tan^2 x$$
$$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$
$$\tan^2 x = \tan^2 x$$

d)
$$\cos^2 x \tan^2 x + \cos^2 x = 1$$
$$\cos^2 x \frac{\sin^2 x}{\cos^2 x} + \cos^2 x = 1$$
$$\sin^2 x + \cos^2 x = 1$$
$$1 = 1$$

e)
$$\frac{\cos^2 x \tan x}{\cot x} = \sin^2 x$$
$$\frac{\cos^2 x \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \sin^2 x$$
$$\cos^2 x \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \sin^2 x$$
$$\sin^2 x = \sin^2 x$$

f)
$$\sin^2 x \cot^2 x + \sin^2 x = 1$$
$$\sin^2 x \frac{\cos^2 x}{\sin^2 x} + \sin^2 x = 1$$
$$\cos^2 x + \sin^2 x = 1$$
$$1 = 1$$

g)
$$\frac{\sin x}{\sec x} \operatorname{cosec} x = \cos x$$
$$\frac{\sin x}{\sec x} \frac{1}{\sin x} = \cos x$$
$$\frac{1}{\sec x} = \cos x$$
$$\frac{1}{\frac{1}{\cos x}} = \cos x$$
$$\cos x = \cos x$$

h)
$$(1 - \cos^2 x) \cot^2 x = \cos^2 x$$
$$\sin^2 x \cot^2 x = \cos^2 x$$
$$\sin^2 x \frac{\cos^2 x}{\sin^2 x} = \cos^2 x$$
$$\cos^2 x = \cos^2 x$$

19. $a = \frac{2\pi}{3}$ and $b = \tan^{-1}\frac{4}{5} = 0.6435011088$

a)
$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$
$$\tan(a - b) = \frac{-\sqrt{3} - \frac{3}{4}}{1 + -\sqrt{3} \cdot \frac{3}{4}}$$
$$\tan(a - b) = \frac{48 + 25\sqrt{3}}{11}$$

b)
$$\sin(a + b) = \sin a \cos b + \sin b \cos a$$
$$\sin(a + b) = \frac{\sqrt{3}}{2} \cdot 0.8 + 0.6 \cdot -\frac{1}{2}$$
$$\sin(a + b) = \frac{2\sqrt{3}}{5} + -\frac{3}{10}$$
$$\sin(a + b) = \frac{-3 + 4\sqrt{3}}{10}$$

c)
$$\cos(b - a) = \cos b \cos a + \sin b \sin a$$
$$\cos(b - a) = 0.8 \cdot -\frac{1}{2} + 0.6 \cdot \frac{\sqrt{3}}{2}$$
$$\cos(b - a) = -\frac{2}{5} + \frac{3\sqrt{3}}{10}$$
$$\cos(b - a) = \frac{-4 + 3\sqrt{3}}{10}$$

d)
$$\tan 2a = \tan(a + a)$$
$$\tan 2a = \frac{\tan a + \tan a}{1 - \tan a \tan a}$$
$$\tan 2a = \frac{-\sqrt{3} + -\sqrt{3}}{1 - (-\sqrt{3})(-\sqrt{3})}$$
$$\tan 2a = \frac{-2\sqrt{3}}{-2}$$
$$\tan 2a = \sqrt{3}$$

20. The altitude (in km) of the rocket corresponds to the tangent of the angle of elevation. You must show that the statement "When the measure of the angle of elevation O is doubled, the tangent of this angle doubles." To show that the statement is wrong, you must find a counter-example.

Let the angle of elevation be $\frac{\pi}{6}$ rad. The tangent of $\frac{\pi}{6}$ is $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$. The double of $\frac{\pi}{6}$ rad is $\frac{\pi}{3}$ rad.

The tangent of $\frac{\pi}{3}$ is $\tan\frac{\pi}{3} = \sqrt{3}$. Since $\sqrt{3}$ is not double $\frac{\sqrt{3}}{3}$, but actually triple, you can confirm that the statement is false.

SPECIAL FEATURES 5

Chronicle of the past

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1. a) $\frac{2-\sqrt{3}}{2}$ **b)** $\frac{3}{2}$ **c)** $\frac{2-\sqrt{2}}{2}$

2. $\cos\left(\frac{\pi}{2} - a\right) = \cos\frac{\pi}{2}\cos a + \sin\frac{\pi}{2}\sin a$

$\cos\left(\frac{\pi}{2} - a\right) = 0\cos a + 1\sin a$

$\cos\left(\frac{\pi}{2} - a\right) = \sin a$

3. a) $\frac{1}{2}$ **b)** $\frac{12}{17}$ **c)** $\frac{32}{37}$

4. $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x = \cos^2 x - \cos 2x$

$\sin^2 x = (1 - \sin^2 x) - \cos 2x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

In the workplace

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1.

Refraction index of Medium 1 (n_1)	Refraction index of Medium 2 (n_2)	Measure of the angle of incidence (rad)	Measure of the angle of refraction (rad)
1.46	1.001	$\frac{\pi}{18}$	**≈ 0.2561**
2.01	1.02	**≈ 0.1317**	$\frac{\pi}{12}$
3.26	**≈ 2.54**	$\frac{\pi}{18}$	$\frac{\pi}{14}$
≈ 1.33	1	$\frac{\pi}{24}$	$\frac{\pi}{18}$

2. a) The phenomenon of total reflection is produced when the angle of refraction is obtuse. The critical angle is therefore the angle of incidence that generates an angle of refraction of $\frac{\pi}{2}$ rad. Therefore, once the critical angle is reached, you know that $n_1 \times \sin\theta_1 = n_2 \times \sin\frac{\pi}{2}$.

Since $\sin\frac{\pi}{2} = 1$, you obtain rule $n_1 \times \sin\theta_1 = n_2$.

By isolating $\sin\theta_1$ you obtain $\sin\theta_1 = \frac{n_2}{n_1}$, which brings you to say that $\theta_1 = \arcsin\frac{n_2}{n_1}$.

b) The domain of arcsin is limited to interval [-1, 1]. In the case of problems of refraction, only the angles that are not oriented are used. Therefore, their value is always greater than 0. Thus, for your needs, the domain of arcsin becomes [0, 1]. In order for $\frac{n_2}{n_1}$ to be included in this interval, you must have $0 < \frac{n_2}{n_1} < 1$.

By solving this inequality, you obtain $n_2 > 0$ and $n_2 < n_1$.

3. a) ≈ 0.7484 rad **b)** ≈ 0.68 rad **c)** ≈ 0.67 rad **d)** ≈ 0.43 rad

Overview

1. a) $30° = \frac{\pi}{6}$ rad b) $\frac{35\pi}{18}$ rad $= \mathbf{350°}$ c) $75° = \frac{5\pi}{12}$ rad

 d) $27° = \frac{3\pi}{20}$ rad e) $-\frac{5\pi}{8}$ rad $= \mathbf{-112.5°}$ f) $270° = \frac{3\pi}{2}$ rad

 g) $-36° = -\frac{\pi}{5}$ rad h) $\frac{8\pi}{5}$ rad $= \mathbf{288°}$ i) $-78° = -\frac{13\pi}{30}$ rad

2. a) 1st quadrant. b) 3rd quadrant. c) 2nd quadrant. d) 1st quadrant.
 e) 4th quadrant. f) 4th quadrant. g) 1st quadrant. h) 2nd quadrant.

3. a) 1) Domain: $\mathbb{R}$ 2) Range: $[1, 7]$ 3) Period: 10
 b) 1) Domain: $\mathbb{R} \setminus \left\{x \in \mathbb{R} \mid x = n\frac{\pi}{2}, n \in \mathbb{Z}\right\}$ 2) Range: $\mathbb{R}$ 3) Period: π
 c) 1) Domain: $\mathbb{R}$ 2) Range: $[-7, 3]$ 3) Period: $\frac{2\pi}{3}$
 d) 1) Domain: $\mathbb{R}$ 2) Range: $[17, 19]$ 3) Period: 4
 e) 1) Domain: $\mathbb{R}$ 2) Range: $[-6, 8]$ 3) Period: π
 f) 1) Domain: $\mathbb{R} \setminus \left\{x \in \mathbb{R} \mid x = \frac{11}{6} + \frac{n}{3}, n \in \mathbb{Z}\right\}$ 2) Range: $\mathbb{R}$ 3) Period: $\frac{1}{3}$

4. a)
$$\frac{(\sin x \cot x)^2}{1 + \sin x} = 1 - \sin x$$
$$\frac{\sin^2 x \frac{\cos^2 x}{\sin^2 x}}{1 + \sin x} = 1 - \sin x$$
$$\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$$
$$\frac{1 - \sin^2 x}{1 + \sin x} = 1 - \sin x$$
$$\frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} = 1 - \sin x$$
$$1 - \sin x = 1 - \sin x$$

b)
$$1 - 2\sin^2 x = 2\cos^2 x - 1$$
$$1 - \sin^2 x - \sin^2 x = 2\cos^2 x - 1$$
$$\cos^2 x - \sin^2 x = 2\cos^2 x - 1$$
$$\cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$
$$\cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$$
$$2\cos^2 x - 1 = 2\cos^2 x - 1$$

c)
$$\tan x + \cot x = \sec x \operatorname{cosec} x$$
$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \operatorname{cosec} x$$
$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \sec x \operatorname{cosec} x$$
$$\frac{1}{\cos x \sin x} = \sec x \operatorname{cosec} x$$
$$\frac{1}{\cos x}\frac{1}{\sin x} = \sec x \operatorname{cosec} x$$
$$\sec x \operatorname{cosec} x = \sec x \operatorname{cosec} x$$

d)
$$(\tan x - \cot x)\sin x \cos x = \sin^2 x - \cos^2 x$$
$$\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right)\sin x \cos x = \sin^2 x - \cos^2 x$$
$$\left(\frac{\sin^2 x - \cos^2 x}{\cos x \sin x}\right)\sin x \cos x = \sin^2 x - \cos^2 x$$
$$\sin^2 x - \cos^2 x = \sin^2 x - \cos^2 x$$

e)
$$\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x}{\sin x + \cos x} \times \frac{\sec x}{\sec x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x \sec x}{(\sin x + \cos x)\sec x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x \frac{1}{\cos x}}{(\sin x + \cos x)\frac{1}{\cos x}} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\tan x}{1 + \tan x} = \frac{\tan x}{1 + \tan x}$$

f)
$$\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2\sec x$$
$$\frac{\cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} + \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} = 2\sec x$$
$$\frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{1 - \sin^2 x} = 2\sec x$$
$$\frac{\cos x + \cos x}{\cos^2 x} = 2\sec x$$
$$\frac{2\cos x}{\cos^2 x} = 2\sec x$$
$$2\frac{1}{\cos x} = 2\sec x$$
$$2\sec x = 2\sec x$$

g)
$$(1+\tan x)^2+(1-\tan x)^2=2\sec^2 x$$
$$1+2\tan x+\tan^2 x+1-2\tan x+\tan^2 x=2\sec^2 x$$
$$1+\tan^2 x+1+\tan^2 x=2\sec^2 x$$
$$\sec^2 x+\sec^2 x=2\sec^2 x$$
$$2\sec^2 x=2\sec^2 x$$

h)
$$\frac{\tan^2 x}{1+\tan^2 x}\times\frac{1+\cot^2 x}{\cot^2 x}=\sin^2 x\sec^2 x$$
$$\frac{\tan^2 x}{\sec^2 x}\times\frac{\operatorname{cosec}^2 x}{\cot^2 x}=\sin^2 x\sec^2 x$$
$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}\times\frac{\frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}}=\sin^2 x\sec^2 x$$
$$\frac{\sin^2 x}{\cos^2 x}\times\cos^2 x\times\frac{1}{\sin^2 x}\times\frac{\sin^2 x}{\cos^2 x}=\sin^2 x\sec^2 x$$
$$\sin^2 x\frac{1}{\cos^2 x}=\sin^2 x\sec^2 x$$
$$\sin^2 x\sec^2 x=\sin^2 x\sec^2 x$$

Overview (*cont'd*)

5. **a)** $\left\{x\in\mathbb{R}\mid x=\frac{10}{3}+4n \vee x=\frac{14}{3}+4n,\ n\in\mathbb{Z}\right\}$ **b)** $\{x\in\mathbb{R}\mid x=2+8n,\ n\in\mathbb{Z}\}$

c) $\{x\in\mathbb{R}\mid x=6n,\ n\in\mathbb{Z}\}$ **d)** $\left\{x\in\mathbb{R}\mid x=\frac{2}{9}+\frac{2}{3}n \vee x=\frac{4}{9}+\frac{2}{3}n,\ n\in\mathbb{Z}\right\}$

e) $\left\{x\in\mathbb{R}\mid x=\frac{\pi+4}{4}+n\pi,\ n\in\mathbb{Z}\right\}$ **f)** $\left\{x\in\mathbb{R}\mid x=\frac{15\pi}{16}+n\pi,\ n\in\mathbb{Z}\right\}$

6. **a)** $f(x)=-\sin x$ or $f(x)=\cos\pi\left(x+\frac{\pi}{2}\right)$ **b)** $f(x)=-3\cos x$ or $f(x)=3\cos(x+\pi)$

c) $f(x)=2\sin 3x+1$ or $f(x)=2\cos 3\left(x-\frac{\pi}{6}\right)+1$ **d)** $f(x)=\sin\pi x-1$ or $f(x)=\cos\pi\left(x-\frac{1}{2}\right)-1$

e) $f(x)=\sin x$ or $f(x)=\cos\left(x-\frac{\pi}{2}\right)$ **f)** $f(x)=-10\sin x-20$ or $f(x)=10\cos\left(x+\frac{\pi}{2}\right)-20$

Overview (*cont'd*)

7. **a)** No solution.

b) $\left\{-\frac{25\pi}{9},-\frac{23\pi}{9},-\frac{19\pi}{9},-\frac{17\pi}{9},-\frac{13\pi}{9},-\frac{11\pi}{9},-\frac{5\pi}{9},-\frac{\pi}{9},\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{17\pi}{9},\frac{19\pi}{9},\frac{23\pi}{9},\frac{25\pi}{9}\right\}$

c) $-\frac{5\pi}{4}$

d) No solution.

e) $\left\{-\frac{71\pi}{24},-\frac{67\pi}{24},-\frac{47\pi}{24},-\frac{43\pi}{24},-\frac{23\pi}{24},-\frac{19\pi}{24},\frac{\pi}{24},\frac{5\pi}{24},\frac{25\pi}{24},\frac{29\pi}{24},\frac{49\pi}{24},\frac{53\pi}{24}\right\}$

f) $\left\{-\frac{11\pi}{4},-\frac{9\pi}{4},-\frac{7\pi}{4},-\frac{5\pi}{4},-\frac{3\pi}{4},-\frac{\pi}{4},\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4}\right\}$

8. **a)** $\ldots\cup\left]-\frac{35\pi}{4},-\frac{25\pi}{4}\right[\cup\left]\frac{5\pi}{4},\frac{15\pi}{4}\right[\cup\left]\frac{45\pi}{4},\frac{55\pi}{4}\right[\cup\ldots$ **b)** No solution.

c) $\ldots\cup\,]-1,0[\,\cup\,]3,4[\,\cup\,]7,8[\,\cup\ldots$ **d)** $\ldots\cup\left[-\frac{7\pi}{4},-\frac{\pi}{4}\right]\cup\left[\frac{\pi}{4},\frac{7\pi}{4}\right]\cup\left[\frac{9\pi}{4},\frac{15\pi}{4}\right]\cup\ldots$

e) No solution. **f)** $\ldots\cup\left[\frac{\pi}{3},0\right[\cup\left[\frac{5\pi}{6},\frac{\pi}{2}\right[\cup\left[\frac{4\pi}{3},\pi\right[\cup\ldots$

9. **a)** $\frac{5}{13}$ **b)** $\frac{13}{12}$ **c)** $\frac{5}{12}$ **d)** $\frac{13}{5}$ **e)** $\frac{12}{5}$

10. **a)** $-\frac{1}{2}$ **b)** $\frac{\sqrt{3}}{3}$ **c)** $-\frac{2\sqrt{3}}{3}$ **d)** $-\frac{\sqrt{3}}{2}$ **e)** -2

11. **a)** $\frac{\sqrt{3}}{2}$ **b)** $\frac{\pi}{3}$ **c)** 1 **d)** $\frac{\sqrt{2}}{2}$ **e)** $\frac{\pi}{2}+n\pi$ where $n\in\mathbb{Z}$ **f)** $-\frac{\pi}{4}$

12. The time taken for the wheel to make one complete turn corresponds to the period of the rule $h=14\sin 15(t-15)+8$; therefore $\frac{2\pi}{15}$ s, which is approximately 0.42 s.

Overview (*cont'd*)

13. **a)** $f(x)=2\tan\frac{1}{2}x-2$ **b)** $f(x)=-\frac{1}{2}\tan\left(x-\frac{\pi}{2}\right)+\frac{1}{2}$ **c)** $f(x)=\frac{1}{2}\tan\pi x+0.5$

d) $f(x)=-2\tan\frac{\pi}{2}x$ **e)** $f(x)=4\tan\frac{\pi}{2}x$ **f)** $f(x)=\tan\frac{1}{2}(x+\pi)+2$

14. **a)** $x \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ **b)** $x \in \left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$ **c)** $x \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

d) $x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **e)** $x = \frac{3\pi}{2}$ **f)** $x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

Overview (*cont'd*)

15. **a)** $\frac{2\sqrt{10}}{7}$ **b)** $\frac{2\sqrt{10}}{3}$ **c)** $\frac{7}{3}$ **d)** $\frac{7\sqrt{10}}{20}$ **e)** $\frac{3\sqrt{10}}{20}$ **f)** $\frac{3}{4}$

g) $\frac{\sqrt{7}}{3}$ **h)** $\frac{4}{3}$ **i)** $\frac{4\sqrt{7}}{7}$ **j)** $\frac{3\sqrt{7}}{7}$ **k)** $\frac{6\sqrt{10} + 3\sqrt{7}}{28}$ **l)** $\frac{9 + 2\sqrt{70}}{28}$

16. **a)** $f(x) \geq 0$ if $x \in \left[-2, -\frac{5}{3}\right] \cup \left[-\frac{1}{3}, \frac{1}{3}\right] \cup \left[\frac{5}{3}, 2\right]$,
$f(x) \leq 0$ if $x \in \left[-\frac{5}{3}, -\frac{1}{3}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right]$.

b) $f(x) \geq 0$ if $x \in \left[-\frac{23\pi}{6}, -\frac{7\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{17\pi}{6}\right]$,
$f(x) \leq 0$ if $x \in \left[-4\pi, -\frac{23\pi}{6}\right] \cup \left[-\frac{7\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{17\pi}{6}, 4\pi\right]$.

c) $f(x) \geq 0$ if $x \in \left[-4\pi, -\frac{29\pi}{12}\right] \cup \left[-\frac{13\pi}{12}, \frac{19\pi}{12}\right] \cup \left[\frac{35\pi}{12}, 4\pi\right]$,
$f(x) \leq 0$ if $x \in \left[-\frac{29\pi}{12}, -\frac{13\pi}{12}\right] \cup \left[\frac{19\pi}{12}, \frac{35\pi}{12}\right]$.

d) $f(x) \geq 0$ if $x \in \left[-\frac{7\pi}{4}, -\pi\right[\cup \left[-\frac{3\pi}{4}, 0\right[\cup \left[\frac{\pi}{4}, \pi\right[\cup \left[\frac{5\pi}{4}, 2\pi\right[$,
$f(x) \leq 0$ if $x \in \left]-2\pi, -\frac{7\pi}{4}\right] \cup \left]-\pi, -\frac{3\pi}{4}\right] \cup \left]0, \frac{\pi}{4}\right] \cup \left]\pi, \frac{5\pi}{4}\right]$.

e) $f(x) \geq 0$ if $x \in \left]-3\pi, -\frac{5\pi}{2}\right] \cup \left]-\pi, -\frac{\pi}{2}\right] \cup \left]\pi, \frac{3\pi}{2}\right] \cup \left]3\pi, \frac{7\pi}{2}\right]$,
$f(x) \leq 0$ if $x \in [-4\pi, -3\pi[\cup \left[-\frac{5\pi}{2}, -\pi\right[\cup \left[-\frac{\pi}{2}, \pi\right[\cup \left[\frac{3\pi}{2}, 3\pi\right[\cup \left[\frac{7\pi}{2}, 4\pi\right]$.

f) $f(x) \geq 0$ if $x \in \left[-2, -\frac{11}{6}\right[\cup \left[-\frac{3}{2}, -\frac{7}{6}\right[\cup \left[-\frac{5}{6}, -\frac{1}{2}\right[\cup \left[-\frac{1}{6}, \frac{1}{6}\right[\cup \left[\frac{1}{2}, \frac{5}{6}\right[\cup \left[\frac{7}{6}, \frac{3}{2}\right[\cup \left[\frac{11}{6}, 2\right]$,
$f(x) \leq 0$ if $x \in \left]-\frac{11}{6}, -\frac{3}{2}\right] \cup \left]-\frac{7}{6}, -\frac{5}{6}\right] \cup \left[-\frac{1}{2}, -\frac{1}{6}\right[\cup \left]\frac{1}{6}, \frac{1}{2}\right] \cup \left]\frac{5}{6}, \frac{7}{6}\right] \cup \left]\frac{3}{2}, \frac{11}{6}\right]$.

Overview (*cont'd*)

17. **a)** $\left\{x \in \mathbb{R} \mid x = n\pi \vee x = \frac{2\pi}{3} + 2n\pi \vee x = \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}\right\}$

b) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

c) $\{x \in \mathbb{R} \mid x \approx 0.6749 + 2n\pi \vee x \approx -0.6749 + 2n\pi, n \in \mathbb{Z}\}$

d) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

e) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

f) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

g) $\left\{x \in \mathbb{R} \mid x = \frac{\pi}{3} + n\pi \vee x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}\right\}$

h) $\{x \in \mathbb{R} \mid x \approx 1.9979 + 2n\pi \vee x \approx -1.9979 + 2n\pi, n \in \mathbb{Z}\}$

i) $\left\{x \in \mathbb{R} \mid x = \frac{2\pi}{3} + 2n\pi \vee x = \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}\right\}$

18. Circular arc: 3450 km; Circumference of circle: $(2\pi \cdot 1520)$ km; therefore the length of the circular arc is $\frac{345}{152}$ rad.

Overview (*cont'd*)

19. **a)** $f(x) = 3\cos\pi(x - 1) + 1.5$ or $f(x) = 3\sin\pi\left(x - \frac{1}{2}\right) + 1.5$ **b) 1)** $\{1, 3, 5\}$ **2)** $\left\{\frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3}, \frac{13}{3}, \frac{17}{3}\right\}$

20. During the simulations, an explosion occurs at the following moments: 0.75 s, 2.75 s, 4.75 s, 6.75 s, 8.75 s, 10.75 s, 12.75 s and 14.75 s.

21. The device is saturated at approximately 0.52 s, 0.62 s, 0.72 s, 0.82 s and 0.92 s.

22. The length of this belt is approximately 43.62 cm.

Overview (*cont'd*) **Page 155**

23. You must find the zero of $y = \cos x$ for $x \in [0, 2]$, which is $x = \frac{\pi}{2} \approx 1.57$. The length A of the blade is therefore approximately 1.57 mm.

Solve $\tan x = \cos x$ for $x \in [0, 2]$.

You find $x \approx 0.67$.

$y \approx \tan 0.67$

$y \approx 0.79$

The height B is approximately 0.79 mm.

24. You must find the zeros of $h = 250 \cos \frac{\pi t}{15} + 125$ for $t \in [0, 30]$ s.

$t = 20$ s and $t = 10$ s. Therefore 20 s − 10 s = 10 s.

The water bomber takes 10 s to fill its tank.

Overview (*cont'd*) **Page 156**

25. a) 1) ≈ 194.67 m **2)** ≈ 169.64 m

b) The general formula is $P = \frac{v\cos\theta}{g}(v\sin\theta + \sqrt{(v\sin\theta)^2 + 2gy_0})$.

By replacing y_0 by 0, you obtain: $P = \frac{v\cos\theta}{g}(v\sin\theta + \sqrt{(v\sin\theta)^2 + 2g \times 0})$

By reducing this expression, you get:

$$P = \frac{v\cos\theta}{g}(v\sin\theta + \sqrt{(v\sin\theta)^2})$$
$$= \frac{v\cos\theta}{g}(v\sin\theta + v\sin\theta)$$
$$= \frac{v\cos\theta}{g}2v\sin\theta$$
$$= \frac{2v^2\sin\theta\cos\theta}{g}$$

Since $2\sin\theta\cos\theta = \sin 2\theta$, then $P = \frac{v^2\sin 2\theta}{g}$.

c) At a velocity of approximately 44.29 m/s.

d) The angle of projection must be approximately 0.63 rad.

e) At a velocity of approximately 98.43 m/s.

Overview (*cont'd*) **Page 157**

26. a) 1) $P = 30\cos\frac{\pi}{4}(x - 4) + 210$ or $P = 30\sin\frac{\pi}{4}(x - 2) + 210$ where P represents the population of deer and x represents the time elapsed since 2000 (in years).

2) $P = 4\cos\frac{\pi}{4}(x - 5) + 20$ or $P = 4\sin\frac{\pi}{4}(x - 3) + 20$ where P represents the population of coyotes and x represents the time elapsed since 2000 (in years).

b) 1) In 2021, the population of deer would be approximately 231 animals.

2) In 2027, the population of coyotes would be 20 animals.

c) 1) From September 1, 2010 to April 30, 2013; from September 1, 2018 to April 30, 2021 and from September 1, 2026 to April 30, 2029.

2) Since 24 is the maximum number of coyotes, the population of coyotes is always less than or equal to 24 animals.

Bank of problems **Page 158**

1. The length of segment AC corresponds to one times the tangent of angle ABC.

If angle ABC measures $\frac{\pi}{12}$ rad, segment AC measures m $\overline{AC} = \tan\frac{\pi}{12} = 2 - \sqrt{3}$, which is approximately 0.2679.

If the measure of angle ABC doubles, this measure would be $\frac{\pi}{6}$ rad. In this case, segment AC measures m $\overline{AC} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$, which is approximately 0.5774.

In this particular case, once the measure of angle AC doubles, the length of segment AC is found multiplied by $1 + \sqrt{3}$, which is approximately 2.73.

2. By using a cosine instead to express the rule of this function, you obtain the rule $f(x) = {}^{-}2\cos\frac{\pi}{2}x + 1$. Nelly-Ann's statement is false.

3. *Several answers possible. Example:*

   You can separate the area below the curve into little rectangles and triangles, calculate the areas of each part and add them.

Bank of problems (*cont'd*)

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4. Statement C is the most precise. Statement A is false because by not limiting the domain of the function, its inverse would allow a value of the independant variable to be associated to more than one value of the dependent variable. The same thing is produced in the case of Statement B, if both chosen asymptotes are not consecutive.

5.

Tide table		Tide table	
Date	**Not recommended**	**Date**	**Not recommended**
July 1	From 1:20 a.m. to 4:24 a.m. From 12:50 p.m. to 3:54 p.m.	**July 15**	From 2:14 a.m. to 5:30 a.m. From 1:44 p.m. to 5:00 p.m.
July 2	From 12:20 a.m. to 3:24 a.m. From 11:50 a.m. to 2:54 p.m. From 11:20 p.m. to midnight.	**July 16**	From 1:14 a.m. to 4:30 a.m. From 12:44 p.m. to 4:00 p.m.
July 3	From midnight to 2:24 a.m. From 10:50 a.m. to 1:54 p.m. From 10:20 p.m. to midnight.	**July 17**	From 12:14 a.m. to 3:30 a.m. From 11:44 a.m. to 3:00 p.m. From 11:14 p.m. to midnight.
July 4	From midnight to 1:24 a.m. From 9:50 a.m. to 12:54 p.m. From 9:20 p.m. to midnight.	**July 18**	From midnight to 2:30 a.m. From 10:44 a.m. to 2:00 p.m. From 10:14 p.m. to midnight.
July 5	From midnight to 12:24 a.m. From 8:50 a.m. to 11:54 a.m. From 8:20 p.m. to 11:24 p.m.	**July 19**	From midnight to 1:30 a.m. From 9:44 a.m. to 1:00 p.m. From 9:14 p.m. to midnight.
July 6	From 7:50 a.m. to 10:54 a.m. From 7:20 p.m. to 10:24 p.m.	**July 20**	From midnight to 12:30 a.m. From 8:44 a.m. to 12:00 p.m. From 8:14 p.m. to 11:30 p.m.
July 7	From 6:50 a.m. to 9:54 a.m. From 6:20 p.m. to 9:24 p.m.	**July 21**	From 7:44 a.m. to 11:00 a.m. From 7:14 p.m. to 10:30 p.m.
July 8	From 5:48 a.m. to 9:03 a.m. From 5:33 p.m. to 8:48 p.m.	**July 22**	From 6:45 a.m. to 9:52 a.m. From 6:00 p.m. to 9:07 p.m.
July 9	From 5:18 a.m. to 8:33 a.m. From 5:03 p.m. to 8:18 p.m.	**July 23**	From 5:15 a.m. to 8:22 a.m. From 4:30 p.m. to 7:37 p.m.
July 10	From 4:48 a.m. to 8:03 a.m. From 4:33 p.m. to 7:48 p.m.	**July 24**	From 3:45 a.m. to 6:52 a.m. From 3:00 p.m. to 6:07 p.m.
July 11	From 4:18 a.m. to 7:33 a.m. From 5:03 p.m. to 7:18 p.m.	**July 25**	From 2:15 a.m. to 5:22 a.m. From 1:30 p.m. to 4:37 p.m.
July 12	From 3:48 a.m. to 7:03 a.m. From 3:33 p.m. to 6:48 p.m.	**July 26**	From 12:45 a.m. to 3:52 a.m. From 12:00 p.m. to 3:07 p.m. From 11:15 p.m. to midnight.
July 13	From 3:18 a.m. to 6:33 a.m. From 3:03 p.m. to 6:18 p.m.	**July 27**	From midnight to 2:22 a.m. From 10:30 a.m. to 1:37 p.m. From 9:45 p.m. to midnight.
July 14	From 2:48 a.m. to 6:03 a.m. From 2:33 p.m. to 5:48 p.m.	**July 28**	From midnight to 12:52 a.m. From 9:00 a.m. to 12:07 p.m. From 8:15 p.m. to 11:22 p.m.

Bank of problems (*cont'd*)

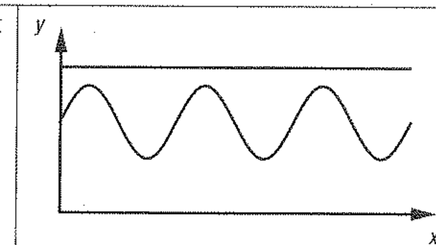
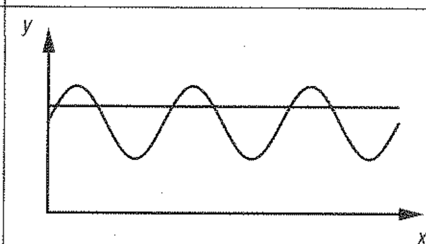
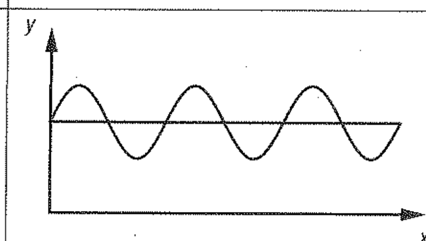
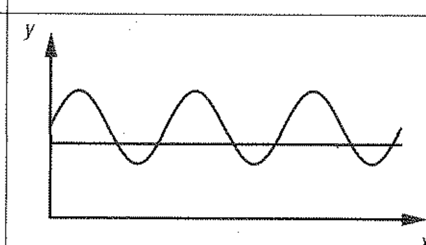
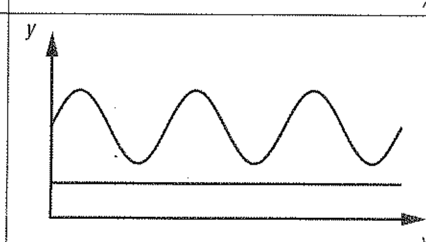
6. A cycle corresponds to a period. 20 cycles/s means that there is a cycle every $\frac{1}{20}$ s; however, 20 000 cycles/s means that there is a cycle every $\frac{1}{20\,000}$ s. Since $P = \frac{2\pi}{|b|}$, parameter **b** can be any value found between 40π and $40\,000\pi$.

7. The computer is at risk of crashing at the following moments during the update: from 2 min to 4 min, from 8 min to 10 min, from 14 min to 16 min, from 20 min to 22 min, from 26 min to 28 min, from 32 min to 34 min, from 38 min to 40 min, from 44 min to 46 min, from 50 min to 52 min and from 56 min to 58 min.

Bank of problems (*cont'd*)

8. This person would have fever every 4 h, in periods of 1 h 20 min, in other words: from 0 h 20 min to 1 h 40 min, from 4 h 20 min to 5 h 40 min, from 8 h 20 min to 9 h 40 min, from 12 h 20 min to 13 h 40 min , from 16 h 20 min to 17 h 40 min and from 20 h 20 min to 21 h 40 min.

9. You must take note that when the pressure of the device is less than the ambient atmospheric pressure, the lungs of the person kept on life-support are filled with air and then emptied once the pressure of the device is higher than the ambient atmospheric pressure. In order for the person to be able to breathe, there must be equilibrium between the time when the lungs are filled with air and the time when the lungs eject the air.

In each of the graphs below, the pressure of the device is represented by the curve; however, the ambient atmospheric pressure is represented by the line.	
In the situation represented in the adjacent graph, the pressure of the device is always less than the ambient atmospheric pressure. The lungs never eject air, and the person suffocates.	
In the situation represented in the adjacent graph, the pressure of the device is more often less than the ambient atmospheric pressure. The quantity of air inhaled by the person is greater than the quantity of air ejected. The lungs can inflate to the point where the person can be at risk of suffocating.	
In the situation represented in the adjacent graph, the pressure of the device is as often inferior as superior to the ambient atmospheric pressure. The quantity of air inhaled by the person is equal to the quantity of air ejected. The person breathes adequately.	
In the situation represented in the adjacent graph, the pressure of the device is more often greater than the ambient atmospheric pressure. The quantity of air inhaled by the person is less than the quantity of air rejected. The person is at risk of suffocating.	
In the situation represented in the adjacent graph, the pressure of the device is always greater than the ambient atmospheric pressure. The quantity of air inhaled by the person is zero and the person is at risk of suffocating.	

In order for there to be some equilibrium, the ambient atmospheric pressure must pass through the points of inflection of the sinusoidal function represented, which corresponds, among others, to point (h, k). Based on the rule $P = \sin 12\pi x + c$, k is equal to c. The ambient atmospheric pressure must therefore correspond to parameter **c**.

