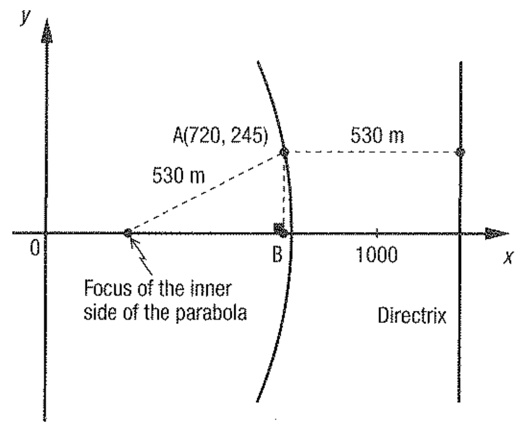
LES 11 | Noise pollution

Page 245

The following is an example of a procedure that allows you to complete the LES.

Zone ①

- Determine the coordinates of the focus and the coordinates of the vertex of the parabola. The equation is in the form $y^2 = 4c(x - h)$. In addition, it is known that the distance between the directrix and a point on the curve is equal to the distance of this point to the focus of the parabola. Therefore, the adjacent graphical representation is obtained.



  The equation of the directrix is $x = 1250$ since it is at a distance of 530 m from point A.

  Based on the Pythagorean theorem, it is determined that

  $d(F, B) = \sqrt{530^2 - 245^2} \approx 470$ m.

  Therefore the distance between the focus and directrix is approximately 470 m + 530 m which is approximately 1000 m.

  It can be deduced that the coordinates of the focus are ($\approx$ 250, 0) and that the coordinates of the vertex of the parabola are ($\approx$ 750, 0).

- Determine the inequality that corresponds to the elliptical region.

  The coordinates of the focus of the ellipse are (739.5, 0) and the coordinates of the vertex of the ellipse correspond to the vertex of the parabola ($\approx$ 750, 0). You can therefore deduce that the value of parameter **b** is approximately 125 because in an ellipse, the following relation exists: $b^2 + c^2 = a^2$. Therefore $b \approx \sqrt{750^2 - 739.5^2} \approx 125$.

  The value of parameter **a** is approximately 750 and the value of parameter **b** is approximately 125. Since the interior region is shaded, it is determined that the inequality of this region is $\frac{x^2}{750^2} + \frac{y^2}{125^2} \leq 1$.

  In conclusion, the sound waves are observed at the focus of the parabolic curve whose coordinates are ($\approx$ 250, 0) and the region considered at risk of noise pollution corresponds to the inequality $\frac{x^2}{750^2} + \frac{y^2}{125^2} \leq 1$.

Zone ②

- Determine the coordinates of the focus of the hyperbola.

  The equation of the hyperbola is in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$. The graphical representation of the curve allows you to deduce that the coordinates of the vertex of the curve are (0, 300) and that the curve passes through the point whose coordinates are (300, 375). By substituting these coordinates in the equation of the curve, you obtain:

  $$\frac{300^2}{a^2} - \frac{375^2}{300^2} = -1$$

  Determine the value of parameter **a**:

  $$\frac{300^2}{a^2} = -1 + \frac{375^2}{300^2}$$

  $$\frac{300^2}{a^2} = 0.5625$$

  $$a^2 = 160\ 000$$

  $$a = 400$$

  It is possible to determine the value of parameter **c** since in a hyperbola, the following relation exists $a^2 + b^2 = c^2$.
  You have $400^2 + 300^2 = c^2$; therefore, $c = 500$.

  The coordinates of the focus where the wave sounds are observed are (0, -500).

- Determine the inequality that corresponds to the circular region.

  Since the coordinates of the focus of the hyperbola are (0, -500) and that this point passes through the curve, you can deduce that the radius of the circle is 500 m. Since the interior region is shaded, it is determined that the inequality of this region is $x^2 + y^2 \leq 500^2$.

  In conclusion, the wave sounds are observed at the focus of the hyperbolic curve whose coordinates are (0, -500), and the region considered at risk of noise pollution corresponds to the inequality $x^2 + y^2 \leq 500^2$.

LES 12 | Elliptical cross-trainers

Page 246

The following is an example of a procedure that helps to create a ratio.

Evaluation of Model A

- Determine if the elliptical movement of this model is safe.

  The equation associated with the elliptical movement is in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

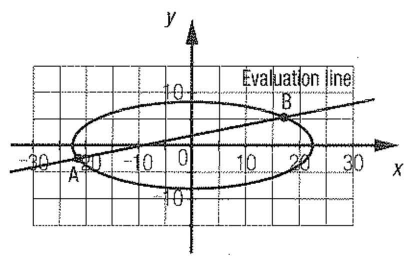
  It can be deduced that the value of parameter **a** is 22.5 since the length of the stride is 45 cm (major axis of the ellipse).

  It can be deduced that the value of parameter **b** is 8 since the height of the stride is 16 cm (minor axis of the ellipse).

  The equation associated with the elliptical movement is therefore $\frac{x^2}{22.5^2} + \frac{y^2}{8^2} = 1$.

  Determine the intersection points A and B in the graphical representation below.

Elliptical movement of feet



The line passes through points (-10, 0) and (15, 5), you can deduce its equation: $y = 0.2x + 2$.

Determine the intersection points by solving the system of equations:

$$\frac{x^2}{22.5^2} + \frac{y^2}{8^2} = 1$$
$$y = 0.2x + 2$$

Therefore, you have: $$\frac{x^2}{22.5^2} + \frac{(0.2x + 2)^2}{8^2} = 1$$

$$64x^2 + 20.25x^2 + 405x + 2025 = 32\,400$$
$$84.25x^2 + 405x - 30\,375 = 0$$

The values of x that satisfy this equation are $x_1 \approx 16.74$ and $x_2 \approx -21.54$.

For $x_1 \approx 16.74$, $y_1 \approx 5.35$ because $y \approx 0.2 \times 16.74 + 2$.

For $x_2 \approx -21.54$, $y_2 \approx -2.31$ because $y \approx 0.2 \times (-21.54) + 2$.

The coordinates of point A are ($\approx$ -21.54, $\approx$ -2.31), and the coordinates of point B are ($\approx$ 16.74, $\approx$ 5.35).

The distance between points A and B is $\sqrt{(16.74 + 21.54)^2 + (5.35 + 2.31)^2} \approx 39.04$ cm.

In conclusion, Model **A** is not safe since the distance between points A and B is less than 40 cm, the minimum required by the security standards.

Evaluation of Model B

- Determine if the elliptical movement of this model is safe.

  The equation associated with the elliptical movement is in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

  It can be deduced that the value of parameter **a** is 25 since the length of the stride is 50 cm (major axis of the ellipse). The focus associated with this curve is the same as the one in Model **A.** Since $8^2 + c^2 = 22.5^2$, the value of parameter **c** is therefore approximately 21.03 and since $b^2 + 21.03^2 \approx 25^2$, parameter **b** associated with Model **B** is approximately 13.52.

  The equation associated with the elliptical movement is therefore $\frac{x^2}{25^2} + \frac{y^2}{13.52^2} \approx 1$.

  Determine the intersection points of the evaluation line with this elliptical curve. These intersection points are determined by solving the system of equations:

  $\frac{x^2}{25^2} + \frac{y^2}{13.52^2} \approx 1$

  $y = 0.2x + 2$

  The coordinates of the intersection points are therefore ($\approx$ -24.42, $\approx$ -2.89) and ($\approx$ 22.02, $\approx$ 6.4). The distance between these points is approximately 47.36 cm.

  The elliptical movement of Model **B** respects the security standards.

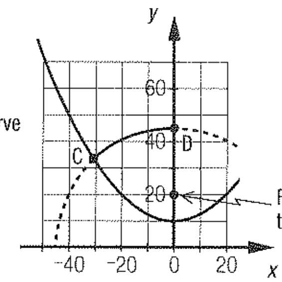
- Determine if the circular movement of this model is safe.

  Since the radius of the circle associated with this movement measures 45 cm, the equation of the circle centred at the origin associated with this movement is $x^2 + y^2 = 45^2$.

  It can be deduced that the equation of the evaluation parabola is $x^2 = 4 \times 10 \times (y - 10)$ since the coordinates of the vertex are (0, 10) and the coordinates of the focus are (0, 20).

  Determine the coordinates of the intersection point C in the graphical representation below.

Circular movement of the arms



The coordinates of the intersection point are determined by solving the system of equations:

$$x^2 + y^2 = 45^2$$
$$x^2 = 4 \times 10 \times (y - 10)$$

You therefore have: $40y - 400 + y^2 = 2025$

$$y^2 + 40y - 2425 = 0$$

The values of y that satisfy this equation are $y_1 \approx 33.15$ and $y_2 \approx -73.15$.

Reject the value of y_2 because it is not part of the solution.

For $y_1 \approx 33.15$, two x-values are possible:

$x^2 \approx 4 \times 10 \times (33.15 - 10)$

$x^2 \approx 926$

$x \approx \pm 30.43$

You therefore have: $x_1 \approx 30.43$ and $x_2 \approx -30.43$.

The only possible solution is the point whose coordinates are ($\approx$ -30.43, $\approx$ 33.15).

The coordinates of point C are ($\approx$ -30.43, $\approx$ 33.15) and the coordinates of point D are (0, 45). The distance between points C and D is approximately 32.66 cm because: $\sqrt{(0 + 30.43)^2 + (45 - 33.15)^2} \approx 32.66$.

The circular movement of Model **B** respects the security standards.

In conclusion, Model **B** is safe since the distance between points A and B of the elliptical movement, which is approximately 47.36 cm, is included in the security interval which is at least 40 cm and at most 55 cm. In addition, the distance between points C and D of the circular movement, which is approximately 32.66 cm, is included in the security interval which is at least 25 cm and at most 35 cm.

Prior learning 1

Page 170

a. **1)** (0, 10) **2)** (40, 30) **3)** (40, 10)

b. It is a right triangle.

c. **1)** 40 km **2)** 20 km **3)** $\sqrt{40^2 + 20^2}$ km or $\approx$ 44.72 km.

d. $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 10}{40 - 0} = \frac{1}{2}$

e. **1)** $y = \frac{x}{2} + 10$ **2)** It suffices to manipulate the equation $y = \frac{x}{2} + 10$:

$2y = 2 \times \left(\frac{x}{2} + 10\right)$

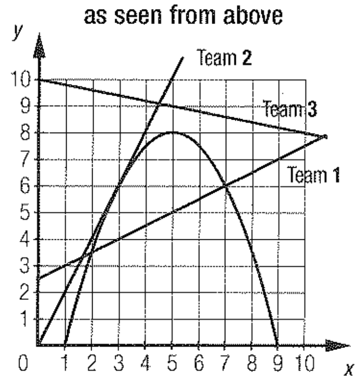
$2y = x + 20$

$x - 2y + 20 = 0$

Prior learning 2

Page 171

a. Movement of the robot as seen from above



It can be noticed that Team **1**'s egg can touch the robot in two places, Team **2**'s egg can touch it in only one place and Team **3**'s egg cannot touch it.

b. The comparison method.

c. **1)** A second-degree equation.

2) The equation $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used and you obtain:

$x_1 = \frac{-4.5 + \sqrt{4.5^2 - 4 \times (-0.5) \times (-7)}}{2 \times (-0.5)} = 2$ and $x_2 = \frac{-4.5 - \sqrt{4.5^2 - 4 \times (-0.5) \times (-7)}}{2 \times (-0.5)} = 7$.

The values that would satisfy this equation are therefore 2 and 7.

d. The x-variable is substituted by the values that were previously obtained:

$y_1 = 0.5 \times 2 + 2.5 = 3.5$

$y_2 = 0.5 \times 7 + 2.5 = 6$

The coordinates of the possible point(s) of impact are (2, 3.5) and (7, 6).

e. **1)** The coordinates of the point of impact are (3, 6).

2) There are no points of impact.

Knowledge in action

Page 174

1. **a)** $y = 2x - 1$ **b)** $y = -4x + 2$ **c)** $x = 2$

d) $y = \frac{5}{2}x - \frac{7}{2}$ **e)** $y = -\frac{2}{3}x + \frac{1}{3}$ **f)** $y = -2$

2. **a)** $x_1 \approx 5.52$ and $x_2 \approx 0.48$. **b)** $x_1 = 12$ and $x_2 = -12$. **c)** $x_1 = -5$ and $x_2 = 2$.

d) $x_1 \approx 0.2$ and $x_2 \approx -10.2$. **e)** $x_1 \approx 2.43$ and $x_2 \approx -0.93$. **f)** $x_1 = 5$

Knowledge in action (*cont'd*)

3. m $\overline{AB}$ = 5 u m $\overline{CD}$ = 7 u m $\overline{EF} = \sqrt{29}$ u m $\overline{GH} = \sqrt{90}$ u

4. a) 5 u **b)** 10 u **c)** 13 u **d)** ≈ 13.34 u **e)** ≈ 4.47 u **f)** ≈ 6.32 u

5.

Equation of the line	Slope	x-intercept	y-intercept
$y = -25x - 75$	**-25**	**-3**	**-75**
$y = 3x - 8$	3	$\frac{8}{3}$	-8
$y = 12$	**0**	**None**	**12**
$y = -\frac{3}{4}x + 60$	$-\frac{3}{4}$	80	60
$x = -5$	**Undefined**	**-5**	**None**
$y = -2$	**0**	None	-2
$x = -2$	**Undefined**	-2	None

6. a) 2 solutions. **b)** 2 solutions. **c)** 2 solutions. **d)** 2 solutions.
e) No solution. **f)** No solution. **g)** 1 solution. **h)** No solution.
i) 1 solution. **j)** 1 solution.

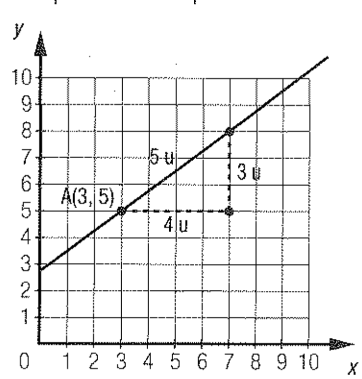
Knowledge in action (*cont'd*)

7. a) $y = 5x + 2$ **b)** $y = -4x + 12$ **c)** $y = \frac{2}{3}x - 5$
d) $y = 2x$ **e)** $y = -\frac{5}{4}x + 10$ **f)** $y = \frac{5}{4}x - \frac{9}{2}$

8. The equation of the orange line is $y = -2x + 60$, the equation of the green line is $y = 2.5x - 52.5$ and the equation of the pink line is $y = 0.25x + 26.25$.

9. a) (≈ 5.79, ≈ 5.58) and (≈ 1.21, ≈ -3.58). **b)** (-3, -4) and (-1, -4). **c)** (-1, -6)
d) No solution. **e)** No solution. **f)** (≈ -9.51, ≈ -1.65) and (≈ 0.71, ≈ 1.41).
g) (2, 16) **h)** (-2, 13) and (2, 13). **i)** No solution.

10. *Several answers possible. Example:*
There are two coordinates that are (15, 14) and (-9, -4).
It is possible to represent this situation using the following graph.



Since the coordinates of this point are at a distance of 15 u from point A, the x-coordinates increases or decreases by 12 u and the y-coordinates increase or decrease by 9 u.

11. a) $\sqrt{(6 + 2)^2 + (8 + 4)^2}$ km or ≈ 14.42 km. **b)** $\sqrt{(6 + 6)^2 + (8 - 2)^2}$ km or ≈ 13.42 km.
c) $\sqrt{(-2 + 6)^2 + (-4 - 2)^2}$ km or ≈ 7.21 km.

Knowledge in action (*cont'd*)

12. Solve the system of equations:

$y = -0.02x^2 + 0.4x$

$y = 0.75$

Therefore, $x_1 \approx 2.09$ and $x_2 \approx 17.91$.

The batter hits the ball after it falls; therefore, the only possible value is the second. The horizontal distance covered by the ball is approximately 17.91 m.

13. a) 1) The equation of the line that passes through the shortest string is $y = -0.5x + 0.6$. The x- and y-values that solve the system of equations are determined:

$y = -0.5x + 0.6$

$y = 2x^2 - 4x + 2$

You obtain the points whose coordinates are (≈ 0.62, ≈ 0.29) and (≈ 1.13, ≈ 0.034).

The length of the shortest string is therefore $\sqrt{(0.62 - 1.13)^2 + (0.29 - 0.034)^2}$ m or approximately 0.57 m.

2) The equation of the line that passes through the longest string is $y = -0.5x + 0.8$. The x- and y-values that solve the system of equations are determined:

$y = -0.5x + 0.8$

$y = 2x^2 - 4x + 2$

You obtain the points whose coordinates are (≈ 0.47, ≈ 0.57) and (≈ 1.28, ≈ 0.16).

The length of the longest cord is therefore $\sqrt{(0.47 - 1.28)^2 + (0.57 - 0.16)^2}$ m or approximately 0.91 m.

b) The coordinates of point A are (0.4, 0.72) since $x = 0.4$; therefore,

$y = 2 \times 0.4^2 - 4 \times 0.4 + 2 = 0.72$.

The coordinates of point B are (1.4, 0.32), since $x = 1.4$; therefore,

$y = 2 \times 1.4^2 - 4 \times 1.4 + 2 = 0.32$.

The distance that separates the extremities A and B of the harp is therefore $\sqrt{(0.4 - 1.4)^2 + (0.72 - 0.32)^2}$ m or approximately 1.08 m.

Knowledge in action (*cont'd*)

Page 178

14. a) 1) Perimeter P of the lot corresponds to $P = 2(3x + 5 + 2x + 4) = 10x + 18$.

2) The area A of the lot corresponds to $A = (3x + 5)(2x + 4) = 6x^2 + 22x + 20$.

b) 1) $P = 10x + 18$

$48 = 10x + 18$

$x = 3$

The sides measure: $3 \times 3 + 5 = 14$ and $2 \times 3 + 4 = 10$.

The dimensions of the lot is 14 m by 10 m.

2) $A = 6x^2 + 22x + 20$

$315 = 6x^2 + 22x + 20$

$0 = 6x^2 + 22x - 295$

$x_1 \approx 5.41$ and $x_2 \approx -9.08$

You must reject the value of x_2; the sides measure: $3 \times 5.41 + 5 \approx 21.23$ and $2 \times 5.41 + 4 = 14.82$.

The dimensions of the lot are approximately 21.23 m by 14.82 m.

15. a)
- Between Traps **A** and **B**, the slope is $-\frac{11}{3}$, and the distance is approximately 11.4 m.
- Between Traps **B** and **C**, the slope is $\frac{1}{8}$, and the distance is approximately 8.06 m.
- Between Traps **C** and **D**, the slope is $-\frac{4}{3}$, and the distance is 5 km.

b) No. You must also indicate that each trap is found in a different quadrant.

Knowledge in action (*cont'd*)

Page 179

16. a) 1) The slope of the segment that corresponds to Gagné Street is $\frac{4}{3}$, because:

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24 - 0}{18 - 0} = \frac{4}{3}.$$

2) The slope of the segment that corresponds to Rivière Street is $-\frac{3}{4}$, because:

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24 - 18}{18 - 26} = -\frac{3}{4}.$$

b) Both of these streets are perpendicular since the slope of one of them is the negative reciprocal of the other.

17. The equation of the line that corresponds to the ramp on which the marble rolls down is $y = {}^{-}0.1x + 8$.
The x- and y-values that solve the system of equations are determined:

$$y = {}^{-}0.1x + 8$$
$$y = \frac{1}{9}x^2 - \frac{4}{3}x + 8$$

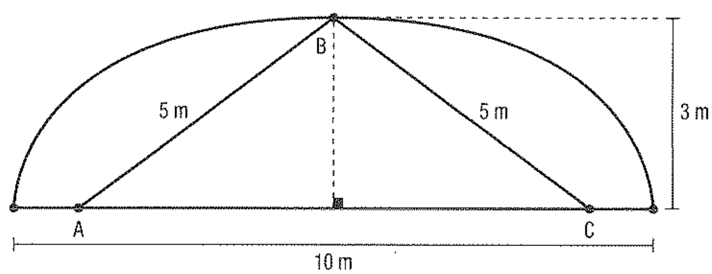
You obtain the points whose coordinates are (0, 8) and (11.1, 6.89).
The marble therefore covers $\sqrt{(11.1 - 0)^2 + (6.89 - 8)^2}$ cm or $\approx$ 11.16 cm.
The marble must cover approximately 11.16 cm in 2 s. Its speed must therefore be approximately 5.58 cm/s.

SECTION 6.1 | Circles and ellipses

Problem

Page 180

Since the distance covered by the sound between Person **A** and Person **C** is constant, regardless of the location of point B, if the positions of Person **A** and Person **C** are fixed and the station allows an axis of symmetry, you obtain the following diagram.

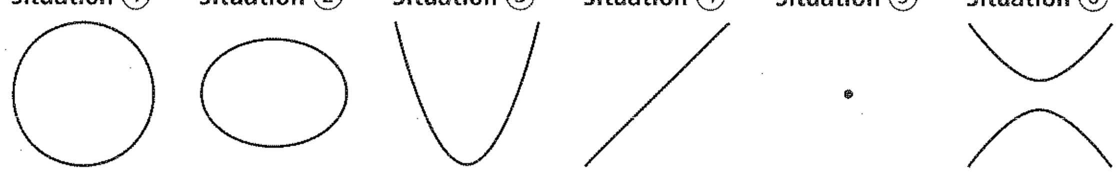


The distance between points A and C are determined based on the Pythagorean theorem: $2 \times \sqrt{5^2 - 3^2} = 8$.
The distance between Person **A** and Person **C** is 8 m.

Activity 1

Page 182

a. Situation ① **Situation ②** **Situation ③** **Situation ④** **Situation ⑤** **Situation ⑥**



b. 1) It is a line. **2)** It is a point.

c. *Several answers possible. Example:*
1) The figure defined in Situation ② is a bound figure whereas the figure defined in Situation ③ is not bound.
2) The figures defined in both situations are not bound. The figure defined in Situation ③ consists of a single curve whereas the figure defined in Situation ⑥ consists of two curves.

Activity 2

Page 183

a. 1) A: 50 u B: 50 u C: 50 u D: 50 u
E: 50 u F: 50 u G: 50 u H: 50 u

2) All these points are found at the same distance from the origin.

Page 177

3) A circle.

b. The radius of the circle.

c. **1)** $d(P, O) = \sqrt{x^2 + y^2}$

2) Since the distance from point P to the origin of the Cartesian plane corresponds to the radius **r** of the circle, you have the following equation:

$r = \sqrt{x^2 + y^2}$

By squaring this expression, you obtain the equation $r^2 = x^2 + y^2$.

d. $x^2 + y^2 = 50^2$ or $x^2 + y^2 = 2500$.

Activity 3

Page 184

a. **1)** $1.94 + 3.06 = 5$ **2)** $2.5 + 2.5 = 5$ **3)** $0.58 + 4.42 = 5$

b. The sum of the distances between point A and any point located on the circumference of the sundial and between this very point and point B is constant.

c. **1)** $\frac{x^2}{2.5^2} + \frac{y^2}{1.5^2} = 1$

2) For point C, whose coordinates are (‑0.7, 1.44): $\frac{(-0.7)^2}{2.5^2} + \frac{1.44^2}{1.5^2} = \frac{0.49}{6.25} + \frac{2.0736}{2.25} = 0.0784 + 0.9216 = 1$.

For point D, whose coordinates are (0, 1.5): $\frac{0^2}{2.5^2} + \frac{1.5^2}{1.5^2} = 0 + \frac{2.25}{2.25} = 0 + 1 = 1$.

For point E, whose coordinates are (‑2.4, ‑0.42): $\frac{(-2.4)^2}{2.5^2} + \frac{(-0.42)^2}{1.5^2} = \frac{5.76}{6.25} + \frac{0.1764}{2.25} = 0.9216 + 0.0784 = 1$.

Technomath

Page 185

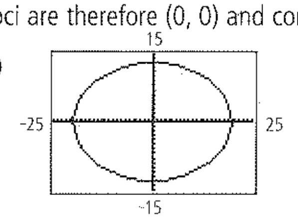
a. **1)** (‑10, 0), (10, 0), (0, ‑2) and (0, 2). **2)** (‑10, 0), (10, 0), (0, ‑5) and (0, 5). **3)** (‑10, 0), (10, 0), (0, ‑8) and (0, 8).

b. **1)** $4\sqrt{6}$ u **2)** $5\sqrt{3}$ u **3)** 6 u

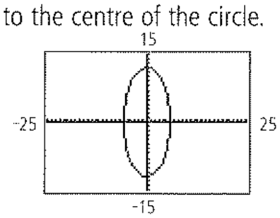
c. **1)** 0 u **2)** A circle.

d. Yes. Let the coordinates of the vertices of an ellipse where parameters **a** and **b** be equal: (4, 0), (‑4, 0), (0, 4) and (0, ‑4). Using the relation $a^2 = b^2 + c^2$ which corresponds to $4^2 = 4^2 + c^2$, therefore $c^2 = 0$ and $c = 0$. The coordinates of the foci are therefore (0, 0) and correspond to the centre of the circle.

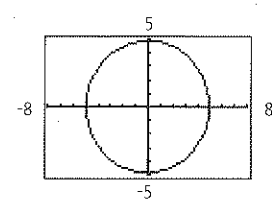
e. **1)**



2)



3)



Practice 6.1

Page 190

1. **a)** $x^2 + y^2 = 64$ **b)** $x^2 + y^2 = 582.5$ **c)** $x^2 + y^2 = 144$
d) $x^2 + y^2 = 625$ **e)** $x^2 + y^2 = 73$ **f)** $x^2 + y^2 = 40$

2. **a)** 7 **b)** 15 **c)** 5.5 **d)** 14 **e)** 5.3 **f)** $\sqrt{5}$

Practice 6.1 *(cont'd)*

Page 191

3. **a)** $\frac{x^2}{64} + \frac{y^2}{36} = 1$ **b)** $\frac{x^2}{64} + \frac{y^2}{256} = 1$ **c)** $\frac{x^2}{289} + \frac{y^2}{196} = 1$
d) $\frac{x^2}{144} + \frac{y^2}{729} = 1$ **e)** $\frac{x^2}{210.25} + \frac{y^2}{110.25} = 1$ **f)** $\frac{x^2}{25} + \frac{y^2}{169} = 1$

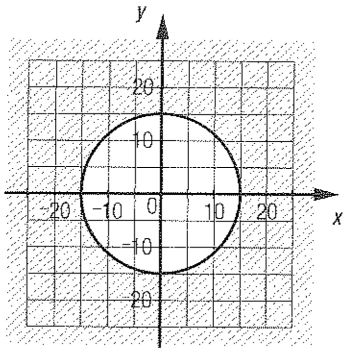
4. a) 1) (13, 0), (−13, 0), (0, 5) and (0, −5). 2) (12, 0) and (−12, 0).
   b) 1) (6, 0), (−6, 0), (0, 10) and (0, −10). 2) (0, 8) and (0, −8).
   c) 1) (8.5, 0), (−8.5, 0), (0, 7.5) and (0, −7.5). 2) (4, 0) and (−4, 0).
   d) 1) (20, 0), (−20, 0), (0, 29) and (0, −29). 2) (0, 21) and (0, −21).
   e) 1) (30, 0), (−30, 0), (0, 18) and (0, −18). 2) (24, 0) and (−24, 0).
   f) 1) (12.5, 0), (−12.5, 0), (0, 3.5) and (0, −3.5). 2) (12, 0) and (−12, 0).

Practice 6.1 (*cont'd*) **Page 192**

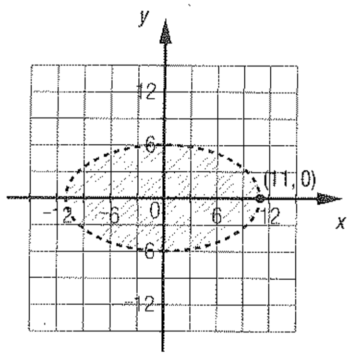
5. a) $x^2 + y^2 = 16$ b) $x^2 + y^2 = 544$ c) $x^2 + y^2 = 169$ d) $x^2 + y^2 = 100$

6. a) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ b) $\frac{x^2}{42.25} + \frac{y^2}{90.25} = 1$ c) $\frac{x^2}{289} + \frac{y^2}{64} = 1$ d) $\frac{x^2}{400} + \frac{y^2}{225} = 1$ e) $\frac{x^2}{11\,025} + \frac{y^2}{21\,025} = 1$

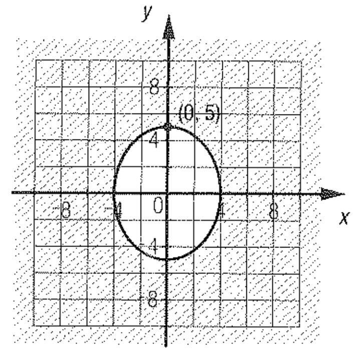
7. a)



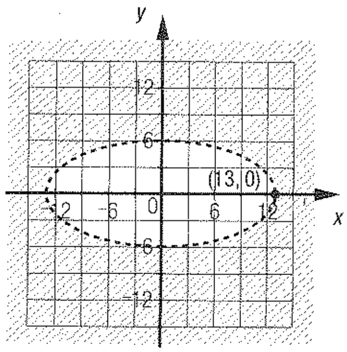
b)



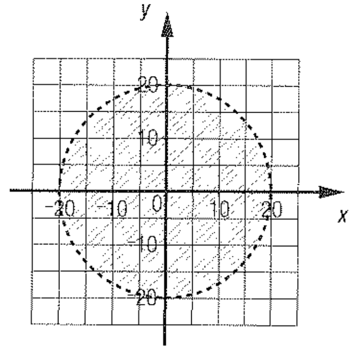
c)



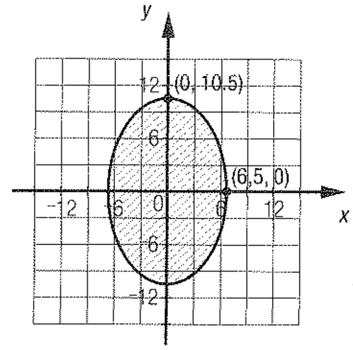
d)



e)



f)



8.

Equation of the ellipse	Coordinates of the vertices	Length of the major axis	Length of the minor axis	Coordinates of the foci
$\frac{x^2}{400} + \frac{y^2}{841} = 1$	**(20, 0), (−20, 0) (0, 29), (0, −29)**	**58 u**	**40 u**	**(0, 21), (0, −21)**
$\frac{x^2}{81} + \frac{y^2}{225} = 1$	(0, 15), (9, 0) (0, −15), (−9, 0)	**30 u**	**18 u**	**(0, 12), (0, −12)**
$\frac{x^2}{169} + \frac{y^2}{25} = 1$	**(13, 0), (−13, 0) (0, 5), (0, −5)**	26 u	**10 u**	(12, 0), (−12, 0)
$\frac{x^2}{100} + \frac{y^2}{210.25} = 1$	**(10, 0), (−10, 0) (0, 14.5), (0, −14.5)**	**29 u**	20 u	(0, 10.5), (0, −10.5)
Two answers possible: $\frac{x^2}{5329} + \frac{y^2}{2304} = 1$ **or** $\frac{x^2}{2304} + \frac{y^2}{5329} = 1$	*Two answers possible:* **(73, 0), (−73, 0) (0, 48), (0, −48) or (48, 0), (−48, 0) (0, 73), (0, −73)**	146 u	96 u	*Two answers possible:* **(55, 0), (−55, 0) or (0, 55), (0, −55)**

Practice 6.1 (*cont'd*)

9. **a)** $x^2 + y^2 \leq 9$ **b)** $\frac{x^2}{100} + \frac{y^2}{36} < 1$ **c)** $\frac{x^2}{64} + \frac{y^2}{36} > 1$

d) $x^2 + y^2 > 13.69$ **e)** $\frac{x^2}{9} + \frac{y^2}{4} \geq 1$ **f)** $\frac{x^2}{49} + \frac{y^2}{625} \leq 1$

10. **a)** Since the perimeter $P \approx \pi[3(a + b) - \sqrt{(a + 3b)(3a + b)}]$ and that the values of parameters **a** and **b** are respectively 48 and ≈ 87.37, you obtain:

$P \approx \pi[3(48 + 87.37) - \sqrt{(48 + 3 \times 87.37)(3 \times 48 + 87.37)}]$

$P \approx \pi[138.24]$

$P \approx 138.24 \times \pi$

The perimeter is approximately 434.31 u.

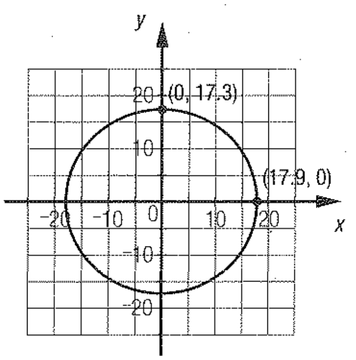
b) Since the area $A = \pi ab$, you obtain:

$A = \pi \times 48 \times 87.37$

The area is approximately 13 174.64 u^2.

Practice 6.1 (*cont'd*)

11. **a)** *Several answers possible. Example:*



b) Based on the graph above, the coordinates of the foci are (≈ 4.6, 0) and (≈ -4.6, 0).

12. **a)** $x^2 + y^2 = 3600$

b) (-60, 0), (-51.96, 30), (-30, ≈ 51.96), (0, 60), (30, ≈ 51.96), (51.96, 30), (60, 0), (51.96, -30), (30, ≈ -51.96), (0, -60), (-30, ≈ -51.96) and (-51.96, -30).

13. **a)** $x^2 + y^2 = 9$ **b)** $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Practice 6.1 (*cont'd*)

14. **a)** It is possible to deduce that the value of parameter **a** is 200 and that parameter **c** is 375. Using the relation $a^2 + c^2 = b^2$, you can deduce that the value of parameter **b** is 425. The inequality that corresponds to the surface of the lake is $\frac{x^2}{40\,000} + \frac{y^2}{180\,625} \leq 1$.

b) The minimum distance is 425 m − 375 m = 50 m.

c) The distance that separates the coach from each of the buoys is 425 m.

15. **a)** Pool **A** has the form of a circle and Pool **B** has the form of an ellipse.

b) Pool **A**:

Since the extended string measures 4 m, the radius of the circle is 4. The equation that corresponds to the circumference of this pool is $x^2 + y^2 = 16$.

Pool **B**: *Several answers possible. Example:*

It is possible to deduce the parameters **a** and **b** based on the parameter **c** which is equal to 3 because 6 m ÷ 2 and based on the parameter **a** which is equal to 5 because 10 m ÷ 2. The equation that corresponds to the circumference of this pool is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

c) 1) The maximum width of Pool **A** is 8 m (diameter of the circle).
2) The maximum width of Pool **B** is 10 m (major axis of the ellipse).

Practice 6.1 (*cont'd*)

Page 196

16. a) The inequality that corresponds to surface of this lot is $x^2 + y^2 \leq 2500$.

b) The circumference C of the lot is $C = \pi d$, $C = \pi \times 100 \approx 314.16$ m.
The total cost of the fence is: \$12/m $\times$ 314.16 m $\approx$ \$3769.92.

c) The area A of the lot is: $A = \pi r^2$, $A = \pi \times 25^2 \approx 7853.98$ m^2.
The total cost of the sod is: \$8/m^2 $\times$ 7853.98 m^2 $\approx$ \$62 831.85.

17. a) The equation of the small ellipse is: $\frac{x^2}{1190.25} + \frac{y^2}{361} = 1$.
The equation of the large ellipse is: $\frac{x^2}{4422.25} + \frac{y^2}{2550.25} = 1$.

b) Using the relation $b^2 + c^2 = a^2$, it is possible to deduce the coordinates of each focus.
Flag **A**: ($\approx$ -43.27, 0) Flag **B**: ($\approx$ -28.8, 0)
Flag **C**: ($\approx$ 28.8, 0) Flag **D**: ($\approx$ 43.27, 0)

c) 1) The distance between Flags **A** and **B** is approximately 14.47 m.
2) The distance between Flags **B** and **C** is approximately 57.6 m.
3) The distance between Flags **A** and **D** is approximately 86.54 m.

Practice 6.1 (*cont'd*)

Page 197

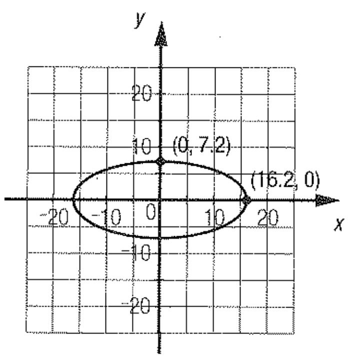
18. a) The equation of the small circle is: $x^2 + y^2 = 9$.
The equation of the ellipse is: $\frac{x^2}{64} + \frac{y^2}{9} = 1$.
The equation of the large circle is: $x^2 + y^2 = 64$.

b) $x^2 + y^2 < 9$

c) 1) $\frac{x^2}{64} + \frac{y^2}{9} < 1$
$x^2 + y^2 > 9$

2) $x^2 + y^2 < 64$
$\frac{x^2}{64} + \frac{y^2}{9} > 1$

19. a) *Several answers possible. Example:*



The equation of the circle associated with this situation is $x^2 + y^2 \approx 81$ because $6.5^2 + 6.23^2 \approx 81$. The radius of the coin measures approximately 9 mm.
The major axis measures approximately 32.4 mm since it is 1.8 times longer than the diameter of the circle:
$9 \times 2 \times 1.8 \approx 32.4$.
The minor axis measures approximately 14.4 mm since it is 1.25 times shorter than the diameter of the circle:
$9 \times 2 \div 1.25 \approx 14.4$.

b) $\frac{x^2}{262.44} + \frac{y^2}{51.84} \approx 1$

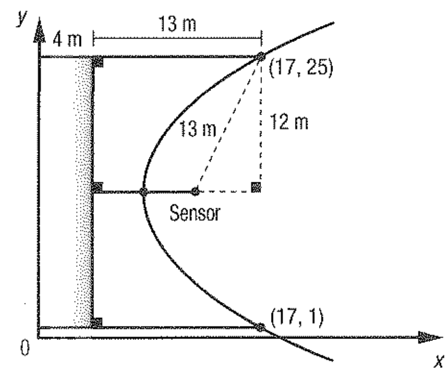
c) Determine the coordinates of each focus by using the relation $a^2 + c^2 = b^2$.
The coordinates of the foci are: ($\approx$ -14.51, 0) and ($\approx$ 14.51, 0).

Hyperbolas and parabolas

Problem

Page 198

The sensor is located on the axis of symmetry of the parabola, and the distance of each of the points of the parabola to the receiver box is equal to the distance of this point to the sensor, which can be represented as follows:



Based on the Pythagorean theorem, the horizontal length of the right triangle formed is: 5 m.

Based on the coordinates of the point, you can deduce the coordinates of the sensor: (17 – 5, 25 – 12) = (12, 13).

Activity 1

Page 199

a. 1) 11.25 u 2) 3.25 u 3) 8 u

b. 1) 2.25 u 2) 10.25 u 3) 8 u

c. The absolute value of the difference of the distances between any point located on the curve and point F_1 and between this very point and point F_2 is constant.

d. 1) $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

2) For point A whose coordinates are (5.8, 3.15): $\frac{5.8^2}{4^2} - \frac{3.15^2}{3^2} = \frac{33.64}{16} - \frac{9.9225}{9} = 2.1025 - 1.1025 = 1$

For point B whose coordinates are (-5, -2.25): $\frac{(-5)^2}{4^2} - \frac{(-2.25)^2}{3^2} = \frac{25}{16} - \frac{5.0625}{9} = 1.5625 - 0.5625 = 1$

e. 1) $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$. 2) $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

f. 1) 5 2) $\sqrt{a^2 + b^2}$

Activity 2

Page 200

a. 1) 6 u 2) 3 u 3) 9.75 u 4) 15 u

b. 1) 6 u 2) 3 u

3) $\sqrt{(6.75 - 3)^2 + (9 - 0)^2} = 9.75$ u 4) $\sqrt{(12 - 3)^2 + (12 - 0)^2} = 15$ u

c. For each of the points, the distance is the same.

d. *Several answers possible. Example:*
The parabola is a curve in which all the points are located at equal distance from a fixed line, called the directrix, and a fixed point, called the focus.

e. You can proceed:
- from ① to ②, by substituting the coordinates of a point on the curve by *x* and *y*
- from ② to ③, by simplifying the expression obtained
- from ③ to ④, by isolating the variable *c*

f. 1) It is identical. 2) It is identical.

Activity 2 (*cont'd*)

Page 201

g. **1)** $(x, -c)$ **2)** (x, c) **3)** (c, y) **4)** $(-c, y)$

h. **1)**

$$\begin{aligned} d(P, d) &= d(P, F) \\ \sqrt{(x-x)^2 + (y+c)^2} &= \sqrt{(x-0)^2 + (y-c)^2} \\ (x-x)^2 + (y+c)^2 &= (x-0)^2 + (y-c)^2 \\ 0 + y^2 + 2cy + c^2 &= x^2 + y^2 - 2cy + c^2 \\ x^2 &= 4cy \end{aligned}$$

2)

$$\begin{aligned} d(P, d) &= d(P, F) \\ \sqrt{(x-x)^2 + (y-c)^2} &= \sqrt{(x-0)^2 + (y+c)^2} \\ (x-x)^2 + (y-c)^2 &= (x-0)^2 + (y+c)^2 \\ 0 + y^2 - 2cy + c^2 &= x^2 + y^2 + 2cy + c^2 \\ x^2 &= -4cy \end{aligned}$$

3)

$$\begin{aligned} d(P, d) &= d(P, F) \\ \sqrt{(x-c)^2 + (y-y)^2} &= \sqrt{(x+c)^2 + (y-0)^2} \\ (x-c)^2 + (y-y)^2 &= (x+c)^2 + (y-0)^2 \\ x^2 - 2cx + c^2 + 0 &= x^2 + 2cx + c^2 + y^2 \\ y^2 &= -4cx \end{aligned}$$

4)

$$\begin{aligned} d(P, d) &= d(P, F) \\ \sqrt{(x+c)^2 + (y-y)^2} &= \sqrt{(x-c)^2 + (y-0)^2} \\ (x+c)^2 + (y-y)^2 &= (x-c)^2 + (y-0)^2 \\ x^2 + 2cx + c^2 + 0 &= x^2 - 2cx + c^2 + y^2 \\ y^2 &= 4cx \end{aligned}$$

Activity 3

Page 202

a. **1)** 19.5 m **2)** 6 m **3)** 12 m **4)** 19.5 m

b. **1)** 19.5 m **2)** 6 m **3)** 12 m **4)** 19.5 m

c. It can be noticed that the distance that separates the bridge's deck from a point is equal to the distance that separates this very point to the cable anchor point.

d. **1)** **i)** $h = 0$ **ii)** $k = 13.5$ **iii)** $c = -6$

2) The values of **h** and **k** correspond to the coordinates of the vertex of the parabola, and the absolute value of **c** corresponds to the distance between the vertex and focus or between the vertex and bridge deck (directrix).

e. Point A(-18, 0): $(-18)^2 = -24(0 - 13.5)$
$324 = 324$

Point S(0, 13.5): $0^2 = -24(13.5 - 13.5)$
$0 = 0$

Point E(12, 7.5): $12^2 = -24(7.5 - 13.5)$
$144 = 144$

Point F(18, 0): $18^2 = -24(0 - 13.5)$
$324 = 324$

Practice 6.2

Page 207

1. **a)** **1)** $(-2, 0)$ **2)** $x = 2$ **b)** **1)** $(0, 0.5)$ **2)** $y = -0.5$

c) **1)** $\left(\frac{33}{4}, -3\right)$ **2)** $x = \frac{7}{4}$ **d)** **1)** $\left(12, \frac{-25}{4}\right)$ **2)** $y = \frac{25}{4}$

e) **1)** $(5, 14)$ **2)** $x = -21$ **f)** **1)** $(0, -2.1)$ **2)** $y = -8.3$

2. **a)** $y^2 = -20x$ **b)** $x^2 = -80y$ **c)** $(y - 8)^2 = 0.4(x + 3)$

d) *Several answers possible. Example:*
$(x - 5)^2 = 32(y + 10)$ or $(x - 5)^2 = -32(y + 10)$.

e) $(y - 12)^2 = -24(x - 9)$

3. **a)** **1)** (9, 0) and (-9, 0). **2)** (15, 0) and (-15, 0). **b)** **1)** (24, 0) and (-24, 0). **2)** (30, 0) and (-30, 0).

c) **1)** (0, 5) and (0, -5). **2)** (0, 13) and (0, -13). **d)** **1)** (4, 0) and (-4, 0). **2)** (8.5, 0) and (-8.5, 0).

e) **1)** (0, 20) and (0, -20). **2)** (0, 20.5) and (0, -20.5). **f)** **1)** (0, 11) and (0, -11). **2)** (0, 61) and (0, -61).

4. **a)** $\frac{x^2}{100} - \frac{y^2}{576} = 1$ **b)** $\frac{x^2}{225} - \frac{y^2}{64} = -1$ **c)** $\frac{x^2}{441} - \frac{y^2}{400.15} \approx 1$ **d)** $\frac{x^2}{5.0625} - \frac{y^2}{100} = -1$

e) Since the equation of the asymptote is $y = -\frac{7}{24}x$, you can present the following proportion:

$\frac{7}{24} = \frac{b}{a}$

$\frac{7}{24} = \frac{28}{a}$

$a = 96$

The equation of the hyperbola is therefore $\frac{x^2}{9216} - \frac{y^2}{784} = -1$.

Practice 6.2 (*cont'd*) — Page 208

5. a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ b) $\frac{x^2}{225} - \frac{y^2}{64} = -1$ c) $\frac{x^2}{576} - \frac{y^2}{100} = -1$

d) $\frac{x^2}{1089} - \frac{y^2}{3136} = 1$ e) $\frac{x^2}{3024.8} - \frac{y^2}{2304} \approx -1$ f) $\frac{x^2}{16} - \frac{y^2}{1} = 1$

6.

Equation of the parabola	Coordinates of the vertex	Coordinates of the focus	Equation of the directrix	Distance between the focus and the directrix
$(y - 8)^2 = -44(x + 2)$	**(-2, 8)**	**(-13, 8)**	**$x = 9$**	**22**
$(x + 13)^2 = 2(y - 10)$	**(-13, 10)**	**(-13, 10.5)**	**$y = 9.5$**	**1**
$(x - 10)^2 = -24(y + 12)$	(10, -12)	**(10, -18)**	$y = -6$	**12**
$(x - 2)^2 = -26(y + 2)$	(2, -2)	(2, -8.5)	**$y = 4.5$**	**13**
$(y - 8)^2 = -32(x - 14)$	(14, 8)	**(6, 8)**	$x = 22$	**16**
$(y + 2)^2 = 10.8(x - 2)$	(2, -2)	(4.7, -2)	**$x = -0.7$**	**5.4**

Practice 6.2 (*cont'd*) — Page 209

7. a) $x^2 = 24y$ b) $(x - 5)^2 = 28.8(y + 9)$ c) $(y + 2)^2 = 2(x + 4)$

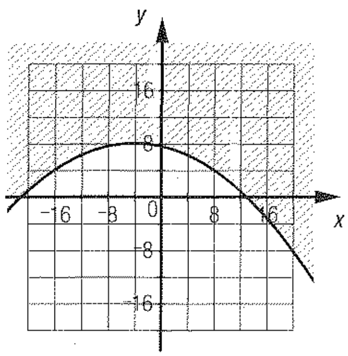
d) $(x + 10)^2 = -80(y - 15)$ e) $y^2 = -8x$ f) $(y - 0.5)^2 = 0.4(x + 1.5)$

8.

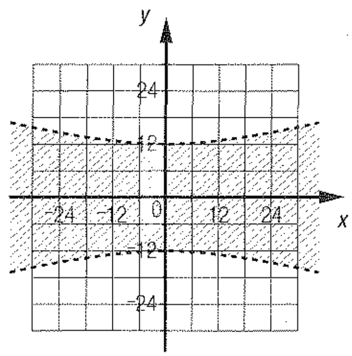
Equation of the hyperbola	Coordinates of the vertices	Coordinates of the foci	Equation of the asymptotes
$\frac{x^2}{5929} - \frac{y^2}{1296} = 1$	**(77, 0)** **(-77, 0)**	**(85, 0)** **(-85, 0)**	**$y = \frac{36}{77}x$** **$y = -\frac{36}{77}x$**
$\frac{x^2}{6400} - \frac{y^2}{324} = -1$	(0, 18) (0, -18)	**(0, 82)** **(0, -82)**	$y = -\frac{9}{40}x$ $y = \frac{9}{40}x$
$\frac{x^2}{4225} - \frac{y^2}{5184} = -1$	(0, 72) (0, -72)	(0, 97) (0, -97)	**$y = \frac{72}{65}x$** **$y = -\frac{72}{65}x$**
$\frac{x^2}{64} - \frac{y^2}{992.25} = -1$	**(0, 31.5)** **(0, -31.5)**	**(0, 32.5)** **(0, -32.5)**	**$y = \frac{63}{16}x$** **$y = -\frac{63}{16}x$**
$\frac{x^2}{2.25} - \frac{y^2}{4} = 1$	(1.5, 0) (-1.5, 0)	(2.5, 0) (-2.5, 0)	**$y = \frac{4}{3}x$** **$y = -\frac{4}{3}x$**

Practice 6.2 (*cont'd*) — Page 210

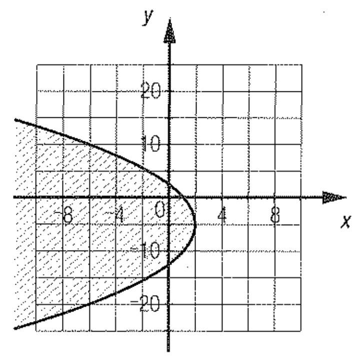
9. a)



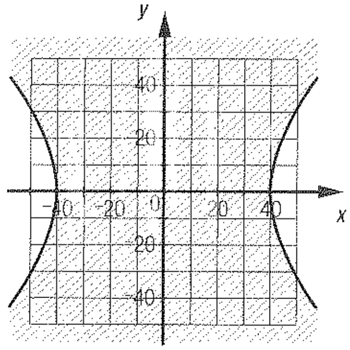
b)



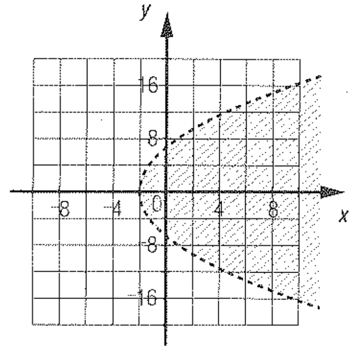
c)



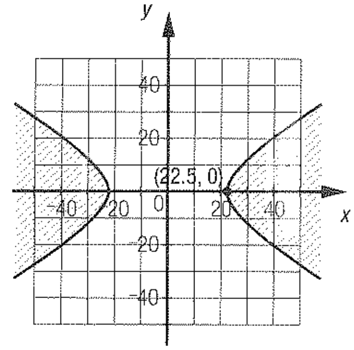
d)



e)



f)



10. a) $\frac{x^2}{81} - \frac{y^2}{144} < 1$ b) $(y - 15)^2 \geq 60(x + 30)$ c) $(x - 8.5)^2 > -40(y - 4.5)$

d) $\frac{x^2}{256} - \frac{y^2}{900} \leq -1$ e) $\frac{x^2}{8100} - \frac{y^2}{3136} \geq 1$ f) $(x + 4)^2 \geq 6y$

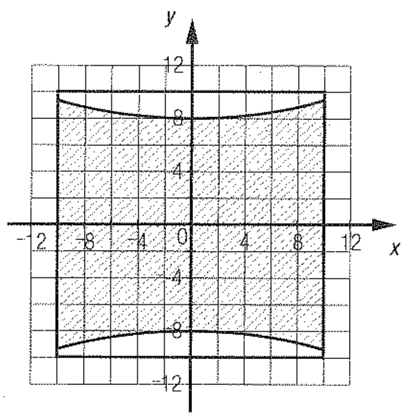
11. Because $\frac{40}{9}$ is a simplified fraction of the expression $\frac{b}{a}$. You can therefore have b = 40 and a = 9 or b = 20 and a = 4.5 or b = 80 and a = 18, etc.

Practice 6.2 (*cont'd*) **Page 211**

12. a) 1) $\frac{x^2}{1} - \frac{y^2}{4} = 1$ 2) $y = 2x$ and $y = -2x$.

b) $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$ since in the hyperbola $a^2 + b^2 = c^2$; therefore, $1 + 4 = 5$.

13. a) This flower garden corresponds to the shaded region of the inside of the square.

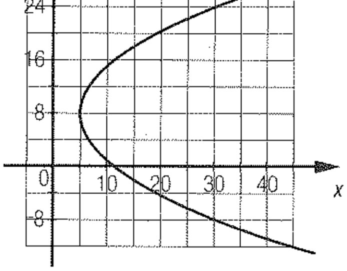


b) The minimum width of the flower garden corresponds to the distance between the two vertices and the hyperbola, which is 16 m.

c) No, because the coordinates of the foci of the hyperbola are (0, 17) and (0, -17). The foci are therefore located outside of the lot.

14. a) Since the equation of the trajectory is $(y - 8)^2 = 10(x - 5)$, it can be deduced that the value of the parameter **c** is $10 \div 4 = 2.5$. Using the coordinates of the vertex, determine the coordinates of the focus: $(5 + 2.5, 8) = (7.5, 8)$. The coordinates of the Sun are therefore (7.5, 8).

b)



c) The minimum distance is 2.5 billion kilometres. It consists of the distance between the vertex of the parabola and the directrix.

Practice 6.2 (*cont'd*) — Page 212

15. a) Since the coordinates of the vertex are (50, 0) and the trajectory passes through point (80, -11.25), the equation of the parabola associated with the trajectory of the submarine is $(x - 50)^2 = -80y$.

b) You must search for the y-value when $x = 0$:

$$(0 - 50)^2 = -80y$$
$$2500 = -80y$$
$$-31.25 = y$$

The maximum depth reached by the submarine is 31.25 m.

16. a) The coordinates of the vertex of the parabola are (0, 0), and the mirror passes through the point (20, 40). The equation of the parabola associated with the concave parabolic mirror is therefore $y^2 = 80x$.

b) The equation of the hyperbola centred at the origin in which one of the branches is associated with the convex hyperbolic mirror is $\frac{x^2}{100} - \frac{y^2}{576} = 1$ since the coordinates of one of the vertices are (0, -10) and the coordinates of one of the foci are (-26, 0).

SECTION 6.3 Intersecting conics

Problem — Page 213

The equation associated with the hyperbola is $\frac{x^2}{25} - \frac{y^2}{625} = 1$ because the equation of one of the asymptotes is $y = 5x$ and the coordinates of one of the vertices are (5, 0).

You must search for the x-value for which $y = -63$.

$$\frac{x^2}{25} - \frac{(-63)^2}{625} = 1$$
$$\frac{x^2}{25} = \frac{4594}{625}$$
$$x^2 = \frac{4594}{25}$$
$$x \approx \pm 13.556$$

The diameter of the circular base of the tower is 2 × 13.556 m, which is approximately 27.11 m.

Activity 1 — Page 214

a. 1) The circle that corresponds to the external circular footpath passes through the point (45, 0) and its radius is 45 m. The equation of this circle is therefore $x^2 + y^2 = 2025$.

2) The line that corresponds to Footpath ① passes through the origin of the Cartesian plane and its slope is $\frac{28 - -28}{21 - -21} = \frac{56}{42} = \frac{4}{3}$. The equation of this line is $y = \frac{4}{3}x$.

b. $x^2 + \left(\frac{4}{3}x\right)^2 = 2025$

c. $x^2 + \frac{16}{9}x^2 = 2025$

$$\frac{25}{9}x^2 = 2025$$
$$x^2 = 729$$

$x_1 = 27$ and $x_2 = -27$.

These two values correspond to the x-coordinates where Person **A** and **C** are found, respectively.

d. 1) The equation of the circle is $x^2 + y^2 = 2025$, solve the equation $27^2 + y^2 = 2025$. The coordinates of the point where Person **A** is found are (27, 36).

2) The equation of the circle is $x^2 + y^2 = 2025$, solve the equation $(-27)^2 + y^2 = 2025$. The coordinates of the point where Person **C** is found is (-27, -36).

e. Solve the equation $x^2 + \left(\frac{-3}{4}x\right)^2 = 2025$. The coordinates of the point where Person **B** is found are (-36, 27).

Activity 2

Page 215

a. The coordinates of the two possible points of impact are (≈ -15, ≈ 7.5) and (≈ 15, ≈ 7.5).

b. 1) First isolate the x^2's in each of the equations. You can then proceed:

- from ① to ②, using the comparison method to solve the system of equations
- from ② to ③, by writing the equation in the form $ax^2 + bx + c = 0$
- from ③ to ④, by finding the zeros using the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2) No, since both trajectories do not cross each other for $y = -12.5$.

3) It is known that $y = 8$; therefore, $x^2 = 28.125 \times 8$

$$x^2 = 225$$
$$x_1 = 15 \text{ and } x_2 = -15.$$

The coordinates of the possible points of impact are (15, 8) and (-15, 8).

Technomath

Page 216

a. There can be 0, 1 or 2 intersection points.

b. There can be 0, 1, 2, 3 or 4 intersection points.

c. 1) (6, 8) and (-8, -6).

2) (≈ 1.46, ≈ 5.9), (≈ 3.14, ≈ -5.52), (≈ 5.09, ≈ -4.63) and (≈ 6.31, ≈ 3.69).

d. 1) (≈ -4.73, ≈ 6.45) and (≈ 2.32, ≈ -7.65).

2) (≈ 2.96, ≈ 8.34) and (≈ 7.22, ≈ 9.87).

3) (≈ -2.12, ≈ 1.25) and (≈ 6.12, ≈ -15.25).

Practice 6.3

Page 218

1. **a)** (-4, 6) **b)** (≈ 21.17, 24) and (≈ -21.17, 24). **c)** (-4.2, -2.9) and (3, 2.5).
d) (≈ 1.9, ≈ 15.8) and (≈ -10.21, ≈ -8.41). **e)** (-30, 18) and (≈ 58.89, ≈ -35.33). **f)** (-12, 0) and (≈ 42.86, ≈ 54.86).

Practice 6.3 *(cont'd)*

Page 219

2. **a)** (15, 8) and (-15, 8). **b)** (-7.8, 4) and (-7.8, -4). **c)** (-7.5, 30), (0, -24) and (7.5, 30).
d) (3.6, 4.8) and (3.6, -4.8). **e)** (≈ 4.74, ≈ -4.9), (≈ -4.74, ≈ -4.9), (≈ 4.24, 4) and (≈ -4.24, 4).
f) (34, 0)

3. **a)** (12, 5) and (12, -5). **b)** (≈ 2.09, ≈ -6.81), (≈ -2.09, ≈ -6.81), (≈ 5.4, ≈ 5.6) and (≈ -5.4, ≈ 5.6).
c) (≈ -20.31, ≈ 14.58) and (≈ -20.31, ≈ -14.58). **d)** (-4, ≈ 6.93) and (-4, ≈ -6.93).
e) (≈ -19.83, -3.77), (0, 29) and (≈ 19.83, -3.77).
f) (≈ 3.74, ≈ -8.25), (≈ -3.74, ≈ -8.25), (≈ 12.81, ≈ 10.52) and (≈ -12.81, ≈ 10.52).

Practice 6.3 (*cont'd*)

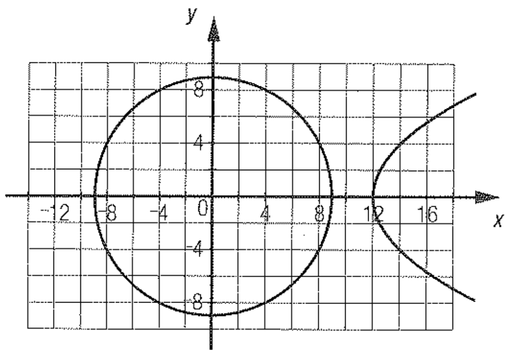
Page 220

4. **a)** (0.4, -10.1) **b)** (-11, -22.5) and (17, -22.5). **c)** (-24, -10) and (24, -10).
   d) (19.2, 15) and (-19.2, -15). **e)** (40, 30) and (-30, -40). **f)** (-2, -11) and (-20, 7).
   g) ($\approx$ 12.71, $\approx$ 91.69) and ($\approx$ -8.04, $\approx$ -74.32). **h)** (4, -6) and ($\approx$ -4.94, $\approx$ -1.53). **i)** (-36, 0) and (60, 20).

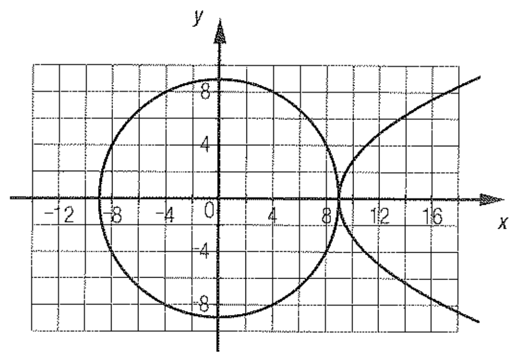
5. **a)** No points. **b)** 2 points. **c)** 3 points. **d)** 1 point. **e)** 4 points. **f)** 2 points.

6. *Several answers possible. Examples:*

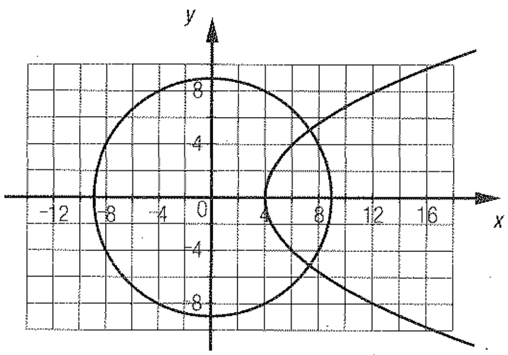
a)



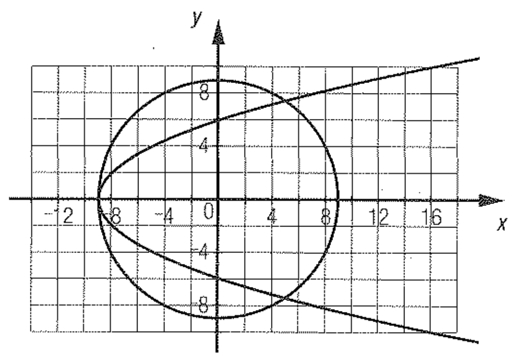
b)



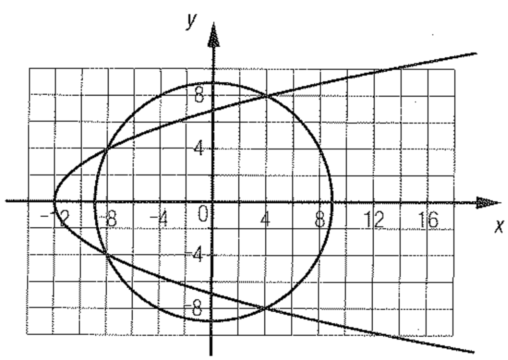
c)



d)



e)



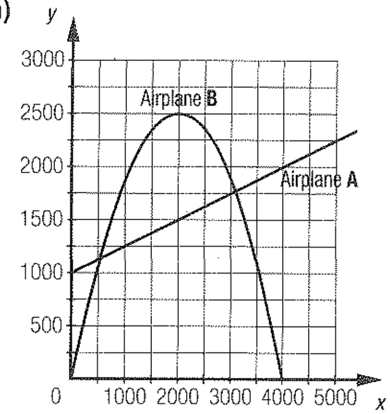
7. **a)** ($\approx$ 11.24, 6) and ($\approx$ -11.24, 6).
   b) ($\approx$ -12.96, $\approx$ 5.18) and (10.2, -6.4).
   c) (0, -8), ($\approx$ 14.12, $\approx$ 4.46) and ($\approx$ -14.12, $\approx$ 4.46).

8. **a) 1)** The equation of the line that passes through the centre of the bridge and points (0, 20) and (10, 0) is $y = -2x + 20$.
   2) The equation of the ellipse associated with the circumference of the lake is $\frac{x^2}{900} + \frac{y^2}{1600} = 1$ since the vertices are (30, 0) and (0, 40).

   b) 1) ($\approx$ -9.07, $\approx$ 38.13) **2)** ($\approx$ 22.91, $\approx$ -25.82)

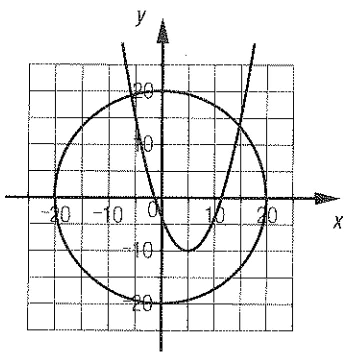
   c) The distance between points A and B is $\sqrt{(22.91 - -9.07)^2 + (-25.82 - 38.13)^2} \approx 71.5$. The length that separates the two piers of the bridge is approximately 71.5 dam.

9. a)



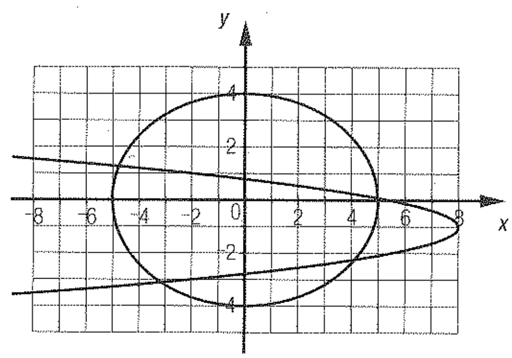
b) The coordinates of the intersection points where Airplane **B** passes exactly over Airplane **A** are ($\approx$ 519.38, $\approx$ 1129.84) and ($\approx$ 3080.62, $\approx$ 1770.16).

10. a)



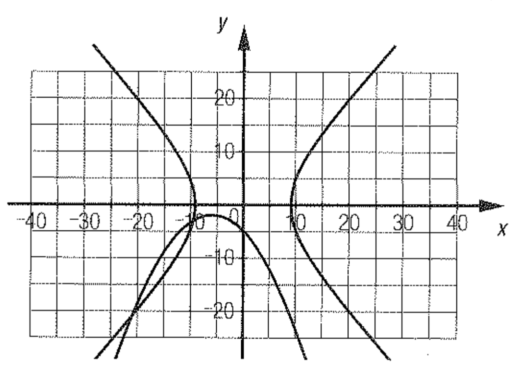
($\approx$ -5.8, $\approx$ 19.1) and ($\approx$ 14.7, $\approx$ 13.6).

b)



($\approx$ -4.7, $\approx$ 1.3), ($\approx$ -3.1, $\approx$ -3.1), ($\approx$ 4.1, $\approx$ -2.2) and ($\approx$ 4.99, $\approx$ 0.1).

c)



($\approx$ -21.3, $\approx$ -21.4) and ($\approx$ -9.4, $\approx$ -3).

11. a) ($\approx$ -86.45, $\approx$ -83.22) and ($\approx$ 118.45, $\approx$ 19.22). **b)** ($\approx$ 56.26, $\approx$ -11.87) and ($\approx$ -68.26, $\approx$ -74.13).

c) (0, 120), (72, -96) and (-72, -96).

12. a) 1) The equation of the line that corresponds to the high wire is $y = \frac{x}{3} - 10$ since the line passes through the points (30, 0) and (60, 10).

2) The equation of the curve that corresponds to the canyon is $(x - 60)^2 = 20(y + 45)$ since the vertex of the parabola is (60, -45) and passes through the point (30, 0).

b) 1) (30, 0) **2)** ($\approx$ 96.67, $\approx$ 22.22)

c) The distance covered by the tightrope walker is $\sqrt{(96.67 - 30)^2 + (22.22 - 0)^2}$ m $\approx$ 70.28 m.

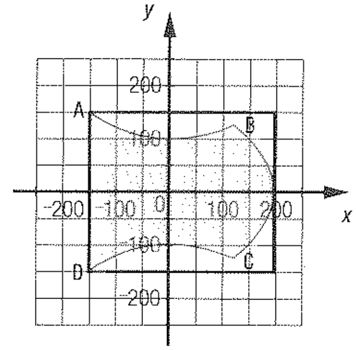
d) The equation of the parabola is $(x - 60)^2 = 20(y + 45)$, therefore $c = 20 \div 4 = 5$.
The coordinates of the vertex are (60, -45); the coordinates of the focus and the location of the camera are therefore (60, -45 + 5) = (60, -40).

13. a) For points A and D, find the intersection points between the hyperbola and the line. For points B and C, find the intersection points between the hyperbola and the parabola.

1) (−150, ≈ 141.42) 2) (≈ 118.69, ≈ 127.52) 3) (≈ 118.69, ≈ −127.52) 4) (−150, ≈ −141.42)

b) The minimum dimensions of the piece of wood are 350 cm by approximately 282.84 cm. The adjacent diagram represents this piece of wood.

Table top



Practice 6.3 (*cont'd*)

Page 223

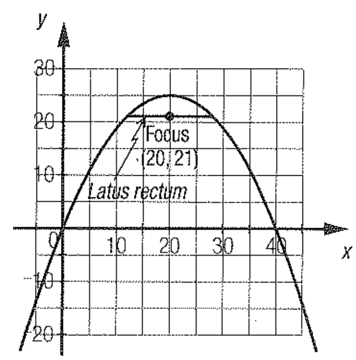
14. a) The equation of the parabola is $(x - 20)^2 = -16(y - 25)$. You can deduce that the parameter $c = -16 \div 4 = -4$ and that the vertex of the parabola is located at point (20, 25). The focus is therefore located at coordinates (20, 21). You can determine the intersection point between the parabola and line of the equation $y = 21$:

$(x - 20)^2 = -16(21 - 25)$

$(x - 20)^2 = 64$

$x_1 \approx 28$ and $x_2 \approx -28$.

The *latus rectum* is the segment that joins points (12, 21) and (28, 21). It therefore measures 16 u.



b) The equation of the ellipse is $\frac{x^2}{64} + \frac{y^2}{100} = 1$. It can be deduced that the parameter $c = \pm 6$, since $a^2 + c^2 = b^2$ and that the foci are therefore located at coordinates (0, 6) and (0, -6). Determine the intersection point between the ellipse and the line of the equation $y = 6$ or $y = -6$:

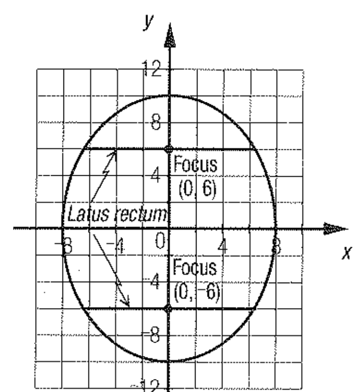
$$\frac{x^2}{64} + \frac{6^2}{100} = 1$$

$$\frac{x^2}{64} = \frac{16}{25}$$

$$x^2 = 40.96$$

$x_1 \approx 6.4$ and $x_2 \approx -6.4$.

The *latus rectum* is the segment that joins points (6.4, 6) and (-6.4, 6) as well as points (6.4, -6) and (-6.4, -6). It therefore measures 12.8 u.



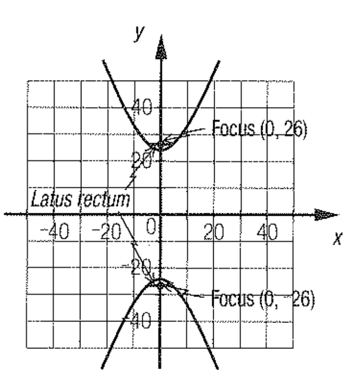
c) The equation of the hyperbola is $\frac{x^2}{100} - \frac{y^2}{576} = -1$. It can be deduced that the parameter $c = \pm 26$, since $a^2 + b^2 = c^2$ and that the foci are therefore located at coordinates (0, 26) and (0, -26). Determine the intersection point between the hyperbola and the line of the equation $y = 26$ or $y = -26$:

$$\frac{x^2}{100} - \frac{26^2}{576} = -1$$

$$\frac{x^2}{100} = \frac{25}{144}$$

$$x^2 = \frac{625}{36}$$

$x_1 \approx 4.17$ and $x_2 \approx -4.17$.



The *latus rectum* is the segment that joins points (≈ 4.17, 26) and (≈ -4.17, 26) as well as the points (≈ 4.17, -26) and (≈ -4.17, -26). It therefore measures approximately 8.33 u.

15. Solve the system of equations:

$$\frac{x^2}{40\,000} + \frac{y^2}{400} = 1$$

$$y = 0.05x + 5$$

this allows you to obtain the equation:

$$\frac{x^2}{40\,000} + \frac{(0.05x + 5)^2}{400} = 1$$

$$400x^2 + 100x^2 + 20\,000x + 1\,000\,000 = 16\,000\,000$$

$$500x^2 + 20\,000x - 15\,000\,000 = 0$$

$$x_1 \approx 154.4 \text{ and } x_2 \approx -194.36,$$

of where: $y_1 \approx 12.72$, since $y_1 \approx 0.05 \times 154.4 + 5$,

$y_2 \approx -4.72$, since $y_2 \approx 0.05 \times -194.36 + 5$.

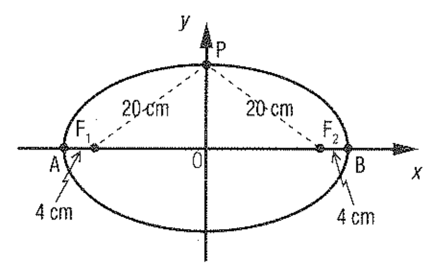
The coordinates of the probe's potential landing points are (≈ 154.36, ≈ 12.72) and (≈ -194.36, ≈ -4.72).

SPECIAL FEATURES 6

Chronicle of the past

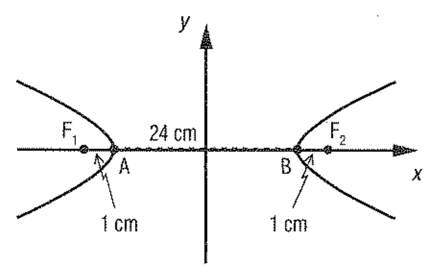
Page 225

1. In Figure ①, it is known that m $\overline{PD}$ = 29,6 cm and that m $\overline{PC}$ = 10.4 cm; therefore, m $\overline{CD}$ = 10.4 + 29.6 = 40 cm. The definition of the ellipse allows you to determine the length of the major axis since m $\overline{PF_1}$ + m $\overline{PF_2}$ = 2a; therefore, 2a = 40 cm. The adjacent graphical representation can be drawn.



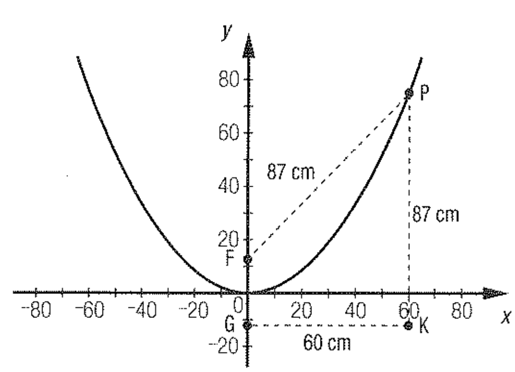
It can be deduced that the minor axis measures 24 cm because $20^2 = b^2 + (20 - 4)^2$, b = 12 cm. Therefore, the length of the major axis is 40 cm, and the length of the minor axis is 24 cm.

2. In Figure ②, it is known that m $\overline{PD}$ = 32.8 cm and that m $\overline{PC}$ = 8.8 cm; therefore, m $\overline{CD}$ = 32.8 − 8.8 = 24 cm. The definition of the hyperbola allows you to determine the distance between the two vertices of the hyperbola since m $\overline{PF_2}$ − m $\overline{PF_1}$ = 2a; therefore, 2a = 24 cm. The adjacent graphical represenation can be drawn.



It can be deduced that the distance between the two foci of the hyperbola is 26 cm.

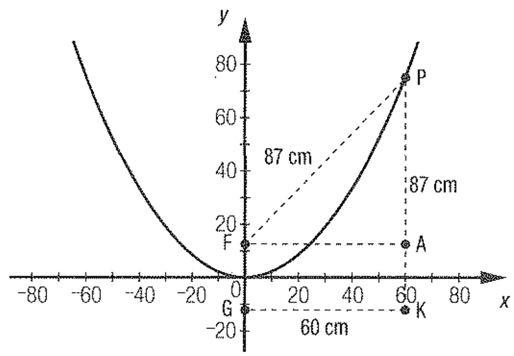
3. Since the parabola is a curve in which all the points are located at an equal distance of a fixed line, called the directrix, and of a fixed point, called the focus, it is possible to draw the adjacent graphical representation.



Based on the Pythagorean theorem, determine the distance between point A and point P of the adjacent representation.

$d(A, P) = \sqrt{87^2 - 60^2} = 63$ cm

The distance between the directrix and the focus is 87 cm − 63 cm = 24 cm; therefore, the distance between the focus and the vertex of the parabola is 24 cm ÷ 2 = 12 cm.

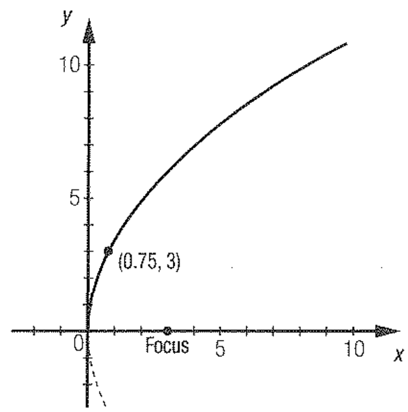


In the workplace

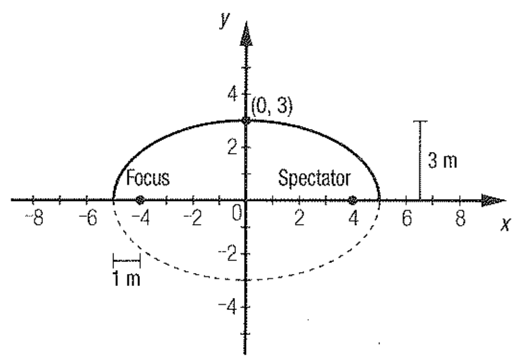
Page 227

1. a) For each case, the optimal location is the focus of the ceiling that has a conic shape.

   b) 1) The minimum distance that separates the focus and the parabola from the wall in Theatre ① is 3 m, which corresponds to the value of parameter **c** in the adjacent graphical representation.

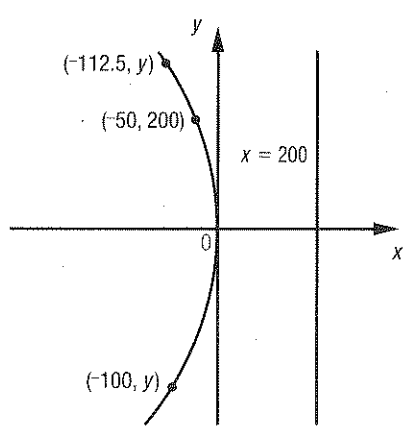


   2) The distance that separates the two foci is 8 m, which corresponds to double the value of parameter **c** in the adjacent graphical representation.

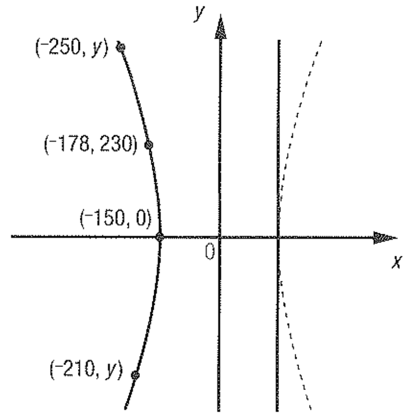


   You obtain the relation $(c + 1)^2 = 3^2 + c^2$ where $c = 4$.

2. a) It is possible to represent this situation using the adjacent graph, where the equation of the parabola is $y^2 = -800x$. The coordinates of the north and south extremities are (−112.5, 300) and (−100, ≈ −282.84). The distance that separates the two extremities is approximately $\sqrt{(-112.5 + 100)^2 + (300 + 282.84)^2}$ m or approximately 582.97 m.



b) It is possible to represent this situation using the adjacent graph, of the hyperbola is $\frac{x^2}{150^2} - \frac{y^2}{360^2} \approx 1$. The coordinates of the north and south extremities are (-250, ≈ 480) and (-210, ≈ -352.73). The distance that separates the two extremities is approximately $\sqrt{(-250 + 210)^2 + (480 + 352.73)^2}$ m or approximately 833.69 m.



Overview

Page 228

1. a) $x^2 + y^2 = 1600$ **b)** $\frac{x^2}{2025} + \frac{y^2}{2809} = 1$ **c)** $(y + 3)^2 = 2(x + 4)$ **d)** $\frac{x^2}{64} - \frac{y^2}{36} = 1$

e) $x^2 + y^2 = 5625$ **f)** $(x + 10)^2 = -20(y - 20)$ **g)** $\frac{x^2}{841} + \frac{y^2}{441} = 1$ **h)** $\frac{x^2}{576} - \frac{y^2}{49} = -1$

Overview (*cont'd*)

Page 229

2. a) 1) A parabola.
2) i) (20, 10) **ii)** (20, 10.25) **iii)** Does not apply.

b) 1) An ellipse.
2) i) (0, 12.5), (0, -12.5), (3.5, 0) and (-3.5, 0). **ii)** (0, 12) and (0, -12). **iii)** Does not apply.

c) 1) A hyperbola.
2) i) (0, 36) and (0, -36). **ii)** (0, 39) and (0, -39). **iii)** $y = \frac{12}{5}x$ and $y = -\frac{12}{5}x$.

d) 1) A parabola.
2) i) (-2, -9) **ii)** (-26, -9) **iii)** Does not apply.

e) 1) A hyperbola.
2) i) (22.5, 0) and (-22.5, 0). **ii)** (26.5, 0) and (-26.5, 0). **iii)** $y = \frac{28}{45}x$ and $y = -\frac{28}{45}x$.

f) 1) An ellipse.
2) i) (0, 61), (0, -61), (11, 0) and (-11, 0). **ii)** (0, 60) and (0, -60). **iii)** Does not apply.

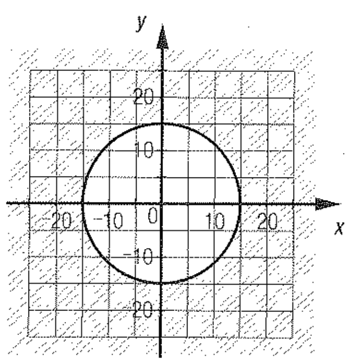
3. a) $x^2 + y^2 = 676$ **b)** $\frac{x^2}{576} + \frac{y^2}{1089} = 1$ **c)** $\frac{x^2}{400} + \frac{y^2}{1241} = 1$ **d)** $(y - 8)^2 = 8(x + 3)$

e) $(x - 12)^2 = 12(y + 4)$ **f)** $\frac{x^2}{576} - \frac{y^2}{4900} = -1$ **g)** $\frac{x^2}{196} - \frac{y^2}{506.25} = 1$

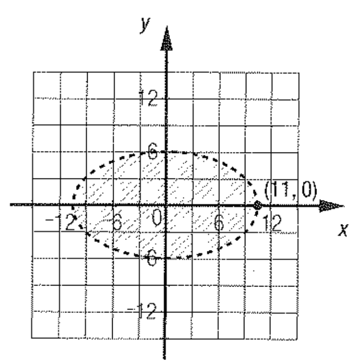
4. a) (35, 0) and (-35, 0). **b)** (0, 10) and (0, -10). **c)** (2, -4)

d) (-3, -0.5) **e)** (0, 4) and (0, -4). **f)** (13, 0) and (-13, 0).

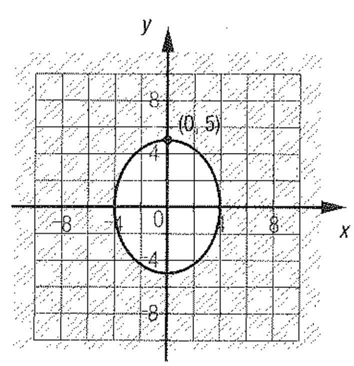
5. a)



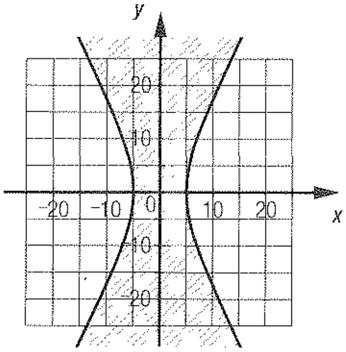
b)



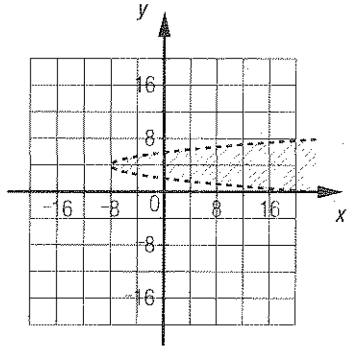
c)



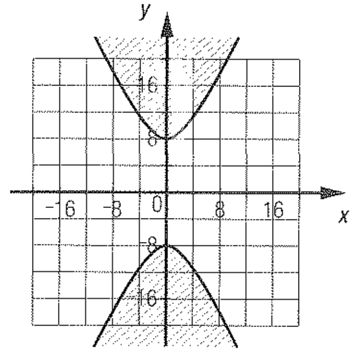
d)



e)



f)

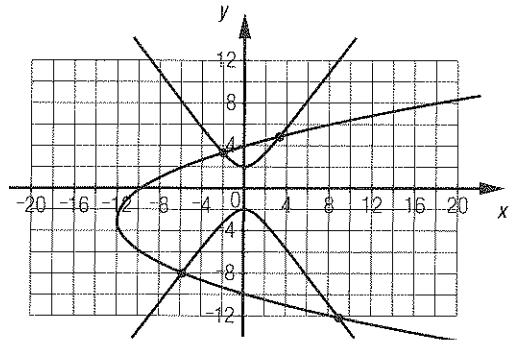


Overview (*cont'd*)

Page 230

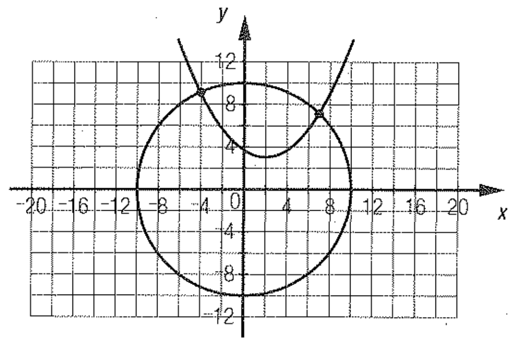
6. a) $\frac{x^2}{841} + \frac{y^2}{400} \geq 1$
   b) $x^2 + y^2 \leq 289$
   c) $\frac{x^2}{576} - \frac{y^2}{100} > 1$
   d) $(y + 5)^2 < -2(x - 7)$
   e) $\frac{x^2}{12.25} - \frac{y^2}{144} \geq -1$
   f) $(x + 10)^2 > 15(y - 5)$

7. a)



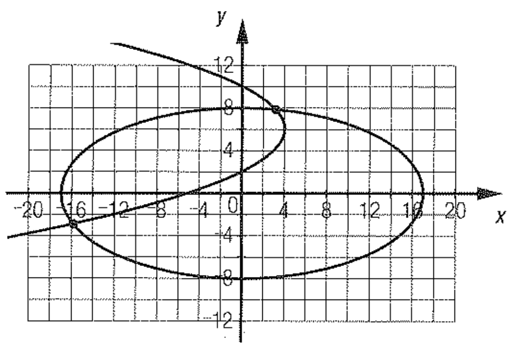
The coordinates of the intersection points are ($\approx$ -5.8, $\approx$ -7.98), ($\approx$ -1.99, $\approx$ 3.33), ($\approx$ 3.29, $\approx$ 4.82) and ($\approx$ 9, $\approx$ -12.17).

b)



The coordinates of the intersection points are ($\approx$ -4.07, $\approx$ 9.14) and ($\approx$ 6.99, $\approx$ 7.15).

c)



The coordinates of the intersection points are ($\approx$ -15.84, $\approx$ -2.91) and ($\approx$ 3.13, $\approx$ 7.86).

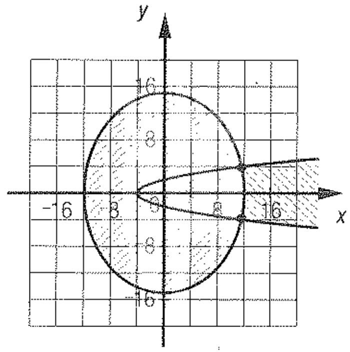
Overview (*cont'd*)

Page 231

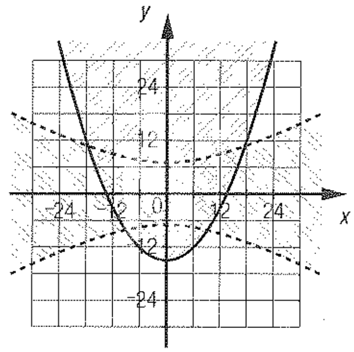
8.

Equation of the conic	Coordinates of the vertex or vertices	Coordinates of the focus or foci
$\frac{x^2}{6724} + \frac{y^2}{324} = 1$	**(82, 0) and (-82, 0)** **(0, 18) and (0, -18)**	**(80, 0)** **(-80, 0)**
$(y - 12)^2 = -9(x + 4)$	**(-4, 12)**	**(-6.25, 12)**
$\frac{x^2}{256} - \frac{y^2}{3969} = -1$	**(0, 63) and (0, -63)**	**(0, 65)** **(0, -65)**
$(x + 9)^2 = 0.5(y + 5)$	**(-9, -5)**	**(-9, -4.875)**
$\frac{x^2}{196} - \frac{y^2}{506.25} = 1$	**(14, 0) and (-14, 0)**	**(26.5, 0)** **(-26.5, 0)**

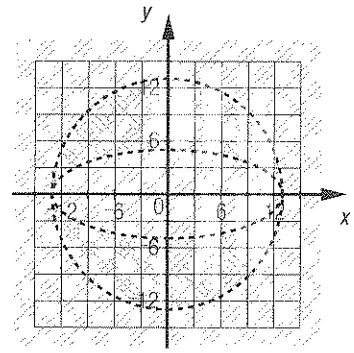
9. a)



b)



c)



10. The inequality associated with the secure swimming zone is in the form $(x - h)^2 \leq 4c(y - k)$. Using the graphical representation, it is possible to determine:

- the value of c: -5
- a part of the coordinates of the vertex: (*x*, 25)
- the coordinates of a point through which the curve passes: (40, 20)

You therefore obtain $(40 - h)^2 = 4 \times -5 \times (20 - 25)$ where h = 30 since based on the graphical representation, it is the only accepted value. The inequality associated with the secure swimming zone is therefore $(x - 30)^2 \leq -20(y - 25)$.

Overview (*cont'd*)

Page 232

11. a) It is possible to deduce that the coordinates of the focus of the ellipse and of the vertex of the parabola are (20, 0) since a = 29 and b = 21 and that $a^2 + c^2 = b^2$.
The equation of the directrix of the parabola is $x = 29$ since it passes through the vertex of the ellipse. It is possible to deduce that parameter c = -9 since the vertex of the parabola is (20, 0). The equation of the parabola associated with the trajectory of the swimmer is therefore $y^2 = -36(x - 20)$.

b) To determine the coordinates of the swimmer's entry point and exit point, find the intersection points between the parabola of the equation $y^2 = -36(x - 20)$ and the ellipse of the equation $\frac{x^2}{841} + \frac{y^2}{441} = 1$.
You obtain: $\frac{x^2}{841} + \frac{-36(x - 20)}{441} = 1$ where $x \approx 8.9$ since the second value obtained does not correspond to the situation.
Since $y^2 \approx -36(8.9 - 20)$, $y \approx \pm 19.99$.

1) The coordinates of the swimmer's entry point are ($\approx$ 8.9, $\approx$ 19.99).
2) The coordinates of the swimmer's exit point are ($\approx$ 8.9, $\approx$ -19.99).

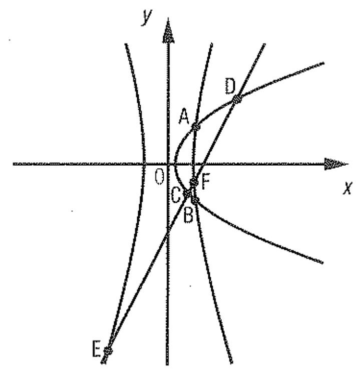
12. a) 1) Since the parabola has an axis of symmetry, based on the coordinates of points (2, 6) and (6, 6) you can deduce that the coordinates of the vertex are (6 − 2, -2) = (4, -2).
The equation of this parabola is in the form $(x - h)^2 = 4c(y - k)$. By substituting the known data by these variables, the value of parameter **c** is determined:
$(2 - 4)^2 = 4c(6 + 2)$
$c = 0.125$
The equation of the parabola is $(x - 4)^2 = 0.5(y + 2)$.

2) The line passes through the focus whose coordinates are (4, -2 + 0.125) = (4, -1.875).
Since the line passes through points (4, -1.875) and (2, -1), its equation is $y = -0.4375x - 0.125$.

b) Solve the system of equations:
$(x - 4)^2 = 0.5(y + 2)$
$y = -0.4375x - 0.125$
From this system, you obtain:
$(x - 4)^2 = 0.5(-0.4375x - 0.125 + 2)$
$x^2 - 7.78125x + 15.0625 = 0$, from where $x_1 \approx 3.62$ and $x_2 \approx 4.16$.
Therefore, $y_1 \approx -0.4375 \times 3.62 - 0.125$, or $y_1 \approx -1.71$, and $y_2 \approx -0.4375 \times 4.16 - 0.125$, or $y_2 \approx -1.95$.
The coordinates of the points where the osprey could catch the fish are ($\approx$ 3.62, $\approx$ -1.71) and ($\approx$ 4.16, $\approx$ -1.95).

13. Determine the coordinates of 6 intersection points.

Obtain the coordinates of points A and B by solving the system of equations $\frac{x^2}{49} - \frac{y^2}{576} = 1$ and $y^2 = 20(x - 2)$.
The coordinates of points A and B are respectively ($\approx$ 7.66, $\approx$ 10.64) and ($\approx$ 7.66, -10.64).
Obtain the coordinates of points C and D by solving the system of equations $y = 2x - 20$ and $y^2 = 20(x - 2)$. The coordinates of points C and D are respectively ($\approx$ 5.7, $\approx$ -8.6) and ($\approx$ 19.3, 18.6).
Obtain the coordinates of points E and F by solving the system of equations $\frac{x^2}{49} - \frac{y^2}{576} = 1$ and $y = 2x - 20$.
The coordinates of points E and F are respectively ($\approx$ -17.51, $\approx$ -55.01) and ($\approx$ 7.19, -5.62).



14. a) Based on the definition of the ellipse, it is known that d(B, A) + d(A, C) = 2a.
Therefore, $2a = \sqrt{(7.2 + 5.66)^2 + (4.2 - 0)^2} + \sqrt{(7.2 - 5.66)^2 + (4.2 + 0)^2} \approx 18$.
The value of parameter a $\approx 18 \div 2 \approx 9$.
The value of parameter **b** is determined using the relation $b^2 + c^2 = a^2$:
$b^2 + 5.66^2 \approx 9^2$
$b^2 \approx 48.96$
$b \approx 7$
The inequality that corresponds to the region associated with this archipelago is $\frac{x^2}{81} + \frac{y^2}{48.96} \leq 1$.

b) The length of the major axis is approximately 18 km and the length of the minor axis is approximately 14 km.

c) Determine the intersection points between the ellipse of the equation $\frac{x^2}{81} + \frac{y^2}{48.96} = 1$ and the line of the equation $y = -0.4x - 1$:
$\frac{x^2}{81} + \frac{(-0.4x - 1)^2}{48.96} = 1$
$x_1 \approx -8.46$ and $x_2 \approx 7.42$, and $y_1 \approx 2.39$ and $y_2 \approx -3.97$.
The distance separating these two points is $\sqrt{(-8.46 - 7.42)^2 + (2.39 + 3.97)^2}$ km $\approx$ 17.11 km. The distance covered by the plane is approximately 17.11 km.

15. Note the following:
- Since the height of the viaduct is 8 m, parameter **b** is 8.
- Since the width of the viaduct is 34 m, parameter $a = 34 \div 2 = 17$.

Therefore, the equation of the ellipse is $\frac{x^2}{289} + \frac{y^2}{64} = 1$.
Note the following:
- Since the *x*-axis is superimposed onto the directrix of the parabola, $x = 0$.
- The coordinates of the focus of the parabola and those of the vertices of the ellipse are the same, which is (0, 8).

Therefore, $c = 8 \div 2 = 4$; the vertex is $(0, 8 - 4) = (0, 4)$, and the equation of the parabola is $x^2 = 16(y - 4)$.
To find the intersection points, solve the equation:
$\frac{16(y - 4)}{289} + \frac{y^2}{64} = 1$
$y_1 \approx 7.25$ and $y_2 \approx -10.79$.
The value of y_2 rejected based on the context.
$x^2 \approx 16(7.25 - 4)$
$x^2 \approx 52$
$x \approx \pm 7.21$
The coordinates of the two anchor points are therefore ($\approx$ -7.21, $\approx$ 7.25) and ($\approx$ 7.21, 7.25).

16. a) 1) The eccentricity of this ellipse is $\frac{m\,\overline{FA}}{m\,\overline{AB}} = \frac{2.6}{3.25} = 0.8$.

2) The coordinates of the vertex of the ellipse, which are $(x, 0)$, are presented. Since the eccentricity is 0.8, you have $\frac{-4 - x}{x + 6.25} = 0.8$.
The value of the x-variable is determined:
$-4 - x = 0.8x + 5$
$-1.8x = 9$
$x = -5$
The coordinates of the vertex over the ellipse are (-5, 0).

b) The value of the ratio is 0.8. It corresponds to the eccentricity of this ellipse.

c) The coordinates of one of the foci are (0, 7) since $a^2 + c^2 = b^2$.
In addition, the coordinates of one of the vertices are (0, 25).
Therefore, the eccentricity of the ellipse $\frac{\text{is distance of the origin of the focus}}{\text{distance of the origin of the vertex V}} = \frac{7}{25} = 0.28$.

17. Using the graphical representation and the equation of the asymptote, it is possible to deduce that the equation of the hyperbola is $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$.
In addition, the equation of the line passing through the greater segment is $y = 0.5x + 4$ since the segment passes through the points (0, 4) and (2, 5).
The equation of the parabola passing through the top of the door is $x^2 = -12(y + 3)$ since the equation of its directrix is $x = 0$ and the coordinates of its vertex is (0, -3).

a) The coordinates of the upper right corner of the front (≈ 2.29, ≈ 5.14) are determined by calculating the intersection point between the hyperbola of the equation $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and the line of the equation $y = 0.5x + 4$.
The maximum height of the building is approximately 6 + 5.14 m, which is approximately 11.14 m.

b) The coordinates of the doors' anchor points are determined by solving the two following systems:
- $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and $y = -6$.
- $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and $x^2 = -12(y + 3)$.

The coordinates of the doors' four anchor points are (-2.6, -6), (2.6, -6), (≈ 1.62, ≈ -3.22) and (≈ -1.62, ≈ -3.22).

18. The equation of the parabola is $(y - 30)^2 = 20(x - 10)$ since the coordinates of its vertex are (10, 30) and the equation of the directrix is $x = 5$.
Find the y-values when $x = 20$.
$(y - 30)^2 = 20(20 - 10)$
$(y - 30)^2 = 200$
$y_1 \approx 15.86$ and $y_2 \approx 44.14$.
The diameter of this car's headlight is approximatey (44.14 – 15.86) cm, which is approximately 28.28 cm.

19. *Several answers possible. Example:*
Let the equation of a circle be $x^2 + y^2 = r^2$. It is possible to transform this equation into the equation of the ellipse:
$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$ in which parameters **a** and **b** are equal.
Since in an ellipse, you have the relation $a^2 = b^2 + c^2$, you therefore obtain $r^2 = r^2 + c^2$; therefore $c = 0$.
Since the value of parameter **c** is 0, the foci are therefore located at the origin of the Cartesian plane.
Joseph is therefore correct.

20. a) The tulips are planted in the region defined by $x^2 + y^2 \leq 36$. It can be deduced using the point (3.6, 4.8):
$3.6^2 + 4.8^2 = 36$.

b) The roses are planted in the region defined by $\frac{x^2}{20.25} - \frac{y^2}{36} \leq -1$. It can be deduced using the coordinates of one of the vertices of the hyperbola (0, 6), the coordinates of one of the foci (0, 7.5) and the relation $a^2 + b^2 = c^2$.

c) The lilies are planted in the region defined by $y^2 \leq 12(x - 6)$. It can be deduced using the coordinates of the vertex (6, 0) and the coordinates of the focus (9, 0).

d) The daisies are planted in the region defined by $y^2 \leq -12(x - 6)$. It can be deduced using the coordinates of the vertex (-6, 0) and the coordinates of the focus (-9, 0).

21. The equation of the ellipse associated with this situation is $\frac{x^2}{36} + \frac{y^2}{100} = 1$ since the coordinates of one of the vertices are (0, 6) and those of the foci are (0, -8).

The equation of the parabola associated with this situation is $x^2 = -8(y - 8)$ since the coordinates of its vertex are (0, 8) and the equation of its directrix is $y = 10$

The equation of the line associated with this situation is $y = 0.25x - 8$ since the line passes through the points whose coordinates are (0, -8) and (2, -7.5).

To determine the coordinates of points A and B, solve the system of equations $\frac{x^2}{36} + \frac{y^2}{100} = 1$ and $x^2 = -8(y - 8)$.

To determine the coordinates of points C and D, solve the system of equations $\frac{x^2}{36} + \frac{y^2}{100} = 1$ and $y = 0.25x - 8$.

The coordinates of point A are ($\approx$ -5.4, $\approx$ 4.35). The coordinates of point B are ($\approx$ 5.4, $\approx$ 4.35).

The coordinates of point C are ($\approx$ 4.33, $\approx$ -6.92). The coordinates of point D are ($\approx$ -2.93, $\approx$ -8.73).

Overview (*cont'd*)

22. a) Since the piece of wood measures 6.5 m by 10 m, it can be deduced that the coordinates of the foci are (-3.25, 0) and (3.25, 0).

Using this information and the equation of one of the asymptotes, the values of parameters **a** and **b** are determined.

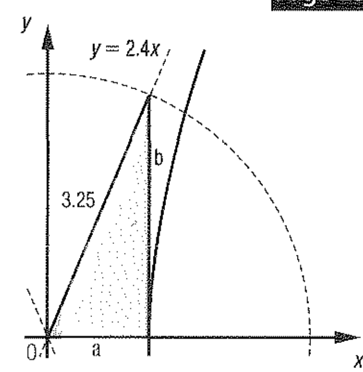
It is known that $\frac{b}{a} = 2.4$, therefore $b = 2.4a$ which allows you to obtain $b^2 = 5.76a^2$.

It is also known that, in the hyperbola, $a^2 + b^2 = c^2$;

therefore $a^2 + 5.76a^2 = 3.25^2$.

It is determined that $a^2 = 1.5625$ and $b^2 = 9$.

The equation of the hyperbola is therefore $\frac{x^2}{1.5625} - \frac{y^2}{9} = 1$.



b) 1) The minimum width of the tabletop corresponds to the distance between the two vertices (-1.25, 0) and (1.25, 0) of the hyperbola, which is 2.5 m.

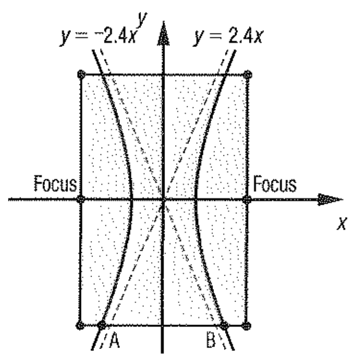
2) In the adjacent graphical represenation, it consists of the distance between points A and B.

To determine these coordinates, solve the system of equations $\frac{x^2}{1.5625} - \frac{y^2}{9} = 1$ and $y = -5$.

The coordinates of point A are ($\approx$ -2.43, -5),

and those of point B are ($\approx$ 2.43, -5).

The maximum width of the tabletop is therefore $2.43 + 2.43 \approx 4.86$ m.



23. a) The relation $b^2 + c^2 = a^2$ allows to deduce the value of parameter **c**: $56^2 + c^2 = 106^2$, $c = \pm 90$.

The coordinates of the foci are (-90, 0) and (90, 0).

Since the equation of the elliptical fence is $\frac{x^2}{106^2} + \frac{y^2}{56^2} = 1$ and you must find the coordinates of a point whose coordinates are (90, *y*), you can substitute *x* by 90 and isolate the *y*-variable:

$$\frac{90^2}{106^2} + \frac{y^2}{56^2} = 1$$
$$\frac{y^2}{3136} = \frac{784}{2809}$$
$$y^2 \approx 875.27$$
$$y \approx \pm 29.58$$

The coordinates of the four corners of the stage are (90, $\approx$ 29.58), (90, $\approx$ -29.58), (-90, $\approx$ 29.58) and (-90, $\approx$ -29.58).

b) Since you must look for the coordinates of a point whose coordinates are $(x, 41)$ you can substitute y by 41 in the equation of the ellipse and isolate the x-variable:

$$\frac{x^2}{106^2} + \frac{41^2}{56^2} = 1$$
$$\frac{x^2}{11\,236} = \frac{1455}{3136}$$
$$x^2 \approx 5213.13$$
$$x \approx \pm 72.2$$

The coordinates of the spectator are therefore $(\approx -72.2, 41)$.
The distance between this point and the origin of the Cartesian plane is $\sqrt{(-72.2)^2 + 41^2} = 83.03$.
The distance separating the spectator from the singer is approximately 83.03 m.

24. a) 1) It can be deduced that the radius of the exterior circle associated with the red crown is $450 - 60 \div 2 = 195$ mm. The equation of this circle is $x^2 + y^2 = 38\,025$.

2) It can be deduced that the radius of the interior circle associated with the red crown is $195 - 41 = 154$ mm. The equation of this circle is $x^2 + y^2 = 23\,716$.

b) You have the equation:
$x^2 + (-x + 30)^2 = 23\,716$
$2x^2 + -60x - 22\,816 = 0$, from where $x_1 \approx 122.86$ and $x_2 \approx -92.86$.
Therefore $y_1 \approx -122.86 + 30$ or ≈ -92.86 and $y_2 \approx 92.86 + 30$ or ≈ 122.86.
The coordinates of the intersection points are $(\approx 122.86, \approx -92.86)$ and $(\approx -92.86, \approx 122.86)$.

Bank of problems

Page 238

1. • Determine the coordinates of the foci of the hyperbola.
You have:

$$\frac{x^2}{a^2} - \frac{y^2}{441} = -1$$
$$\frac{(-51.4)^2}{a^2} - \frac{43.9^2}{441} = -1$$
$$\frac{(-51.4)^2}{a^2} \approx 3.37$$
$$a^2 \approx 783.96$$

The equation of the hyperbola is $\frac{x^2}{783.96} - \frac{y^2}{441} \approx -1$.
For the hyperbola, $a^2 + b^2 = c^2$, the foci are therefore located at points $(0, \approx 35)$, you therefore have:
$35^2 \approx 4c(0 - 20)$
$c \approx -15.3125$
Since the coordinates of the vertex are (0, 20) and the value of parameter **c** is approximately -15.3125, the equation of the directrix is $x \approx 35.3125$.

• Determine the intersection points between the directrix and hyperbola.
Solve the system of equations:
$\frac{x^2}{783.96} - \frac{y^2}{441} \approx -1$
$x \approx 35.3125$,
from where $\frac{35.3125^2}{783.96} - \frac{y^2}{441} \approx -1$
$y \approx \pm 33.8$
The intersection points are $(\approx 35.3125, \approx 33.8)$ and $(\approx 35.3125, \approx -33.8)$.

• Determine the cost of the repairs.
The length of the route to repair is approximately 33.8 + 33.8 km, which is approximately 67.6 km.
The total cost would be approximately 67.6 × \$100,000, which is approximately \$6,760,052.98.

The contractor can proceed with the road repairs since he will make a profit of approximately \$239,947.02.

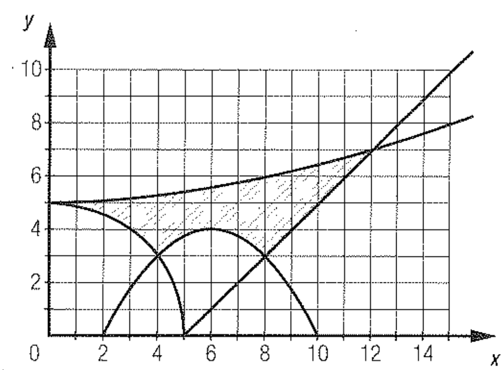
2. *Several answers possible. Example:*

- Determine the equation of the parabola.
  The equation of the parabola is $(x - h)^2 = 4c(y - 3)$ since the ball reaches a maximum height of 3 m.
  It is known that this parabola passes through points (0, 2) and (18, 0).
  By substituting these coordinates in the equation of the parabola, you obtain the two following equations that consist of two unknowns:
  - $(0 - h)^2 = 4c(2 - 3)$, therefore $h^2 = -4c$ and $c = -\frac{h^2}{4}$.
  - $(18 - h)^2 = 4c(0 - 3)$, therefore $324 - 36h + h^2 = -12c$.

  By substitution you obtain:
  $324 - 36h + h^2 = -12\left(-\frac{h^2}{4}\right)$
  $324 - 36h + h^2 = 3h^2$
  $0 = 2h^2 + 36h - 324$
  $h_1 \approx 6.59$ and $h_2 \approx -24.59$ (to refute).
  Therefore, $c \approx -10.85$.
  The equation of the parabola is $(x - 6.59)^2 \approx -43.4(y - 3)$.
- You can determine the height y when $x = 9$.
  $(9 - 6.59)^2 \approx -43.4\ (y - 3)$
  $y \approx 2.87$ cm
  $2.87 - 2.43 = 0.44$
  The serve must therefore pass no more than 44 cm above the net, rather than 40 cm.

The coach did not evaluate the situation properly.

3. The search zone corresponds to the shaded zone in the graph below.



Determine the 4 intersection points.

- The 1st point is located at the intersection of the hyperbola of the equation $\frac{x^2}{144} - \frac{y^2}{25} = -1$ and the circle of the equation $x^2 + y^2 = 25$. The coordinates of the vertex of the hyperbola are (0, 5), and the radius of the circle is 5. The coordinates of the intersection point are therefore (0, 5).
- The 2nd point is located at the intersection of the parabola of the equation $(x - 6)^2 = -4(y - 4)$ and the circle of the equation $x^2 + y^2 = 25$. Solve the system of equations formed this way, and you will obtain the coordinates of the intersection point, which are (4, 3).
- The 3rd point is located at the intersection of the parabola of the equation $(x - 6)^2 = -4(y - 4)$ and the line of the equation $y = x - 5$. Solve the system of equations formed this way, and you will obtain the coordinates of the intersection point, which are (8, 3).
- The 4th point is located at the intersection of the hyperbola of the equation $\frac{x^2}{144} - \frac{y^2}{25} = -1$ and the line of the equation $y = x - 5$. Solve the system of equations formed this way, and you will obtain the coordinates of the intersection point, which is ($\approx$ 12.1, $\approx$ 7.1).

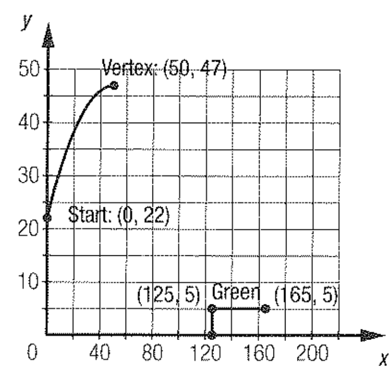
The intersection points of the boundary curves are (0, 5), (4, 3), (8, 3) and ($\approx$ 12.1, $\approx$ 7,1).

4. • Determine the information that concerns the parabola.
The equation of the directrix is $y = -5$, and the coordinates of the vertex are (0, 5) since the distance that separates them is 10 cm.
In addition, the value of parameter **c** is 10 since the distance between the directrix and the vertex is 10 cm.
The coordinates of the focus are therefore (0.5 + 10) = (0, 15).
The equation of the parabola is therefore $x^2 = 40(y - 5)$.

• Determine the information that concerns the ellipse.
The coordinates of the foci are (10, 0) and (-10, 0) since the distance that separates them is 20 cm.
It can be deduced that the curve passes through point (10, 4.5).
Since it is known that the sum of the distances of a point of the curve on both foci is constant, you obtain:
$\sqrt{(-10 - 10)^2 + (0 - 4.5)^2} + \sqrt{(10 - 10)^2 + (0 - 4.5)^2} = 20.5 + 4.5 = 25$
Therefore, $2a = 25$ and $a = 12.5$.
The coordinates of two of the vertices are (12.5, 0) and (-12.5, 0).
Using the relation $b^2 + c^2 = a^2$, you obtain $b^2 = 56.25$. The coordinates of the two other vertices are therefore (0, 7.5) and (0, -7.5).
The equation of the ellipse is $\frac{x^2}{156.25} + \frac{y^2}{56.25} = 1$.

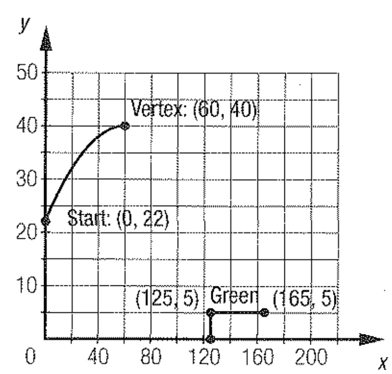
5. *Several answers possible. Example:*

• Analyzing the 8 iron.
The adjacent is a graphical representation of the situation.



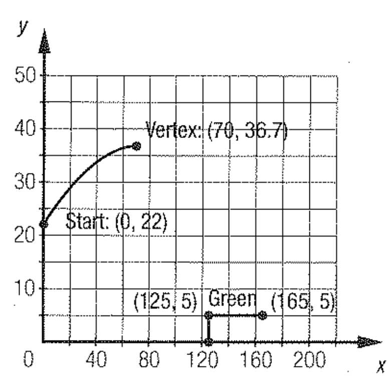
Based on this information, the equation of the trajectory is
$(x - 50)^2 = -100(y - 47)$.
The coordinates of the ball are determined as $y = 5$:
$$(x - 50)^2 = -100(5 - 47)$$
$x^2 - 100x + 2500 = 4200$
$x^2 - 100x - 1700 = 0$
$x_1 \approx 114.81$ m and $x_2 \approx -14.81$ m (to refute).
The ball cannot reach the green with this iron; it will fall in the water.

• Analyzing the 7 iron.
The adjacent is a graphical representation of the situation.



Based on this information, the equation of the trajectory is
$(x - 60)^2 = -200(y - 40)$.
The coordinates of the ball are determined as $y = 5$:
$$(x - 60)^2 = -200(5 - 40)$$
$x^2 - 120x + 3600 = 7000$
$x^2 - 120x - 3400 = 0$
$x_1 \approx 143.67$ m and $x_2 \approx -23.67$ m (to refute).
The ball can reach the green with this iron.

• Analyzing the 6 iron.
The adjacent is a graphical representation of the situation.



Based on this information, the equation of the trajectory is
$(x - 70)^2 = -\frac{1000}{3}(y - 36.7)$.
The coordinates of the ball are determined as $y = 5$:
$$(x - 70)^2 = -\frac{1000}{3}(5 - 36.7)$$
$x^2 - 140x + 4900 \approx 10\,566.67$
$x^2 - 140x - 5666.67 = 0$
$x_1 \approx 172.79$ m and $x_2 \approx -32.79$ m (to refute).
The ball cannot reach the green with this iron; it will pass it.

Bank of problems (*cont'd*)

6. • Determine the zeros of the first jump.

Since the parabola passes through the point whose coordinates are (0, -3) and the coordinates of the vertex are (6, 1), it is possible to deduce that the equation of this jump is $(x - 6)^2 = -9(y - 1)$.

The zeros of this equation are determined:

$(x - 6)^2 = -9(0 - 1)$

$x^2 - 12x + 36 = 9$

$x^2 - 12x + 27 = 0$

$x_1 = 3$ and $x_2 = 9$.

The dolphin propels out of the water at coordinates (3, 0) and enters the water at coordinates (9, 0).

• Determine the zeros of the second jump.

Since the parabola passes through the point whose coordinates are (9, 0) and the coordinates of the vertex are (13.5, -3), it is possible to deduce that the equation of this jump is $(x - 13.5)^2 = 6.75(y + 3)$.

The zeros of this equation are determined:

$(x - 13.5)^2 = 6.75(0 + 3)$

$x^2 - 27x + 182.25 = 20.25$

$x^2 - 27x + 162 = 0$

$x_1 = 9$ and $x_2 = 18$.

The dolphin enters the water at coordinates (9, 0) and propels out of the water at coordinates (18, 0).

• Determine the coordinates of the vertex of the third jump.

Since the parabola passes through the points whose coordinates are (18, 0) and (26, 0), it is possible to deduce that the value of parameter **h** is $(26 + 18) \div 2 = 22$ since a parabola has one axis of symmetry. In addition, the curve passes through another point whose coordinates are (24, 1.875). Considering that the equation of this trajectory is in the form $(x - h)^2 = 4c(y - k)$, you obtain the following system of equations:

$(18 - 22)^2 = 4c(0 - k)$

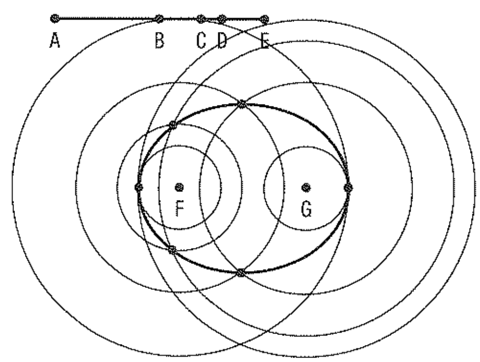
$(24 - 22)^2 = 4c(1.875 - k)$

By solving this system, you obtain $c = -1.6$ and $k = 2.5$.

The coordinates of the point that corresponds to the center of the hoop are (22, 2.5).

7. The adjacent illustration is a drawing made by using the process mentioned below:

It is possible to draw an ellipse whose major axis has a length of 10 cm and whose minor axis has a length of 8 cm. The distance between each focus of the ellipse is 6 cm.



Bank of problems (*cont'd*)

8. • Determine the equations of the circle and ellipse.

The equation of the circle is $x^2 + y^2 = 1000^2$.

The major axis of the ellipse has a length of 4000 m since it is two times longer than the diameter of the circle.

The minor axis has a length of 2000 m since it is 1.5 times longer than the diameter of the circle.

The equation of the ellipse is therefore $\frac{x^2}{1500^2} + \frac{y^2}{2000^2} = 1$.

• Determine the coordinates of the intersection point A.

Solve the system of equations:

$y = \frac{4}{3}x$

$x^2 + y^2 = 1000^2$

The coordinates of point A are (600, 800).

- Determine the coordinates of the intersection points B and C.

  Since the pink trail is perpendicular to the turquoise trail, find the equation of a line whose slope is $-\frac{3}{4}$ and passes through the point (600, 800).

  Obtain $y = -\frac{3}{4}x + 1250$.

  To determine the intersection points, solve the system of equations:

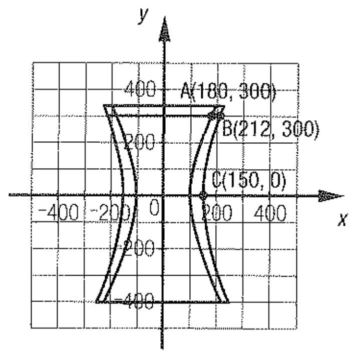
  $\frac{x^2}{1500^2} + \frac{y^2}{2000^2} = 1$

  $y = -\frac{3}{4}x + 1250$

  The coordinates of point B are (≈ (≈ -695.8, ≈ 1772).

  The coordinates of point C are (≈ 1496.96, ≈ 127.28).

9. This situation is represented in the Cartesian plane below where certain measures are deduced from the situation.



- Determine the equation of the external hyperbola.

  It can be deduced that the external hyperbola passes through the vertex whose coordinates are (150, 0) and through point B whose coordinates are (212, 300).

  Using this information, the equation of this hyperbola is determined:

  $\frac{212^2}{22\,500} - \frac{300^2}{b^2} = 1$

  $\frac{300^2}{b^2} \approx 0.9975$

  $b^2 \approx 90\,225$

  The equation of this hyperbola is $\frac{x^2}{22\,500} - \frac{y^2}{90\,225} \approx 1$.

- Determine the diameter of the base of this nozzle.

  Find the width when $y = -400$. By substituting in the equation of the hyperbola, you have:

  $\frac{x^2}{22\,500} - \frac{(-400)^2}{90\,225} \approx 1$

  $x_1 \approx -249.8$ cm and $x_2 \approx 249.8$ cm.

  The width is $249.8 \times 2 \approx 499.6$ cm.

The manufacturer is correct, the diameter of the base is approximately 4.996 m; it is therefore less than 5.2 m.