Support 6.1

Page 1

1. a) Ellipse. b) Circle. c) Circle. d) Ellipse.

2. a) 1) $a = 4$
2) $b = 3$
3) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

b) 1) $a = 1$
2) $b = 2$
3) $x^2 + \frac{y^2}{4} = 1$

c) 1) $a = 3$
2) $b = 5$
3) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

d) 1) $a = 4$
2) $b = 1$
3) $\frac{x^2}{16} + y^2 = 1$

Support 6.1 *(cont'd)*

Page 2

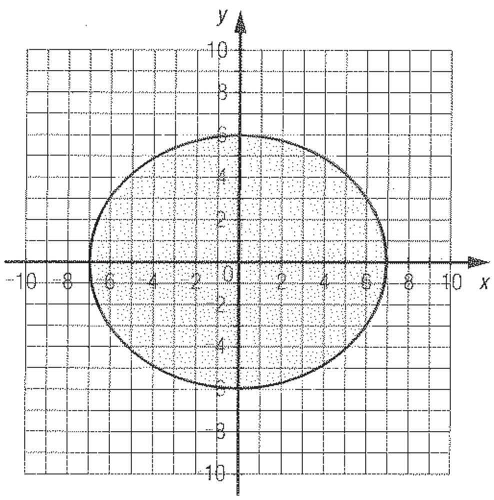
3. a) $V_1(^{-}7, 0)$, $V_2(0, {}^{-}5)$, $V_3(7, 0)$, $V_4(0, 5)$

b) $V_1(^{-}8, 0)$, $V_2(0, {}^{-}6)$, $V_3(8, 0)$, $V_4(0, 6)$

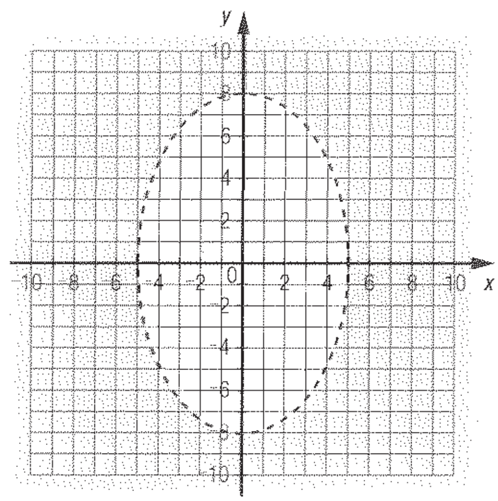
c) $V_1(^{-}1, 0)$, $V_2(0, {}^{-}2)$, $V_3(1, 0)$, $V_4(0, 2)$

d) $V_1(^{-}9, 0)$, $V_2(0, {}^{-}10)$, $V_3(9, 0)$, $V_4(0, 10)$

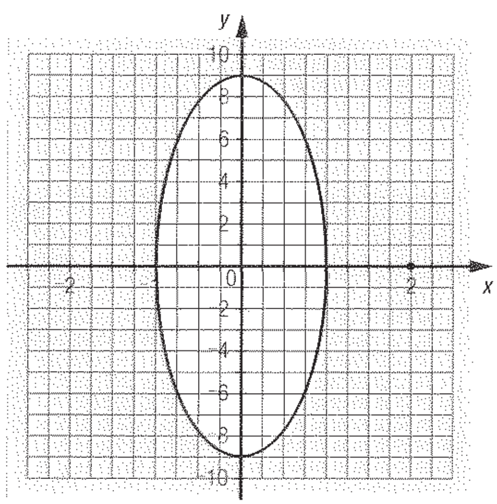
4. a)



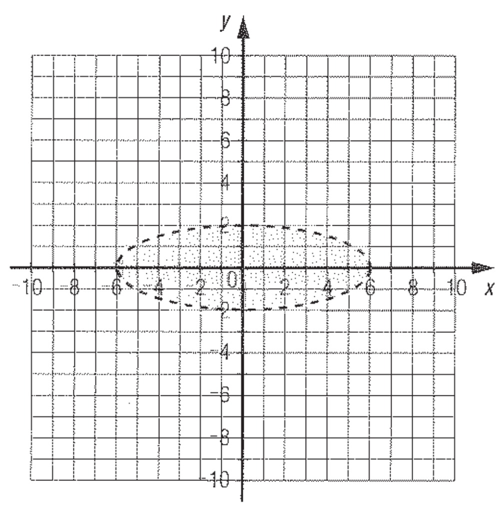
b)



c)



d)



Consolidation 6.1

Page 3

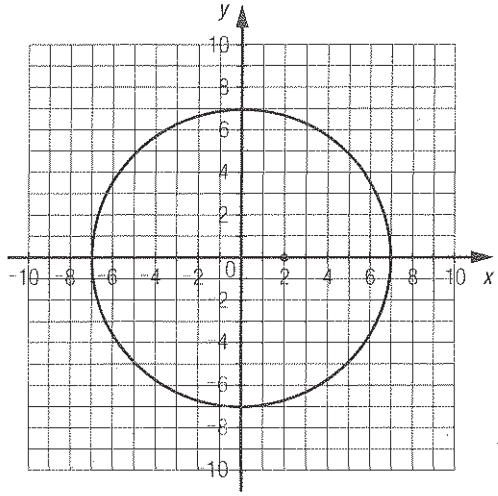
1. a) $x^2 + y^2 = 9$ b) $x^2 + y^2 = 2.25$ c) $\frac{x^2}{10\,000} + \frac{y^2}{22\,500} = 1$

d) $\frac{x^2}{2025} + \frac{y^2}{729} = 1$ e) $\frac{x^2}{1296} + \frac{y^2}{6561} = 1$ f) $\frac{x^2}{4} + \frac{y^2}{2.25} = 1$

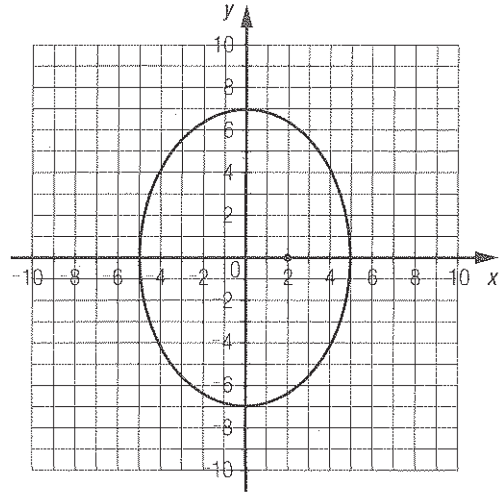
Consolidation 6.1 (*cont'd*)

2. a) $r = 4$ u b) $r = 8$ u c) $r = 4\sqrt{3}$ u

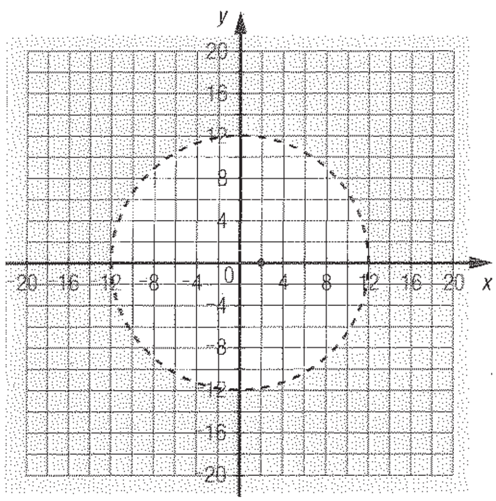
3. a)



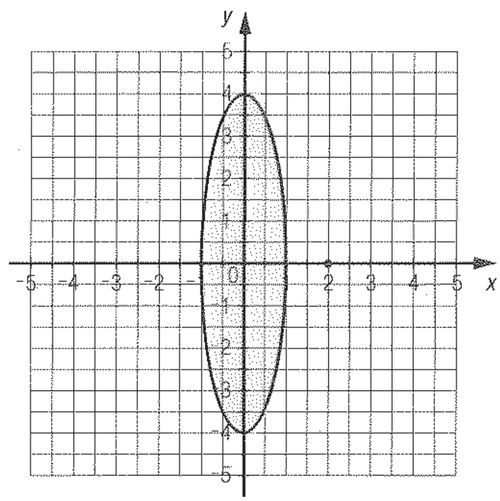
b)



4. a)



b)



Consolidation 6.1 (*cont'd*)

5. a) 1) $V_1(-30, 0)$, $V_2(0, 24)$, $V_3(30, 0)$, $V_4(0, -24)$ 2) $F_1(-18, 0)$, $F_2(18, 0)$
   b) 1) $V_1(-7, 0)$, $V_2(0, -25)$, $V_3(7, 0)$, $V_4(0, 25)$ 2) $F_1(0, -24)$, $F_2(0, 24)$
   c) 1) $V_1(-20, 0)$, $V_2(0, -20.5)$, $V_3(20, 0)$, $V_4(0, 20.5)$ 2) $F_1(0, -4.5)$, $F_2(0, 4.5)$
   d) 1) $V_1(-4, 0)$, $V_2(0, -10)$, $V_3(4, 0)$, $V_4(0, 10)$ 2) $F_1(0, -2\sqrt{21})$, $F_2(0, 2\sqrt{21})$

6. a) $x^2 + y^2 = 65$ b) $\frac{x^2}{169} + \frac{y^2}{81} = 1$ c) $\frac{x^2}{296} + \frac{y^2}{196} = 1$
   d) $x^2 + y^2 = 208$ e) $\frac{x^2}{1156} + \frac{y^2}{289} = 1$

Consolidation 6.1 (*cont'd*)

7. Coordinates of each focus
   In the ellipse, the relation $b^2 + c^2 = a^2$ exists; therefore,
   $46\,656 + c^2 = 129\,600$.
   $c = \pm 288.16$
   Therefore, the coordinates of the foci are $(-288, 0)$ and $(288, 0)$.

Coordinates of speedometers

For each speedometer, it is known that $x = \pm 288$; therefore,

$\frac{288^2}{129\,600} + \frac{y^2}{46\,656} = 1$

$y = \pm 129.6$

The coordinates of Speedometers **A**, **B**, **C** and **D** are respectively (288, 129.6), (288, ≈ ⁻129.6), (⁻288, ⁻129.6) and (⁻288, 129.6).

Total distance between the wires:

$2 \times (129.6 \times 2 + 288 \times 2) \approx 1670.4$ m

The total length of the wire required is 1670.4 m.

8. **a)** The equation of the circle is $x^2 + y^2 = 312.5$.

The maximum width of the elliptical enclosure is 50 m; therefore, the value of parameter **a** is $50 \div 2 = 25$. In the ellipse, the relation $b^2 + c^2 = a^2$ exists, and since the circle passes through one of the vertices and focus of the ellipse, $b = c = r$, where r is the radius of the circle; therefore:

$r^2 + r^2 = 252$

$r^2 = 312.5$

$r \approx 17.68$ m

The equation of the fence that defines the circle is $x^2 + y^2 = 312.5$.

b) Since the circle passes through two vertices of the ellipse and its radius is approximately 17.68 m, then the coordinates of two of the four vertices of the ellipse are (0, ≈ 17.68) and (25, 0). The equation of the fence that defines the ellipse is therefore $\frac{x^2}{625} + \frac{y^2}{312.5} = 1$.

c) The area of the enclosure is approximately 981.75 m².

Since the radius of the circle measures approximately 17.68 m, you can calculate the area.

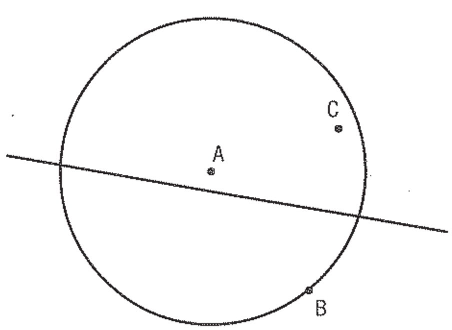
$A = \pi r^2$

$A \approx \pi \times 17.68^2 \approx 981.75$ m²

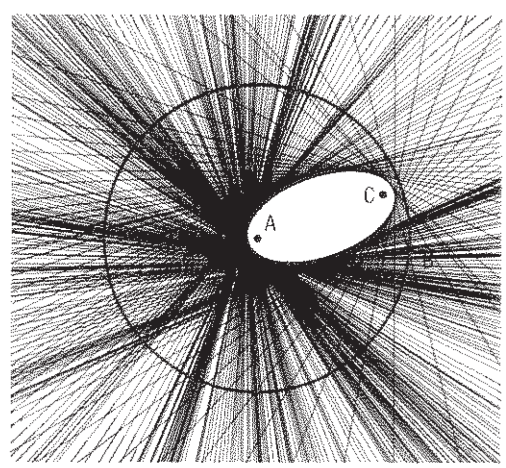
Enrichment 6.1

Page 7

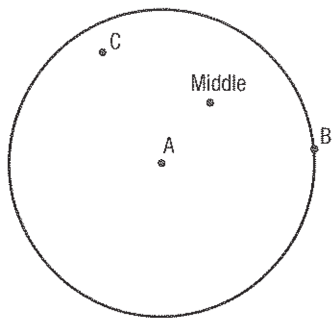
1. **a)** Tracing the perpendicular bisector generates an ellipse. The following is the image of the initial construction:



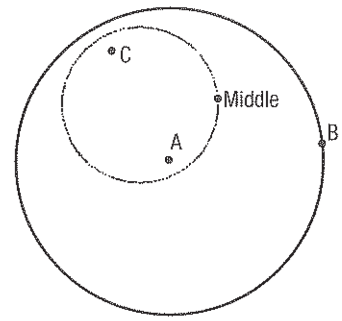
By moving point B, you obtain the following image.



b) Tracing the midpoint results in a circle. The following is the image of the initial construction.



By moving point B, you obtain the following image.



2. a) A = 64, C = 289, F = $-18\ 496$ whereas the values of parameters **B**, **D** and **E** are zero.
   b) A = 1, C = 1, F = -100 whereas the values of parameters **B**, **D** and **E** are zero.
   c) A = 100, C = 36, F = -225 whereas the values of parameters **B**, **D** and **E** are zero.

Support 6.2

Page 8

1. a) F(5, 4) b) $F_1(-6, 0)$ and $F_2(6, 0)$. c) F(-3, -3)
   d) F(-21, -1) e) $F_1(0, -65)$ and $F_2(0, 65)$. f) F(-2, 7)

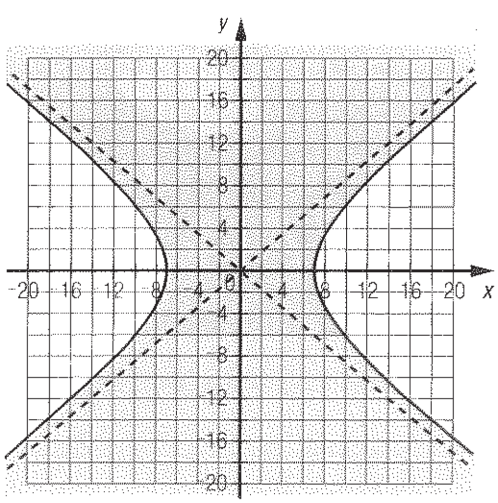
2. a) 1) $y = -6$
      2) F(3, -2)
      3) $(x - 3)^2 = 8(y + 4)$
   b) 1) $x = -1.5$
      2) F(5.5, 5)
      3) $(y - 5)^2 = 14(x - 2)$
   c) 1) $x = 9$
      2) F(1, 1)
      3) $(y - 1)^2 = -16(x - 5)$
   d) 1) $y = 7$
      2) F(-1, -3)
      3) $(x + 1)^2 = -20(y - 2)$

Support 6.2 (*cont'd*)

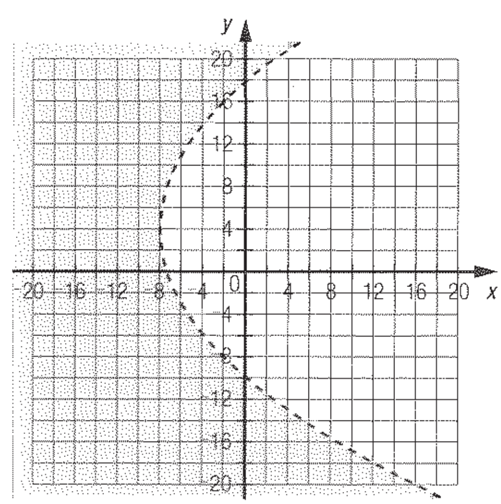
Page 9

3. a) 1) $F_1(-5, 0)$ and $F_2(5, 0)$.
      2) $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$.
      3) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
   b) 1) $F_1(0, -13)$ and $F_2(0, 13)$.
      2) $y = \frac{5}{12}x$ and $y = -\frac{5}{12}x$.
      3) $\frac{x^2}{144} - \frac{y^2}{25} = -1$

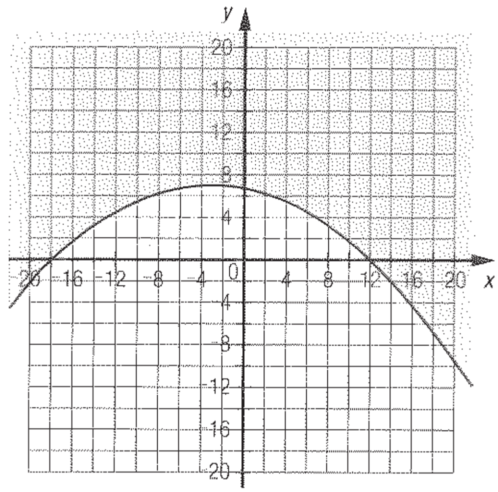
4. a)



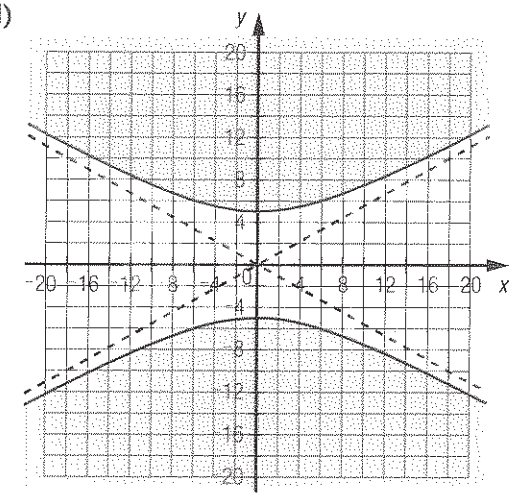
b)



c)



d)



Consolidation 6.2

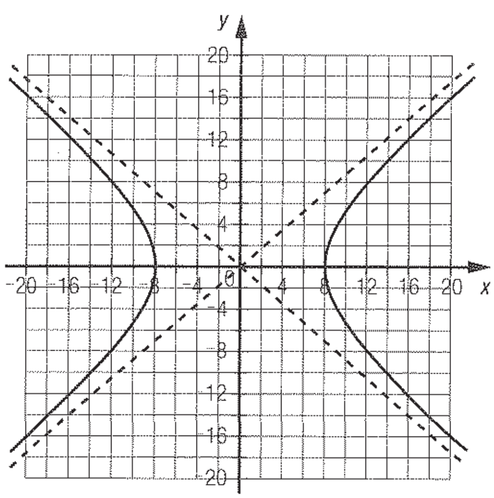
Page 10

1. a) $(y - 6)^2 = -24(x - 4)$
   b) $\frac{x^2}{9} - \frac{y^2}{25} = 1$
   c) $(x - 7)^2 = 16(y + 16)$
   d) $\frac{x^2}{25} - \frac{y^2}{144} = -1$
   e) $(y + 6)^2 = 28(x + 11)$
   f) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

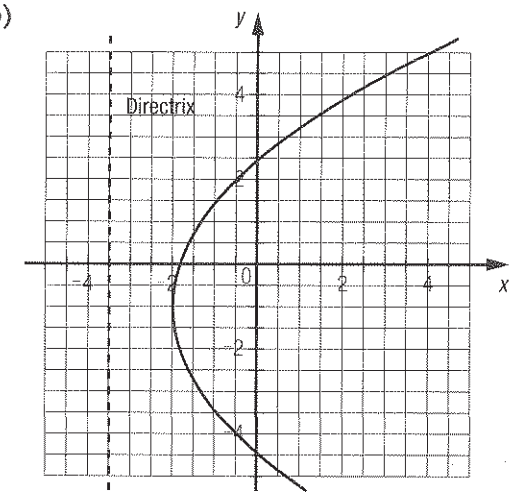
Consolidation 6.2 *(cont'd)*

Page 11

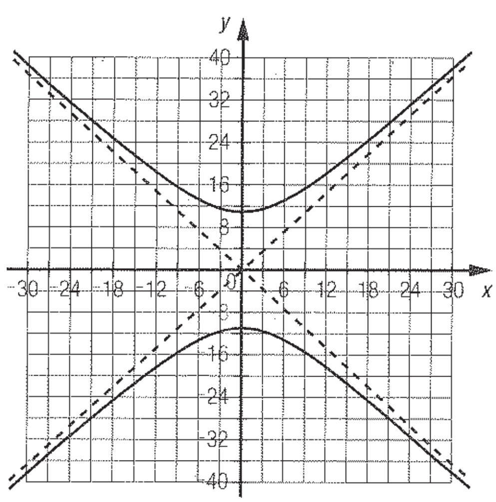
2. a)



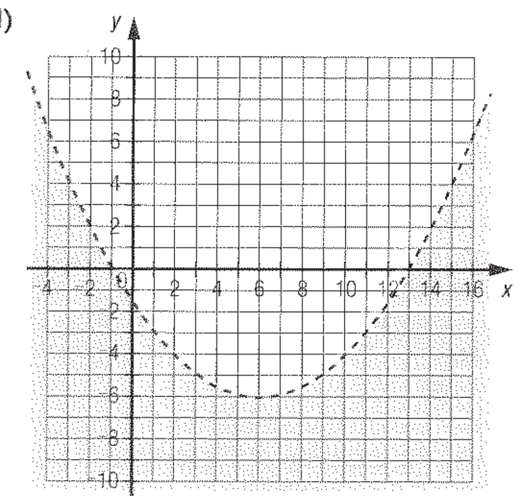
b)



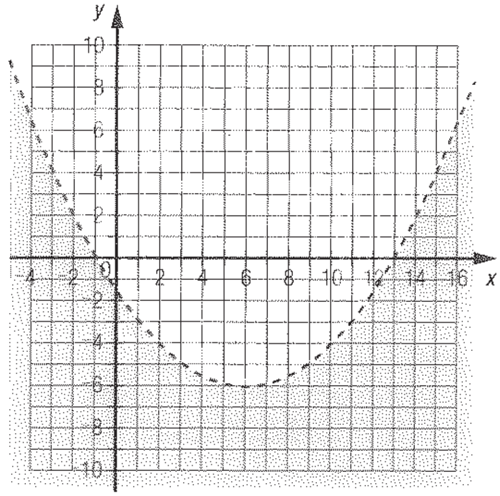
c)



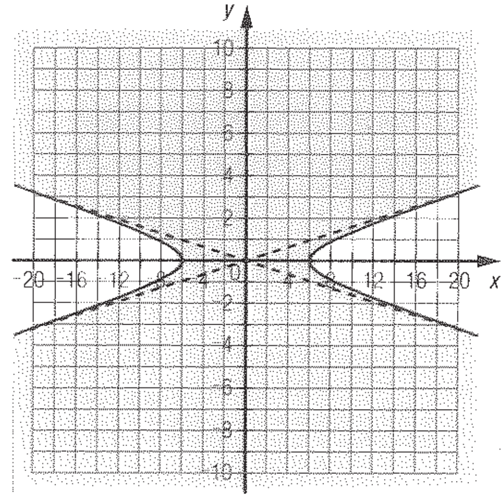
d)



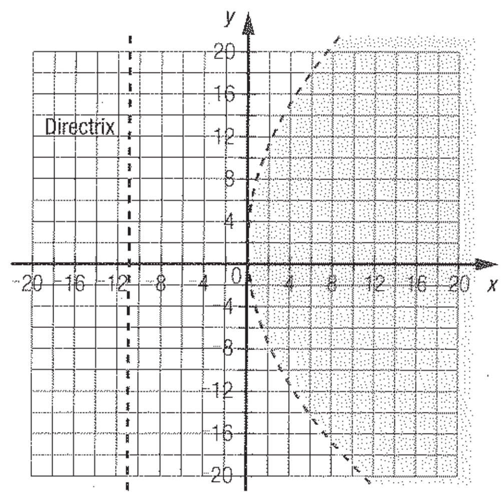
3. a)



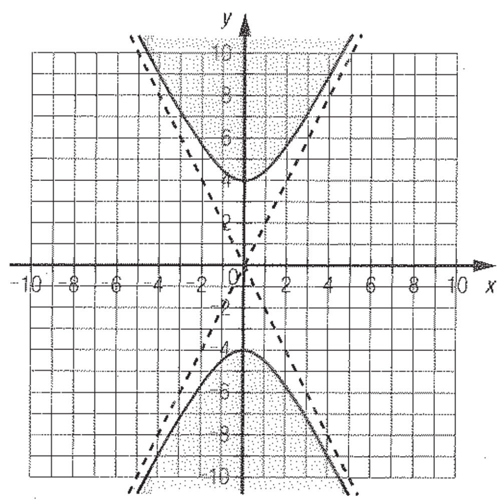
b)



c)



d)



Consolidation 6.2 (*cont'd*)

Page 12

4. a) 1) $V_1(0, -90)$ and $V_2(0, 90)$.
   2) $F_1(0, -106)$ and $F_2(0, 106)$.

b) 1) $V(15, -13)$
   2) $F(15, -25)$

c) 1) $V(8, -3)$
   2) $F(9, -3)$

d) 1) $V_1(-1, 0)$ and $V_2(1, 0)$.
   2) $F_1(-1.25, 0)$ and $F_2(1.25, 0)$.

5. a) $\frac{x^2}{1296} - \frac{y^2}{729} = -1$

b) $(x + 6)^2 = 16(y - 3)$

c) $\frac{x^2}{400} - \frac{y^2}{1600} = 1$

d) $(y - 8)^2 = 12(x + 5)$

e) $\frac{x^2}{25} - \frac{y^2}{144} = 1$

Consolidation 6.2 (*cont'd*)

Page 13

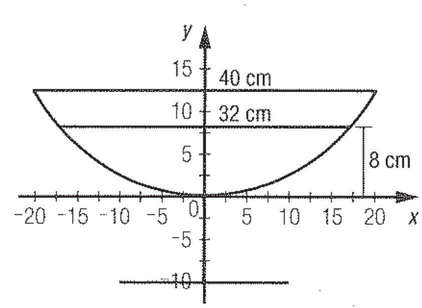
6. a) The height of the stem of the glass is 8 cm.
It is possible to represent this situation in a Cartesian plane as shown in the adjacent illustration.
Therefore, you can determine the equation of this parabola centred at the origin since it passes through the point (16, 8):

$x^2 = 4cy$

$16^2 = 4c(8)$

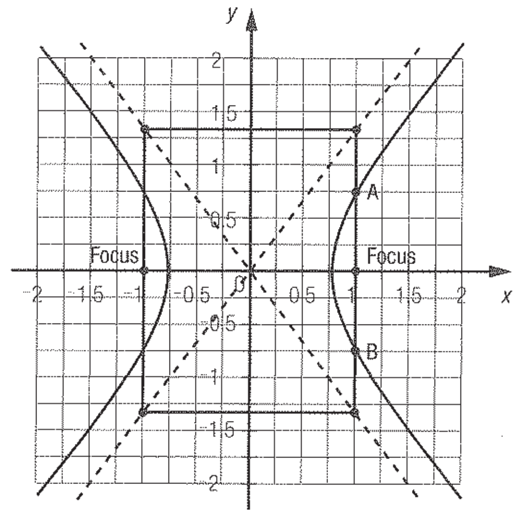
$c = 8$

Therefore, the equation of the directrix is $y = 8$.



b) The total height of the glass is 20.5 cm.
Find the point where $x = 20$ in the equation of the parabola $x^2 = 32y$.
$20^2 = 32y$
$y = \pm 12.5$
Add the height of the stem of the glass.
$12.5 + 8 = 20.5$ cm

7. a) The distance between the vertices is 1.2 m.
It is possible to represent this situation in a Cartesian plane such as in the adjacent illustration.
It is possible to determine the equations of the asymptotes since they pass through the origin and points $\left(1, \frac{4}{3}\right)$ and $\left(-1, \frac{4}{3}\right)$.
$y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.
Since the focus of the hyperbola is (1, 0) and that there is a relationship between the equations of the asymptotes and this focus, you obtain the following proportions:
$\frac{3}{5} = \frac{a}{1}$ and $\frac{4}{5} = \frac{b}{1}$.
Therefore, a = 0.6 and b = 0.8.
The equation of this hyperbola is $\frac{x^2}{0.6^2} - \frac{y^2}{0.8^2} = 1$.
Therefore, the width between the vertices of the hyperbola is $0.6 + 0.6 = 1.2$ m.

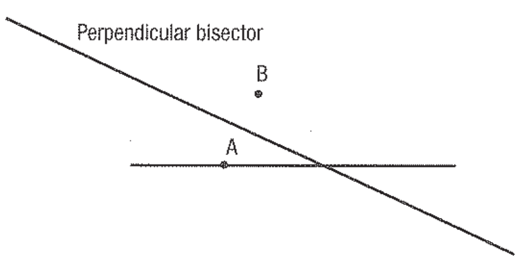


b) The distance that separates point A from point B is approximately 2.13 m.
Find the value of y when $x = 1$:
$\frac{1^2}{0.6^2} - \frac{y^2}{0.8^2} = 1$
$\frac{y^2}{0.64} \approx 1.78$
$y^2 \approx 1.14$
$y \approx \pm 1.067$

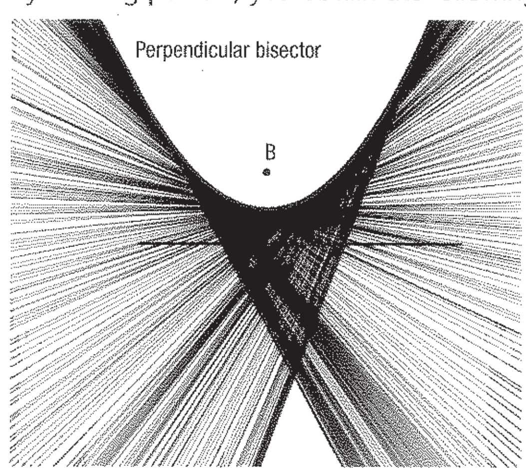
Enrichment 6.2

Page 14

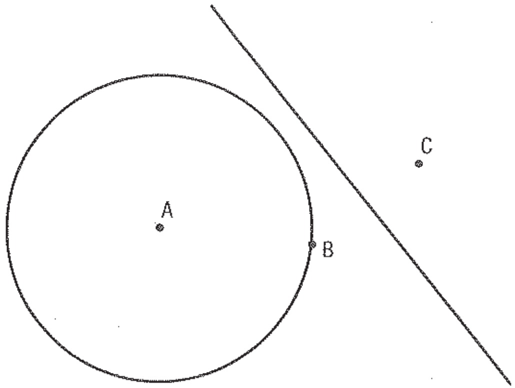
1. a) Tracing the perpendicular bisector generates a parabola. The following is the image of the initial construction:



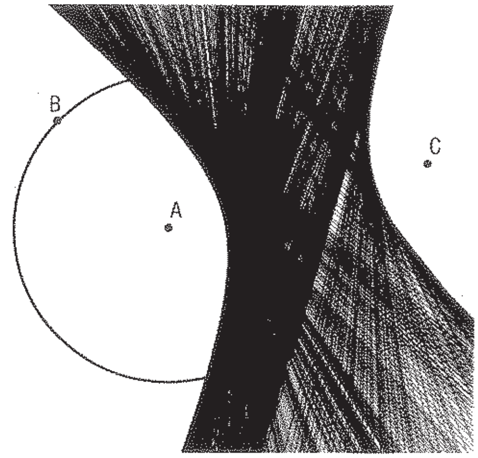
By moving point A, you obtain the following image.



b) Tracing the perpendicular bisector generates a hyperbola. The following is the image of the initial construction:



By moving point B, you obtain the following image.



2. a) A = 225, C = ⁻169, F = ⁻38 025 whereas the values of parameters **B**, **D** and **E** are zero.
   b) A = 1, D = ⁻14, E = ⁻8, F = 33 whereas the values of the parameters **B** and **C** are zero.
   c) A = 144, C = ⁻169, F = 6084 whereas the values of parameters **B**, **D** and **E** are zero.

Support 6.3

Page 15

1. a) $P_1(^-7, 2)$ and $P_2(1, 2)$.
   b) $P_1(^-4, \approx 7.45)$ and $P_2(^-4, \approx {}^-7.45)$.
   c) $P_1(^-10, 12)$ and $P_2(10, 12)$.
   d) $P_1(^-24, ^-10)$ and $P_2(10, 24)$.
   e) $P_1(23.8, 4.6)$ and $P_2(^-19.08, ^-9.69)$.
   f) $P_1(^-4, 8)$ and $P_2(5, ^-10)$.

Support 6.3 *(cont'd)*

Page 16

2. a) For $x = 2$, $y = 8$ and for $x = {}^-0.6$, $y = {}^-5$.
   b) For $x = 4$, $y = 3$ and for $x = 4$, $y = {}^-3$.
   c) For $x = {}^-1$, $y = {}^-9$ and for $x = 3$, $y = {}^-1$.
   d) For $x = 6$, $y = 2$ and for $x = {}^-6$, $y = 2$.
3. a) For $x = {}^-9$, $y = 12$ and for $x = 9$, $y = 12$.
   b) For $x = {}^-2$, $y \approx 6.32$, for $x = {}^-2$, $y \approx -6.32$, for $x = -\frac{2}{9}$, $y \approx 2.11$ and for $x = -\frac{2}{9}$, $y \approx -2.11$.
   c) For $x = 0$, $y = 4$.
   d) For $x = {}^-4$, $y \approx 4.9$, for $x = {}^-4$, $y \approx -4.9$, for $x = 8$, $y \approx 10.95$ and for $x = 8$, $x = {}^-10.95$.

Consolidation 6.3

Page 17

1. a) 1) $(y - 4)^2 = 4(x + 5)$ and $y = {}^-2x - 2$.
      2) $P_1(^-4, 6)$ and $P_2(^-1, 0)$.
   b) 1) $\frac{x^2}{36} - \frac{y^2}{64} = 1$ and $y = 9$.
      2) $P_1(\approx {}^-9.03, 9)$ and $P_2(\approx 9.03, 9)$.
   c) 1) $\frac{x^2}{196} + \frac{y^2}{100} = 1$ and $x = 9$.
      2) $P_1(9, \approx {}^-7.66)$ and $P_2(9, \approx 7.66)$.
   d) 1) $x^2 + y^2 = 35^2$ and $y = {}^-1.5x + 5$.
      2) $P_1(\approx {}^-17.05, \approx 30.57)$ and $P_2(\approx 21.66, \approx {}^-27.49)$.

Consolidation 6.3 (*cont'd*)

2. a) 1) $(y - 10)^2 = 20(x + 40)$
   $(y - 10)^2 = 36(x + 20)$
   2) $P_1(5, -20)$ and $P_2(5, 40)$.

   b) 1) $\frac{x^2}{100} - \frac{y^2}{81} = -1$
   $y^2 = 12(x + 4)$
   2) $P_1(\approx 3.65, \approx -9.58)$, $P_2(\approx 3.65, \approx 9.58)$. $P_3(\approx 11.17, \approx -13.49)$ and $P_4(\approx 11.17, \approx 13.49)$.

   c) 1) $\frac{x^2}{144} + \frac{y^2}{256} = 1$
   $x^2 = -12(y - 16)$
   2) $P_1(0, 16)$, $P_2(\approx -11.31, \approx 5.33)$ and $P_3(\approx 11.31, \approx 5.33)$.

   d) 1) $y^2 = 30(x + 10)$
   $x^2 + y^2 = 500$
   2) $P_1(\approx 5.62, \approx 21.64)$ and $P_2(\approx 5.62, \approx -21.64)$.

Consolidation 6.3 (*cont'd*)

3. a) $P_1(\approx -0.97, \approx -4.93)$ and $P_2(\approx 3.52, \approx 4.05)$.
   b) No intersection point.
   c) $P_1(-1, 0)$ and $P_2(1, 0)$.
   d) $P_1(\approx -1.31, \approx 0.93)$ and $P_2(\approx -1.31, \approx -0.93)$.
   e) $P_1(0, -2)$ and $P_2(2.4, 5.2)$.
   f) $P_1(0, -26)$, $P_2(-24, -10)$ and $P_3(24, 10)$.

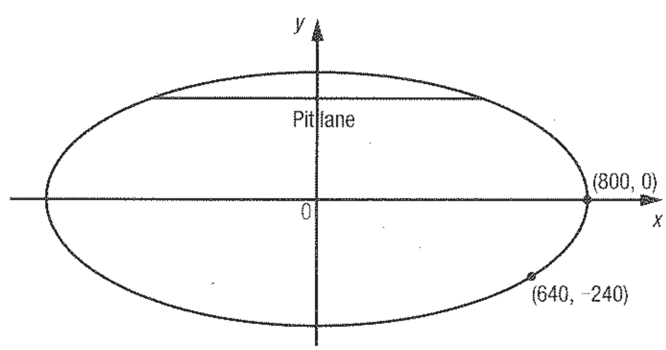
4. a) The equation of the ellipse is $\frac{x^2}{7.5^2} + \frac{y^2}{4.5^2} = 1$.
   The ellipse passes through points (7.5, 0) and (6, 2.7)
   therefore: $\frac{6^2}{7.5^2} + \frac{2.7^2}{b^2} = 1$
   $b^2 = 4.5^2$

   b) The equation of the parabola is $x^2 = -8(y - 4.5)$.
   The coordinates of the vertex are (0, 4.5) and pass through the foci of the ellipse (6, 0); therefore:
   $6^2 = 4c(0 - 4.5)$
   $c = -2$

   c) The distance is 2 u since $c = -2$.

   d) The intersection points are $P_1(0, 4.5)$, $P_2(\approx -7, \approx -1.62)$ and $P_3(\approx 7, \approx -1.62)$.
   You must find the intersection points of the parabola and ellipse.
   $\frac{-8(y - 4.5)}{56.25} + \frac{y^2}{20.25} = 1$
   $56.25y^2 - 162y - 410.0625 = 0$
   $y_1 = 4.5$
   $y_2 \approx -1.62$
   Therefore, $x \approx \pm 7$.

5. It is possible to represent this situation using the adjacent graph.



   a) The length of the major axis is 1600 m, and the length of the minor axis is 800 m.
   Since the ellipse passes through points (800, 0) and (640, -240), determine the value of parameter **b**.
   $\frac{640^2}{800^2} + \frac{(-240)^2}{b^2} = 1$
   $b^2 = 160\ 000$
   $b = 400$

b) The length of the pit lane is 960 m.
The pit lane passes through the line of the equation $y = 320$; therefore:

$$\frac{x^2}{800^2} + \frac{320^2}{400} = 1$$

$$x = \pm 480$$

Consolidation 6.3 (*cont'd*)

Page 20

6. a) The equation that defines the security perimeter is $x^2 + y^2 = 1.69$.
The curve passes through the point (0.5, -1.2); therefore:
$0.5^2 + (-1.2)^2 = r^2$
$r^2 = 1.69$

b) The equation associated with the road is $x^2 = 0.25(y - 1)$.
The curve associated with the road passes through points (0, 1) and (0.5, 2); therefore:
$x^2 = 4c(y - 1)$
$0.5^2 = 4c(2 - 1)$
$c = 0.0625$

c) This hotel must be evacuated since it is located at a distance of 1.0625 km from the centre of the evacuated zone, which has a radius of 1.3 km.

d) The coordinates of where the fence is installed are (≈ 0.26, ≈ 1.27) and (≈ -0.26 ≈ 1.27).
$0.25(y - 1) + y^2 = 1.69$
$y^2 + 0.25y - 1.94 = 0$
Therefore, $y_1 \approx 1.27$ and $y_2 \approx -1.52$ (reject).
Therefore, you have $x^2 \approx 0.25(1.27 - 1)$.
Therefore, $x_1 \approx 0.26$ and $x_2 \approx -0.26$.

7. a) The distance between points A and B is approximately 59.78 cm.
Solve the equation $\frac{200(y - 30)}{225} - \frac{y^2}{400} = 1$.
$-225y^2 + 80\,000y - 2\,400\,000 = 90\,000$
$-225y^2 + 80\,000y - 2\,490\,000 = 0$
Therefore, $y_1 \approx 34.47$ and $y_2 \approx 321.09$ (reject).
You have $x^2 \approx 200(34.47 - 30)$. Therefore, $x_1 \approx 29.89$ and $x_2 \approx -29.89$.

b) The distance between points B and C is approximately 73.14 cm.
Solve the equation $\frac{x^2}{225} - \frac{(0.5x - 55)^2}{400} = 1$.
$400x^2 - 56.25x^2 + 6187.5x - 680\,625 = 90\,000$
$343.75x^2 + 12\,375x - 770\,625 = 0$
Therefore, $x_1 \approx 32.65$ and $x_2 \approx -68.65$ (to refute).
You have $y \approx 0.5 \times 32.65 - 55$; therefore, $y \approx -38.67$.
The distance between points B and C is $38.67 + 34.47 \approx 73.14$ cm.

c) The total height of the shield is approximately 89.47 cm.

Enrichment 6.3

Page 21

1. a) 1) Equation of the ellipse: $\frac{x^2}{64} + \frac{y^2}{16} = 1$
Equation of the hyperbola: $\frac{x^2}{36} - \frac{y^2}{16} = 1$

2) The coordinates of the intersection points are (≈ -6.79, ≈ 2.12), (≈ 6.79, ≈ 2.12), (≈ 6.79, ≈ -2.12) and (≈ -6.79, ≈ 2.12).

b) 1) Equation of the ellipse: $\frac{x^2}{400} + \frac{y^2}{100} = 1$
Equation of the circle: $x^2 + y^2 = 225$

2) The coordinates of the intersection points are (≈ -12.91, ≈ 7.64), (≈ 12.91, ≈ 7.64), (≈ 12.91, ≈ -7.64) and (≈ -12.91, ≈ -7.64).

c) 1) Equation of the first hyperbola:
$\frac{x^2}{36} + \frac{y^2}{16} = 1$
Equation of the second hyperbola:
$\frac{x^2}{36} - \frac{y^2}{9} = -1$

2) The coordinates of the intersection points are
($\approx$ -11.34, $\approx$ 6.41), ($\approx$ 11.34, $\approx$ 6.41),
($\approx$ 11.34, $\approx$ -6.41) and ($\approx$ -11.34, $\approx$ -6.41).

d) 1) Equation of the hyperbola: $\frac{x^2}{20} - \frac{y^2}{4} = 1$
Equation of the circle: $x^2 + y^2 = 64$

2) The coordinates of the intersection points are
($\approx$ -7.07, $\approx$ 3.74), ($\approx$ 7.07, $\approx$ 3.74),
($\approx$ 7.07, $\approx$ -3.74) and ($\approx$ -7.07, $\approx$ -3.74).

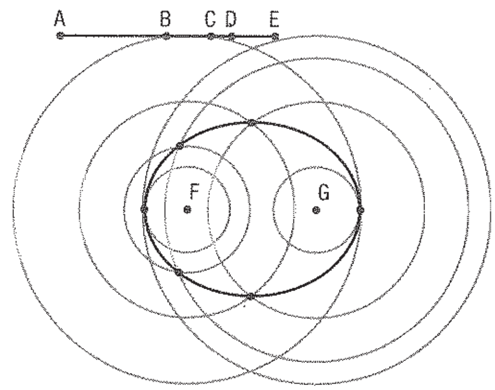
2. The intersection points of these two curves are $(2, \sqrt{6})$ and $(2, -\sqrt{6})$.

Bank of problems

Page 22

7. The adjacent illustration is a drawing made by using the process mentioned below:

It is possible to draw an ellipse whose major axis has a length of 10 cm and whose minor axis has a length of 8 cm. The distance between each focus of the ellipse is 6 cm.



Snapshot 6

Page 23

1. • Determine an equation that represents the trajectory of the Earth.

The minimum distance and maximum distance of the Earth to the Sun allow you to deduce that the major axis of the ellipse measures 299 million kilometres. Therefore, the value of parameter **a** is 149.5.

$2a = 147 + 152 = 299$

$a = \frac{299}{2} = 149.5$

And the value of the parameter **c** is 2.5.

$c = \frac{299 - 2 \times 147}{2} = 2.5$

Therefore, the value of parameter **b** is approximately 149.479.

$b^2 + c^2 = a^2$

$b^2 + 2.5^2 = 149.5^2$

$b \approx 149.479$

The equation that represents the trajectory of the Earth is $\frac{x^2}{149.5^2} + \frac{y^2}{149.479^2} \approx 1$.

• Determine an equation that represents the trajectory of the comet.

The focus of the trajectory of the comet corresponds to one-quarter of the minimum distance between the Sun and the Earth. Therefore, the value of parameter **c** is 36.75 since $147 \div 4 = 36.75$.

The vertex of the trajectory of the comet corresponds to one of the vertices of the ellipse, whose coordinates are (-149.5, 0).

The equation of the trajectory of the comet is $y^2 = 147(x - 149.5)$.

• Determine the intersection points of these two curves.

There are three intersecting points.

$\frac{x^2}{149.5^2} + \frac{y^2}{149.479^2} \approx 1$

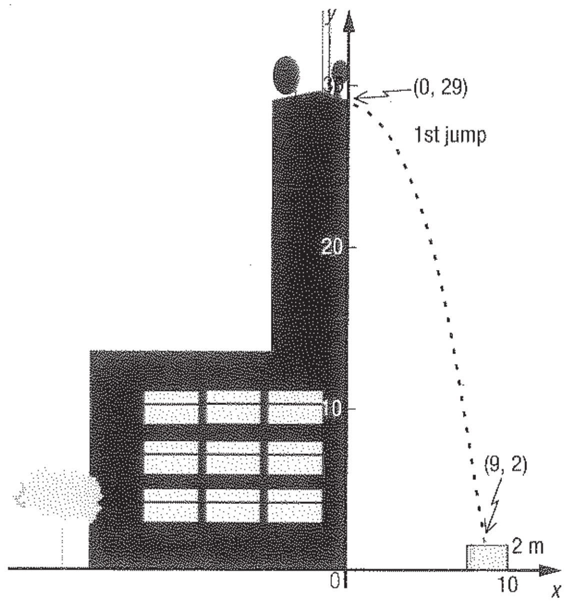
$y^2 = 147(x - 149.5)$

The intersection points are A(-149.5, 0), B($\approx$ 2.46, $\approx$ -149.5) and C($\approx$ 2.46, $\approx$ 149.5).

- Determine the distances that separate the Earth from these intersection points.
  The Earth is located at point (149.5, 0) since it is the end of June.
  Distance from point A: $149.5 + 149.5 = 299$ million kilometres.
  Distance from point B: $\sqrt{(149.5 - 2.46)^2 + 149.5^2} \approx 209.69$ million kilometres.
  Distance from point C: $\sqrt{(149.5 - 2.46)^2 + (-149.5^2)} \approx 209.69$ million kilometres.

Snapshot 6 *(cont'd)* **Page 24**

2. It is possible to represent the first jump in the adjacent Cartesian plane.

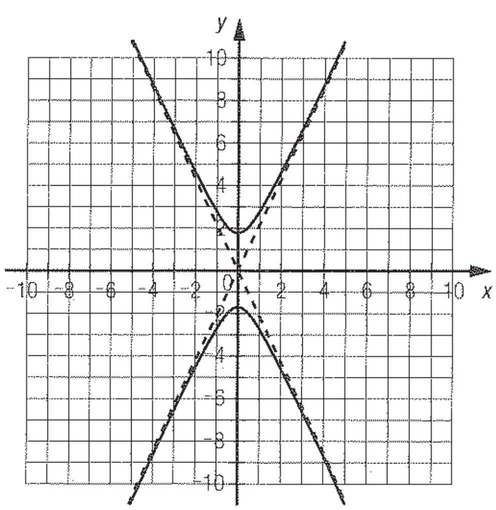


- Determine the equation of the parabola associated with the first jump.
  $(x - h)^2 = 4c(y - k)$
  $(9 - 0)^2 = 4c(2 - 29)$
  $c = -0.75$
  The equation of this trajectory is therefore $x^2 = -3(y - 29)$.
- Determine the equation of the parabola associated with the second jump.
  The stuntman jumps 10 m lower based on the same trajectory. This consists of a vertical translation of 10 units downward. The equation of this trajectory is therefore $x^2 = -3(y - 19)$.
- Determine the position of the centre of the cushion.
  The cushion measures 2 m in height; therefore, you must find the value of x when $y = 2$.
  $x^2 = -3(2 = 19)$
  $x = 7.14$ m
  The stuntman is correct, the centre of the cushion must be located at approximately 7.14 m from the foot of the building.

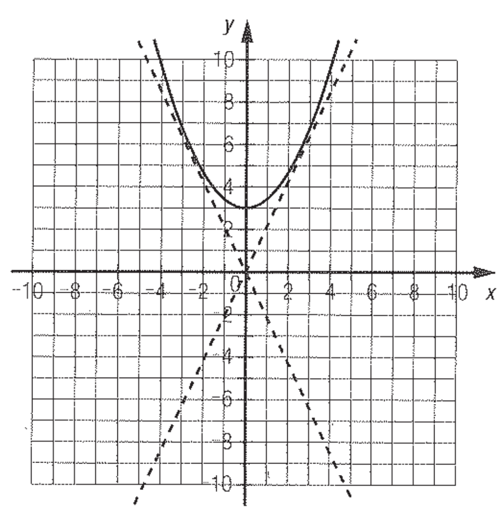
Snapshot 6 *(cont'd)* **Page 25**

3. This student is wrong, because the branches of the hyperbola are asymptotic, in other words the curves become progressively closer to a line without ever touching it, which is not the case for a parabola.

Graph of a hyperbola and its asymptotes



Graph of a parabola and two asymptotes of a hyperbola

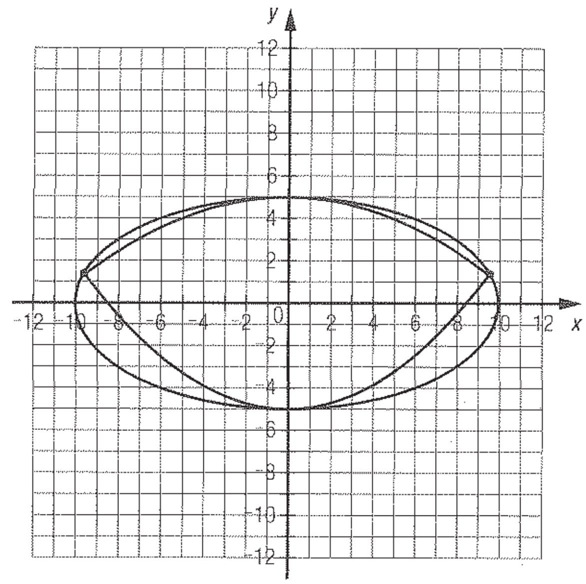


Snapshot 6 (*cont'd*)

4. • Determine the intersection points of both parabolas.
The equations are $x^2 = {}^-25.6(y - 5)$ and $x^2 = 14.4(y + 5)$; you obtain:
$^-25.6(y - 5) = 14.4(y + 5)$
$y = 1.4$
Therefore, $x = \pm 9.6$ since $x^2 = {}^-25.6(1.4 - 5) = 92.16$.
The intersection points of both parabolas are ($^-9.6$, 1.4) and (9.6, 1.4).

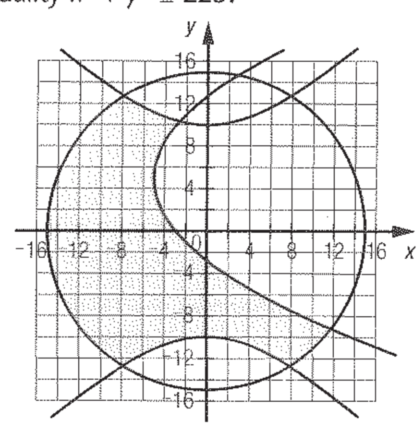
• Determine the equation of the ellipse.
The ellipse, centred at the origin, passes through the intersection points of both parabolas and vertices of each parabola. The coordinates of the vertices of the parabolas are (0, 5) and (0, $^-5$). Since these points are located on the x-axis, they correspond to two vertices of the ellipse, therefore the value of parameter **b** is 5.
By substituting these data in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, you obtain:
$\frac{9.6^2}{a^2} + \frac{1.4^2}{5^2} = 1$
$a^2 = 100$
Therefore, $a = 10$.
The equation of the ellipse is $\frac{x^2}{10^2} + \frac{y^2}{5^2} = 1$.

• Graphically represent this logo.
By using the equations of each conic, draw the adjacent graph.



Snapshot 6 (*cont'd*)

5. • Represent the region where the team would be deployed.
The region does not surpass 30 km in diameter. It consists of a circle whose radius measures 15 km, and this allows you to deduce that the region is bound by the inequality $x^2 + y^2 \leq 225$.
Represent the common region in the following inequalities:
$x^2 + y^2 \leq 225$
$(y - 5)^2 \geq 12(x + 5)$
$\frac{x^2}{100} - \frac{y^2}{100} \geq {}^-1$

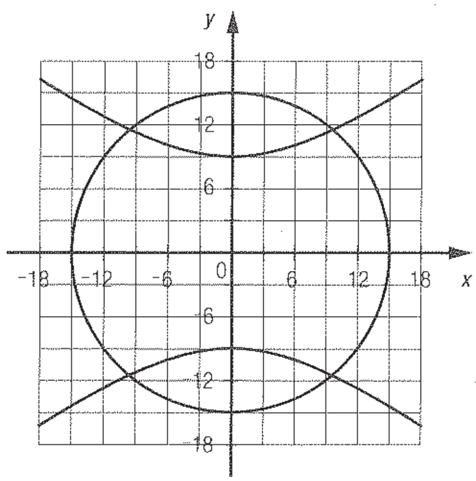


• Determine the intersection points of each conic.
Point A: Intersection of the circle and hyperbola, and the approximate coordinates are ($\approx {}^-8$, ≈ 113).
Point B: Intersection of the parabola and circle, and the approximate coordinates are ($\approx {}^-3$, ≈ 10).
Point C: Intersection of the circle and hyperbola, and the approximate coordinates are (≈ 12, $\approx {}^-9$).
Point D: Intersection of the circle and hyperbola, and the approximate coordinates are (≈ 8, $\approx {}^-113$).
Point E: Intersection of the circle and hyperbola, and the approximate coordinates are ($\approx {}^-8$, ≈ -113).

Snapshot 6 (*cont'd*)

Page 28

6. It is possible to represent this situation in a Cartesian plane and deduce certain information from it.



- Determine the equation of the hyperbola.

  Based on this graphical representation, the equation of the hyperbola is in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

  The vertices of the hyperbola pass through points (0, 9) and (0, −9); therefore, the value of the parameter **b** is 9.

  The value of the parameter **a** is 12 since the equation of an asymptote is $y = -0.75x$. You can present the following proportion.

  $\frac{b}{a} = 0.75$

  $\frac{9}{a} = 0.75$

  $a = 12$

  The equation of the hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = -1$.

- Determine the equation of the circle.

  The circle passes through the foci of the hyperbola of the equation $\frac{x^2}{144} - \frac{y^2}{81} = -1$.
  For the hyperbola, the relation $c^2 = a^2 + b^2$ exists; therefore,

  $c^2 = 144 + 81$

  $c^2 = 225$

  $c = 15$

  The equation of the circle is $x^2 + y^2 = 225$.

- Calculate the area of the ball.

  Since the equation of the circle is $x^2 + y^2 = 225$, the radius of this circle is 15 dm.
  It is possible to calculate the area:

  $A = \pi r^2$

  $A = \pi \times 15^2 \approx 706.86 \text{ dm}^2$

  The total area of this ball is approximately 706.86 dm².