Solving Exponential Equations

Goal:

- to solve exponential equations using logarithms

    - equations involving one exponential term

    - equations involving more than one exponential term

Since logarithmic functions are the inverse of exponential functions we can use logarithms to solve exponential equations.

Ex: Solve:

a) $5^{x}=15$

"Take the log of both sides"

$\ln 5^{x} = \ln 15$

$x \ln 5 = \ln 15$

$x = \frac{\ln 15}{\ln 5} \simeq 1.68$

b) $2(3)^{2x-1} - 1 = 7$

With more complicated equations (ones involving more than one exponential term) we must also apply laws of logarithms.

Ex: Solve:

a) $2^{x}=4^{-x+3}$

$\log 2^{x} = \log 4^{-x+3}$ - log both sides

$x\log 2 = (-x+3)\log 4$ - power rule

$x\log 2 = -x\log 4 + 3\log 4$ - distribute

$x\log 2 + x\log 4 = 3\log 4$

$x(\log 2 + \log 4) = 3\log 4$

$x = \dfrac{3\log 4}{\log 2 + \log 4} = 2$

b) $2(6)^{x}=0.5(3)^{2x}$

$\ln 2(6)^{x} = \ln 0.5(3)^{2x}$

$\ln 2 + \ln 6^{x} = \ln 0.5 + \ln 3^{2x}$

$\ln 2 + x\ln 6 = \ln 0.5 + 2x\ln 3$

$x\ln 6 - 2x\ln 3 = \ln 0.5 - \ln 2$

$x(\ln 6 - 2\ln 3) = \ln 0.5 - \ln 2$

$x = \dfrac{\ln 0.5 - \ln 2}{\ln 6 - 2\ln 3}$ ✓ $= \dfrac{\ln \frac{1}{4}}{\ln \frac{2}{3}} = \log_{\frac{2}{3}} \frac{1}{4}$