

# Solving Logarithmic Equations

Goal:

- to solve logarithmic equations

-to state restrictions of logarithmic equations

Solve:

Domain:  $x - 4 > 0$   
 $x > 4$

a)  $\log_2(x-4)=3$

$$2^3 = x - 4$$

- switch to exponential form

$$8 = x - 4$$

$$x = 12 \checkmark$$

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$$\log_2(x-4)=3$$

$$a^{\log_a m} = m$$

$$2^{\log_2(x-4)} = 2^3$$

$$x - 4 = 8$$

$$x = 12$$

$$\text{b) } \log(x+3) - \log(x-6) = 1$$

$$\log\left(\frac{x+3}{x-6}\right) = 1$$

$$10^1 = \frac{x+3}{x-6}$$

$$10(x-6) = x+3$$

$$10x - 60 = x + 3$$

$$9x = 63$$

$$x = 7 \checkmark$$

$$\text{Domain: } x+3 > 0$$

$$x > -3$$

$$x-6 > 0$$

$$x > 6$$

$$c) \log_2 x = -\log_2(x-2) + 3$$

$$\log_2 x + \log_2(x-2) = 3$$

$$\log_2 x(x-2) = 3$$

$$2^3 = x(x-2)$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$x = 4 \checkmark \quad x = -2 \times$$

Domain:

$$x > 0$$

$$x - 2 > 0$$

$$x > 2$$

$$d) \log_5 x = \log_5 (2x-6)$$

$$\log_5 x - \log_5 (2x-6) = 0$$

$$\log_5 \left( \frac{x}{2x-6} \right) = 0$$

$$5^0 = \frac{x}{2x-6}$$

$$1 = \frac{x}{2x-6}$$

$$2x-6 = x$$

$$x = 6 \checkmark$$

Domain:

$$x > 0$$

$$2x-6 > 0$$

$$x > 3$$

$$c) \log_2 x = -\log_2(x-2) + 3$$

Domain:

$$x > 0$$

$$x - 2 > 0$$

$$x > 2$$

$$\log_2 x + \log_2(x-2) = 3$$

$$\log_2 x(x-2) = 3 \quad \text{OR} \quad \log_2 x(x-2) = \log_2 8$$

$$2^3 = x(x-2)$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$x = 4 \checkmark \quad x = -2 \times$$

p.199

# 5, 7 (logs only)

# 10

$$\log(a+b) \neq \log a + \log b = \log(ab)$$

$$(a+b)\log x = a\log x + b\log x$$