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5.4

#6. a) $\frac{3\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\sin x}$

$\frac{3\sin x}{\cos x} = \frac{5}{\sin x} - \frac{\cos x}{\sin x}$

$\frac{3\sin x}{\cos x} = \frac{5-\cos x}{\sin x}$

$3\sin^2 x = (5-\cos x)\cos x$

$3\sin^2 x = 5\cos x - \cos^2 x$

$3(1-\cos^2 x) = 5\cos x - \cos^2 x$

$3 - 3\cos^2 x = 5\cos x - \cos^2 x$

$0 = 2\cos^2 x + 5\cos x - 3$

$2x^2 + 5x - 3$

$= (2x-1)(x+3)$

$0 = (2\cos x - 1)(\cos x + 3)$

$2\cos x - 1 = 0$ $\qquad$ $\cos x + 3 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -3$ ✗

$x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

6. f) $\cot^2 x + \csc^2 x = 7$

$$\frac{\cos^2 x}{\sin^2 x} + \frac{1}{\sin^2 x} = 7$$

$$\cos^2 x + 1 = 7\sin^2 x$$

$$(1 - \sin^2 x) + 1 = 7\sin^2 x$$

$$2 - \sin^2 x = 7\sin^2 x$$

$$2 = 8\sin^2 x$$

$$\frac{2}{8} = \sin^2 x$$

$$\frac{1}{4} = \sin^2 x$$

$$\pm\sqrt{\frac{1}{4}} = \sin x$$

$$\sin x = \pm\frac{1}{2}$$

OR

$$8\sin^2 x - 2 = 0$$

$$2(4\sin^2 x - 1) = 0$$

$$2(2\sin x + 1)(2\sin x - 1) = 0$$

$2\sin x + 1 = 0$ $\quad$ $2\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = \frac{1}{2}$

$$X = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$$