5.4

#1. $\sec x = \frac{2\sqrt{3}}{3}$ $x \in \left[0, \frac{\pi}{2}\right]$

a) $\sec x = \frac{1}{\cos x}$ $\cos x = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$ $\frac{a}{H}$

Using identity:

$\cos^2 x + \sin^2 x = 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \sin^2 x = 1$

$\sin^2 x = 1 - \frac{3}{4}$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$ $x \in \left[0, \frac{\pi}{2}\right)$ $\sin x = \frac{1}{2}$

Using ratios:

$a = \sqrt{3}$ $c = 2$

$c^2 = a^2 + b^2$

$2^2 = \sqrt{3}^2 + b^2$

$4 = 3 + b^2$

$1 = b^2$

$b = \pm 1$ $\sin x = \frac{O}{H} = \frac{1}{2}$

b) $\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c) $\csc x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$

b) $1+\tan^2 x = \sec^2 x$

$1+\tan^2 x = \left(\frac{2\sqrt{3}}{3}\right)^2$

$1+\tan^2 x = \frac{4 \cdot 3}{9}$

$1+\tan^2 x = \frac{4}{3}$

$\tan^2 x = \frac{4}{3} - 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

4. a) $\sin\frac{\pi}{12} = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\sin(a-b) = \sin a\cos b - \sin b\cos a$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

4. b) $\tan \frac{11\pi}{12} = \tan\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$$= \frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$$

$$= \frac{-\sqrt{3}+1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{-\sqrt{3} + 3 + 1 - \sqrt{3}}{1-3}$$

$$= \frac{4 - 2\sqrt{3}}{-2}$$

$$= \frac{2\sqrt{3} - 4}{2}$$

$$= \sqrt{3} - 2$$