Sum and Difference Identities

Goal:

- to become familiar with the sum and difference identities
- to be able to find exact values of angles not located on the unit circle

What is the exact value of sin(7π/12)? ≈ 0.966

$\frac{7\pi}{12}$ is not a known point on the unit circle

Notice that 7π/12=π/3+π/4.

$\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

Does sin(7π/12)=sin(π/3)+sin(π/4) ?

$$0.966 \stackrel{?}{=} \frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}$$

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$$0.966 \neq 1.57$$

However, mathematicians observed and proved that

Sum of sines:

$$\boxed{\sin(a+b) = \sin a \cos b + \sin b \cos a}$$

So,

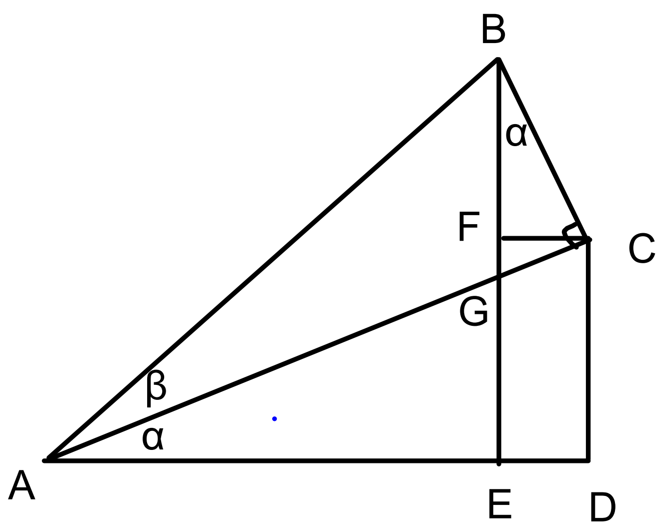
$$\sin(7\pi/12) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4} \approx 0.966$$

Proof:



$$\sin(\alpha+\beta) = \frac{BE}{AB}$$

$$= \frac{CD + BF}{AB}$$

$$= \frac{CD}{AB} + \frac{BF}{AB}$$

$$= \frac{CD}{AB} \cdot \frac{AC}{AC} + \frac{BF}{AB} \cdot \frac{BC}{BC}$$

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$$= \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

Likewise, there is a sum indentity for cosine:

$$\boxed{\cos(a+b) = \cos a \cos b - \sin a \sin b}$$

and for tangent:

$$\boxed{\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}}$$

There are also difference identities for each ratio:

$$\sin(a\overset{+}{-}b) = \sin a \cos b \overset{+}{-} \sin b \cos a$$

$$\cos(a\overset{+}{-}b) = \cos a \cos b \overset{-}{+} \sin a \sin b$$

$$\tan(a\overset{+}{-}b) = \frac{\tan a \overset{+}{-} \tan b}{1 \overset{-}{+} \tan a \tan b}$$

These can be proved by knowing

$$\sin(-b) = -\sin b$$

$$\cos(-b) = \cos b$$

$$\tan(-b) = -\tan b$$

and using the sum formulas with $(a+(-b))$

Find the exact value of $\tan(\pi/12)$

Find the exact value of sin2x and cos2x.

$$\sin 2x = \sin(x+x)$$

$$= \sin x \cos x + \sin x \cos x$$

$$\boxed{\sin 2x = 2\sin x \cos x}$$ double angle

$$\cos 2x = \cos(x+x)$$

$$= \cos x \cos x - \sin x \sin x$$

$$\boxed{\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}}$$ double angle