Word Problems with Exponential Functions

Goal:

- to solve word problems using exponential functions

Many things can be modeled using exponential functions. Often, the horizontal asymptote is y=0 and we can use a simplified form of the equation.

$$f(x) = a(c)^{b(x-h)} + k$$

$$\Downarrow$$

$$f(x) = ac^{bx}$$ with $h=0$ and $k=0$

A bacteria culture doubles every day. Initially, it contained 8 bacteria. How many bacteria will there be after two weeks?

$k = 0$ & $h = 0$ means a: initial value

x: time (days)

$f(x) = ac^{bx}$

$f(x) = 8c^{bx}$

$f(x) = 8(2)^{bx}$ $c = 2$ doubling

$f(x) = 8(2)^{x}$ $b = 1$ doubles once every day

$f(14) = 8(2)^{14}$

$= 131\,072$

A bacteria culture doubles 3 times a day. Initially, it contained 8 bacteria. How many bacteria will there be after two weeks?

$$f(x) = 8(2)^{3x}$$

$$f(14) = 8(2)^{3(14)}$$

$$= 8(2)^{42}$$

$$= 3.52 \times 10^{13}$$

The number of ants in a colony triples every four days. The colony starts with 100 ants. How many ants are there after 16 days? 18 days?

$f(x) = 100(3)^{\frac{1}{4}x}$

$f(16) = 100(3)^{\frac{1}{4}(16)}$

$= 100(3)^4$

$= 8100$

$f(18) = 100(3)^{\frac{1}{4}(18)}$

$= 100(3)^{4.5}$

$= 14029.6$

$= 14029$

A car is purchased for $30 000. It loses 15% of its value every year. How much is the car worth five years after purchase?

$$f(x) = 30\,000(100\% - 15\%)^x$$

$$= 30\,000(85\%)^x$$

$$= 30\,000(0.85)^x$$

how much it is still worth not how much value it loses

$$f(5) = 30\,000(0.85)^5$$

$$= 13\,311.16$$

If the car's scrap metal is worth $400, how does this change the rule describing the value of the car over time?

Exponential "c"

1. Growth or decay by a factor (i.e. doubling, tripling, halving, ...)

c = factor (i.e. $c=2$, $c=3$, $c=\frac{1}{2}$, ...)

2. Growth or decay by percent

C = 100% + percent change for growth

C = 100% − percent change for decay